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LOVELL'S SERIES OF SCHOOL BOOKS.

KEY

SCIENCE & ENGINEERING LIBRARY

TO THE

NATIONAL ARITHMETIC;

CONTAINING

Full Solutions to nearly all the Problems,

DESIGNED FOR THE

E OF TEACHERS AND PRIVATE STUDENTS.

BY JOHN HERBERT SANGSTER, M.A., M.D.,
MATHEMATICAL MASTER AND LECTURER IN CHEMISTRY AND NATURAL
PHILOSOPHY IN THE NORMAL SCHOOL FOR ONTARIO.

THIRD EDITION—CAREFULLY REVISED.

Montreal:
PRINTED AND PUBLISHED BY JOHN LOVELL,
AND FOR SALE AT THE BOOKSTORES.
1870.

QA103
S22

Entered, according to the Act of the Provincial Parliament,
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PREFACE.

It was the original intention of the Author to give, in the Key, merely a series of brief hints upon the Solutions of the more difficult Problems. He was led to modify this plan, and to issue the work in its present form, chiefly from the consideration that as there are in the country many young persons who, from various causes, are unable to avail themselves of the advice and assistance of a teacher, it would be a great boon to these to have access to a book to which they might refer with the certainty of having every doubt removed as to the correctness of their work and methods of solution. He offers the work to his fellow-teachers with the hope that they will accord it the same favorable reception that they have so kindly given to the National Arithmetic.

Toronto, *May*, 1861.

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202
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222
223
231
235
237
238
241
280

KEY TO NATIONAL ARITHMETIC.

Exercise 5—Page 5[illegible].

(1)
d
23328
4
93312 f.

(2)
£
348
20
6960 s.

(3)
£ s.
38 10
20
770 s.
12
9240 d.

(4)
£ s.
58 13
20
1173 s.
12
14076 d.

(5)
£ s.
58 13
20
1173 s.
12
14076 d.
4
56304 f.

(6)
£ s. d.
59 13 6¾
20
1193 s.
12
14322 d.
4
57291 f.

(7)
£ s. d.
63 0 9
20
1260 s.
12
15129 d.

(8)
cwt. qrs. lbs.
16 2 16
4
66 qrs.
25
346
132
1666 lb

(9)
cwt. qrs. lbs.
14 3 16
4
59 qrs.
25
311
118
[illegible] lbs.

(10)
lbs. oz. dwt. grs.
3 5 12 16
12
41 oz.
20
832 dwts.
24
3344
1664
19984 grs.

(11)
lbs. oz. dwt. grs.
7 11 15 14
12
95
20
1915 dwts.
24
7674
3830
45974 grs.

(12)
yrs.
20
365
7300 dys.
24
29200
14600
175200 hrs

(13)
mile.
1
8
8 fur.
40
320 per.
5½
1600
160
1760 yds.
3
5280 ft.

(14)
yrs. d. h. m.
46 21 8 56
365
251
276
138
16811 days.
24
67252
33622
403472 hrs.
60
24208376 min.

(15)
sq. per.
74
30¼
2220
18½
2238½ sq. yds.

(16)
a. r. per.
46 3 12
4
187 r.
40
7492 per.
30¼
224760
1873
226633 sq. yds.

(17)
sq miles.
767
640
30680
4602
490880 sq. a.

(18)
cub. ft.
767
1728
6136
1534
5369
767
1325376 cub. in.

(19)
pks.
767
2
1534 gals.
4
6136 qts.

(20)
pks.
797
2
1594 gals.
4
6376 qts.
2
12752 pts.

Exercise 6—Page 51.

(1)
£.
4)32756
12)8189 d.
20)682s. 5d.
£34 2s. 5d.

(2)
grs.
24)23547
20)981 dwt. 3 grs.
12)49 oz. 1 dwt. 3 grs.
4 lbs. 1 oz. 1 dwt. 3 g.

(3)
yds.
5½)397024
2 2
11)794048
40)72186r. 2hf-yds.=1 yd.
8)1804 fur. 26 r. 1 yd.
225 m. 4 f. 26 r. 1 y.

(4)
sec.
60)28635
60)477 m. 15 sec.
7 hrs. 57 m. 15 sec.

(5)
lbs.
25)1666
4)66 qrs. 16 lbs.
16 cwt. 2 q. 16 lbs.

(6)
lbs.
25)1491
4)59 qrs. 16 lbs.
14 cwt. 3 q. 16 lbs.

(7)
grs.
24)115200
20)4800 dwt.
12)240 oz.
20 lbs.

(8)
oz.
16)107520
6720 lbs.

(9)
cub. in.
1728)1674674
969 ft. 242 in.

(10)
Fl. e.
767
3
4)2301 qrs.
575 yds. 1 qr.

(11)
ft.
3)183810
5½)61270 yds.
2) 2
11)122540
40)11140 per.
8)278 fur. 20 per.
3)34 m. 6 fur. 20 per.
11 lea. 1 m. 6 fur. 20 per.

(12)
cub. in.
1728)138297
27)80 ft. 57 in.
2 c. yds. 26 c. ft. 57 c. in.

(13)
cub. ft.
128)67893
530 cords 53 c. ft.

(14)
sec.
60)3561829
60)59363 m. 49 sec.
24)989 h. 23 m. 49 s.
7)41 d. 5 h. 23 m. 49 s.
5 wks. 6 days 5 hrs. 23 min. 49 sec.

(15)
q'ts.
4)1597
2)399 gals. 1 qt.
4)199 pks. 1 gal. 1 qt.
49 bush. 3 pecks 1 gal. 1 qt.

(16)
c. ft.
8)1000
125 cords.

(17)
seconds.
60)10000
60)166′ 40″
2° 46′ 40″

(18)
sq. links.
10000)70000
7 sq. ch.

(19)
grs.
20)11521
3)576 scr. 1 gr.
8)192 dr. 1 gr.
12)24 oz. 1 gr.
2 lbs. 1 gr.

(20)
sq. ft.
9)26025
30¼)2891 yds. 6 ft.
4) 4
121)11564 quarter yards.
95 per. 69 quar. yds. 6 ft. =
40)95 per. 17 yds. 8 ft. 36 in.
2 r. 15 sq. p. 17 sq. y. 8 sq. ft. 36 sq. in.

EXERCISE 7—Page 58.

(1)
£3×400=1200 cents.
7s.× 20= 140 "
1¼d.=5 far.×5÷12= $2\frac{1}{12}$ "
£3 7s. 1¼d.=$1342\frac{1}{12}$ cts.

(2)
£29×400=$116·00
18s.× 20= 3·60
3½d.=14 far.×5÷12= ·05⅚
£29 18s. 3½d.=$119.65⅚

(3)
11¼d.=45 far.×5÷12=18¾ cts.

(4)
£69×400=$276·00
15s.× 20= 3·00
6d.=24 far.×5÷12= ·10
£69 15s. 6d.=$279·10

(5)
18s.×20=$3·60
8½d.=34 far.×5÷12= ·14⅙
18s. 8½d.=$3·74⅙

(6)

£17×400=$68·00
16s × 20= 3·20
5¾d.=23 far.×5÷12= ·09$\frac{7}{12}$

£17 16s. 5¾d.=$71·29$\frac{7}{12}$

(7)

£87×400=$348·00

(8)

15s.×20=$3·00
11¾d.=47 far.×5÷12= ·19$\frac{7}{12}$

15s. 11¾d.=$3·19$\frac{7}{12}$

(9)

£16×400=$64·00
6s.×20 = 1·20
2d.=8 far.×5÷12 = ·03⅓

£16 6s. 2d.=$65·23⅓

(10)

£2×400=$8·00
9s.×20 = 1·80
11d.=44 far.×5÷12= ·18⅓

£2 9s. 11d.=$9·98⅓

Exercise 13—Page 90.

(1)

36=12 + 3
$169·78
12
2037·36
3
$6112·08

(2)

121=11 × 11
796342.3
11
8759765·3
11
96357418·3

(3)

144=12 × 12
$33460
12
401520
12
$4818240

(4)

648=12 × 9 × 6
735
12
8820
9
79380
6
476280

(5)

18 = 6 × 3

£	s.	d.
3	7	6
		6
20	5	0
		3
60	15	0

(6)

22 = 11 × 2

£	s.	d.
5	14	6½
		11
62	19	11½
		2
125	19	11

(7)

810 = 10 × 9 × 9

£	s.	d.
3	4	7
		10
32	5	10
		9
290	12	6
		9
2615	12	6

(8)

54 = 9 × 6

cwt.	qrs.	lbs.	oz.
11	3	14	7
			9
107	0	4	15
			6
642	1	4	10

(9)
49 = 7 × 7
bush. pks. gal. qt. pt.
26 3 1 1 1
7
188 1 1 2 1
7
1319 0 1 1 1

(10)
63 = 9 × 7
yds. qrs. na. in.
2 2 2 2
9
24 0 2 0
7
168 3 2 0

(11)
288 = 12 × 12 × 2
dys. hrs. m'n. sec.
5 17 33 11
12
68 18 33 12
12
825 7 38 24
2
1650 15 16 48

EXERCISE 14—Page 92.

(1)
83 = 3 × 10 × 8
£ s. d. £ s. d.
12 2 4 × 3 = 36 7 0
10
121 3 4 × 8 = 969 6 8
1005 13 8

(2)
999 = 10 × 10 × 10—1
£ s. d.
963 0 0¾
10
9630 0 7½
10
96300 6 3
10
963003 2 6
963 0 0¾
962040 2 5¼

(3)
3178=8+10×7+10×10×1+10×10×10×3
£ s. d. £ s. d.
3 6 5¼×8= 26 11 6
10
33 4 4½×7= 232 10 7½
10
332 3 9 ×1= 332 3 9
10
3321 17 6 ×3=9965 12 6
10556 18 4½

(4)
678=8+10×7+10×10×6
bush. pk. gal. bush. pk. gal.
16 3 1×8= 135 0 0
10
168 3 0×7= 1181 1 0
10
1687 2 0×6=10125 0 0
11441 1 0

(5)

$$247 = 7+10\times4+10\times10\times2.$$

m.	fur.	rds.	yds.	m.	fur.	rds.	yds.
23	6	33	4×7 =	166	7	36	0½
			10				
238	4	17	1½×4=	954	1	29	0½
			10				
2385	4	12	4 ×2=	4771	0	25	2½
				5892	2	10	3½

(6)

$$721 = 1+10\times2+10\times10\times7$$

S.	deg.	min.	sec.	S.	deg.	min.	sec.
3	16	30	45×1=	3	16	30	45
			10				
35	15	7	30×2=	71	0	15	0
			10				
355	1	15	0×7=	2485	8	45	0
				2559	25	30	45

EXERCISE 15—Page 93.

(6)
7071
556
42426
35355
35355
3931476

(7)
15607
3094
62428
140463
468210
48288058

(8)
39948123
6007
279636861
239688873800
239968374861

(9)
2778588
9867
19450116
16671528
22228704
25007292
27416327796

EXERCISE 16—Page 95.

(4)
3.2517
.023
97551
65034
0747891

(5)
64.001
340
2560040
192003
21760.340

(6)
482000
.37
3374000
1446000
178340.00

(7)
3782.4
.00917
264768
37824
340416
34.684608

(8)
87.96
220
175920
17592
19351.20

Exercise 17—Page 100.

(1)

216=6×6×6

$83469
6

500814
6

3004884
6

$18029304

(2)

$61135.37
229

55021833
12227074
12227074

$13999999.73

(3)

255226
143

765678
1020904
255226

36497318

(4)

176=11×8×2

203736
11

2241096
8

17928768
2

35857536

(5)

116700
235

583500
350100
233400

27424500

(6)

3721
73

11163
26047

271633

(7)

297=11 × 9 × 3
32000
11

352000
9

3168000
3

9504000

(8)

35=7 × 5
9344000
7

65408000
5

327040000

(9)

749=9+10×4+10×10×7

lbs.	oz.	drs.	scr.	gr.		lbs.	oz.	drs.	scrs.	grs.
123	4	7	2	17	×9=	1110	8	7	1	13
				10						
1234	1	7	1	10	×4=	4936	7	6	0	0
				10						
12341	7	3	0	0	×7=	86391	3	5	0	0
						92438	8	2	1	13

(10)

999998=1000000—2

1698732
1000000

1698732000000
3397464

1698728602536

(11)

640 = 10 × 8 × 8

bush.	pk.	gal.	qt.	pt.
123	1	1	1	1
				10
1234	0	1	8	0
				8
9873	3	0	0	0
				8
78990	0	0	0	0

(12)

89
·73
267
623
$64·97

(13)

1143=3+10×4+10×10×1+10×10×10×1

yds.	qrs.	na.	in.		yds.	qrs.	na.	in.
7	3	2	1	× 3 =	23	2	3	0¾
			10					
79	0	0	1	× 4 =	316	0	1	1¾
			10					
790	1	0	1	× 1 =	790	1	0	1
			10					
7902	3	0	1	× 1 =	7902	3	0	1
					9032	3	2	0

(14)

1634·5789
635000
81728945000
49037367
98074734
1037957601·5

(15)

$968·49
3·4
387396
290547
$3292·866

$3292·866
3.7
23050062
9878598
$12183·6042

$12183.6042
3292·866
968·49
$16444·9602

EXERCISE 18—Page 110.

(9)

6423)798965(124$\frac{2513}{6423}$
6423
15666
12846
28205
25692
2513

10

£ s. d.
12)176 14 6
14 14 6$\frac{1}{2}$

(11)

741)56789(76$\frac{473}{741}$
5187
4919
4446
473

(12)

7894)6785158(859$\frac{4212}{7894}$
63152
46995
39470
75258
71046
4212

(13)

£ s. d. £ s. d.
317)4728 16 2(14 18 4$\frac{54}{317}$
317
1558
1268
290
20
5816
317
2646
2536
110
12
1322
1268
54

(15)

6)970763
161793·8333+

(16)

9)71234
7914$\frac{8}{9}$

(14)

429)$97896·64($228·19$\frac{3\cdot13}{429}$
858
1209
858
3516
3432
84·6
42·9
41·74
38·61
3·13

(17)

47600)977076(20$\frac{25076}{47600}$
95200
25076

(18)

lbs. oz. drs. scr. grs. lbs. oz. drs. scr. grs.
498)7289 6 4 2 13(14 7 5 0 12$\frac{437}{498}$
498
2309
1992
317
12
3810
3486
324
8
2596
2490
106
3
320
20
6413
5976
437

(19)

£ s. d. s. d.
487)157 16 7(6 5¾..$\frac{59}{487}$
20
3156
2922
234
12
2815
2435
380
4
1520
1461
59

(20)

9712)7867674(810$\frac{954}{9712}$
77696
9807
9712
954

(21)

m. fur. rds. m. fur. rds.
37)422 3 38(11 3 14
407
15
8
123
111
12
40
518
37
148
148

EXERCISE 19—Page 112.

(1)
25=5×5

5)3766
5)753... 1
150... 3
3×5+1=16
150$\frac{16}{25}$

(2)
42=7×6

7)26406
6)3772... 2
628... 4
4×7+2=30
628$\frac{30}{42}$

(3)
96=12×8

12)25431
8)2119... 3
264 ... 7
7×12+3=87
264$\frac{87}{96}$

(4)
24=12×2

£ s. d.
12)24 17 6
2)2 1 5½
1 0 8¾

(5)
49=7×7
£ s. d.
7)740 13 4
7)105 16 2¼...1
15 2 3¾...$\frac{1}{49}$

(6)
56=8×7
£ s. d.
8)547 12 4
7)68 9 0½
9 15 6¾...$\frac{40}{56}$

(7)
35=7×5
7)6789436
5)969919...3
193983...4
4×7+3=31
193983$\frac{31}{35}$

(8)
147=7×7×3
7)753293
7)107613... 2
3)15373... 2
5124... 1
1×7×7+2×7+2=65
5124$\frac{65}{147}$

(9)
81=9×9
lbs. oz. dwt. grs.
9)1798 6 11 9
9)199 10 1 6...3
22 2 9 0...6
6×9+3=57
22 lbs. 2 oz. 9 dwt. 0$\frac{57}{81}$ grs.

EXERCISE 20—Page 114.

(1)
£ s. d. £ s. d.
491 12 0¼) 8968 13 7½
20 20
9832 179373
12 12
117984 2152483
4 4
471937) 8609934 (18$\frac{115068}{471937}$
471937
3890564
3775496
115068

(2)
m. fur. rds. m. fur. rds.
17 5 27) 1027 1 6
8 8
141 8217
40 40
5667) 328686 (58
28335
45336
45336

```
                  (3)
  £   s.  d.       £    s.   d.
  57  0   7½ )    171   1   10½
  20               20
 ----             ----
 1140             3421
   12               12
 -----            -----
 13687            41062
     4                4
 -----            ------
 54750  )         164250 (3
                  164250
```

```
                     (4)
dwt grs.     lbs. oz. dwt. grs.
 5   9 )       9   9    3   12
 24           12
 ---         ---
 129          117
               20
             ----
             2343
               24
             ----
             9384
            4686
      129)56244(436
           516
           ---
            464
            387
            ---
             774
             774
```

EXERCISE 21—Page 115.

```
                     (5)
 a.   r.  per.       a.    r.  per.
 91   0   6 )      2366    3   36
  4                   4
 ---               ----
 364               9467
  40                 40
 ----              ------
 14566  )          378716 (26
                   29132
                   -----
                    87396
                    87396
```

```
          (6)
47·655 ÷ 4·5 =
45)476·55(10·59
   45
   --
    26·5
    22·5
    ----
     4·05
     4·05
```

```
              (7)
     756·98 ÷ 76·73612 =
7673612)75698000 (9·864 +
        69062508
        --------
         6635492·0
         6138889·6
         ---------
          496602·40
          460416·72
          ---------
           36185·680
           30694·448
           ---------
            5491·232
```

```
              (8)
   47·5782975 ÷ 26·175 =
26175)47578·2975(1·8177
      26175
      -----
       21403·2
       20940·0
       -------
         463·29
         261·75
         ------
          201·547
          183·225
          -------
           18·3225
           18·3225
          (9)
   1 ÷ 7·6845 =
76845) 10000·0(0·1309+
        7684·5
       ————————
        2365·50
        2290·35
       ————————
          75·1500
          68·7105
```

```
            (10)
     75·347 ÷ 0·3829 =
3829)753470(196·7798 +
     3829
     —————
     37057
     34461
     —————
      25960
      22974
      ——————
       2986·0
       2680·3
       ——————
        305·70
        268·03
        ——————
         37·670
         34·461
         ——————
          3·2090
          3·0632
          ——————
           ·1458
```

```
        (11)
·0002÷·00000008=

     8)200000
     ————————
        25000
```

Exercise 22—Page 116.

```
        (1)
95)$3300000($34736·8421
    285
    ———
     450
     380
     ———
      700
      665
      ———
       350
       285
       ———
        650
        570
        ———
         80·0
         76·0
         ————
          4·00
          3·80
          ————
           ·200
           ·190
           ————
            ·100
            ·095
            ————
            ·005
```

```
           (3)  dys.
28800)95270400(3308
      86400
      —————
       88704
       86400
       —————
        230400
        230400

                days.
      365¼)     3308
         4         4
      ————   ——————— yrs. days.
      1461)    13232(9   20¼
               13149
               —————
                4)83
                  ——
                  20¾
```

```
                          (2)
126)$3860000($30634·9206
     378
     ———
      800
      756
      ———
       440
       378
       ———
        620
        504
        ———
        116·0
        113·4
        —————
          2·60
          2·52
          ————
          ·800
          ·756
          ————
          ·044
```

```
              (4)
35781628)$1145012096($32
          107344884
          —————————
           71563256
           71563256
```

(5)
27475271)$3764112127($137
27475271
101658502
82425813
192326897
192326897

(6)
9)$972
$108

(7)
108)$972($9
972

(8)
294)$8526($29
588
2646
2646

(9)
792)340480(429$\frac{89}{99}$ oz.
3168
2368
1584
7840
7128
712
$\frac{712}{792}=\frac{89}{99}$

(10)
1728)1000(·578 oz.
864·0
136·00
120·96
15·040
13·824
1·216

(11)
m. fur.
83 2
8
266
40
10640
5$\frac{1}{2}$
53200
5320
1155)58520(50$\frac{770}{1155}$
5775
770
50$\frac{770}{1155}$=50$\frac{2}{3}$.

(12)
19)4750(250 lbs.
38
95
95

(13)
978·634÷96·34762=
9634762)97863400(10·157
9634762
1515780·0
963476·2
552303·80
481738·10
70565·700
67443·334
3122·366

(14)
bush. pk. gal. qt. pt. bush. pk. gal. qt. pt.
297)729 1 1 1 1 (2 1 1 2 1$\frac{2}{11}$
594
135
4
541
297
244
2
489
297
192
4
769
594
175
2
351
297
54 $\frac{54}{297}=\frac{2}{11}$

```
                 (15)
lbs. oz. dr.     cwt. qr. lbs. oz. dr.
 9    7   8 )    179   3   4   16   0
16                 4
--              ----         (16)
151              719          m. fur. rds.      m.
 16               25          93   4   7     25000
---             ----           8                 8
914             3599          --            ------
151             1438          748           200000
----            -----          40               40
2424            17979         ---           ------- dys.   hrs.
                   16        29927  29927) 8000000(267   7 18295/29927
                -----                      59854
               107890                      ------
                17979                      201460
               ------                      179562
               287680                      ------
                   16                      218980
               ------                      209489
              1726080                      ------
               287680                        9491
              -------                          24
    2424)4602880(1898 266/303                ------
         2424                               37964
         -----                              18982
         21788                             ------
         19392                             227784
         -----                             209489
          23968                            ------
          21816                             18295
          -----
           21520
           19392
           -----
            2128     2128/2424=266/303.
```

EXERCISE 23—Page 118.

(3)

DCCIX, MV̄CCCLXXVI, MX̄CMXCIX, L̄X̄X̄X̄V̄MIV,

M̄M̄M̄C̄M̄X̄L̄V̄MMDXCVI.

(4)

```
72=8×9
  lbs. oz.
  749 10
       8
 ________
 5997  0
       9
 ________
53973  0
```

(5)

```
      17=7+10×1
   s.  d.        £  s.  d.
   4  7¾×7=1  12  6¼
     10
 _________
£2  6  5½+1=2   6  5½
            ___________
             3  18  11¾
```

(6)

$3\cancel{0})28500000\cancel{0}$

```
          ___________ days.   hrs.
      24)95000000(3958333  8
         72
         __                   yrs.
         230      365¼)3958333(10837
         216         4         4
         ___      ____  __________
          140   1461)15833332
          120        1461
          ___        ____
           200       12233
           192       11688
           ___       _____
             80       5453
             72       4383
             __       _____
              80      10702
              72      10227
              __      ______
               80     4)475 quarter days.
               72     _____  days.  hrs.
               __      118¾=118   18
                8 rem.      Add     8
                           _________
                            119     2
```

10837 yrs. 119 days, 2 hrs.

(7)

```
£729×400                =$2916·00
17s.×20                 =     3·40
6¼d.=25 far.×5÷12=           10 5/12
                         __________
                        $2919·50 5/12
```

(8)

```
$10000
  9876·23
 ________
 $123·77
```

(10)

in.
12)7964327

12)663693–11 }
55307– 9 } 119 in.
9)55307 ft. 119 in.

30¼)6145 yds. 2 ft. 119 in.
4 4

121)24580
11)24580

11)2234–6 }
203–1 } 17 qr. yds.

203 p. 4¼ y.=203 p. 4 yds. 2 ft. 36 in.
Add 2 ft. 119 in

40)203 p. 4 yds. 5 ft. 11 in.

4)5 rd. 3 p. 4 yds. 5 ft. 11 in.
1 a. 1 r. 3 p. 4 yds. 5 ft. 11 in.

(11)

$729·43
16·70
976·81
9987·17
429·00
129·19

$12268·30

(12)

429=9+10×2+10×10×4

wks.	dys.	hrs.	min.	wks.	dys.	hrs.	min.
6	4	3	17×9=	59	2	5	33
			10				
65	6	8	50×2=	131	5	17	40
			10				
659	0	16	20×4=	2636	2	17	20
				2827	3	16	33

wks.
52)2827(54 yrs. 19 wks. 3 dys. 16 hrs. 33 min.
260

227
208

19 wks.

(15)

tons.
324
20
cwt. qr. lbs.
13 2 14 6480
4 4
54 25920
25 25
284 129600
108 51840
1364)648000($475\frac{100}{1364}$=
5456 $475\frac{25}{341}$hds.
10240
9548
6920
6820
100

(14)

78·96
·00042
15792
31584
,0331662

(16)

$136
$136×4=544—95=449
1902
2487

$9237–$2847=$6750

(17)

yds.qrs.na. yds.qrs.na.
3 1 2) 39 2 3
4 4
13 158
4 4
54) 635($11\frac{41}{54}$
54
95
54
41

(18)

a.
25
197
156
97
199
674

(19)

a. a. r. per.
732 96 3 17
674 4
58 387
40
15497)$7764·0($0·501
7748·5
15·500
15·497
3

(20)

$
20 $312
75 275
97 $ 37
83
275

(21)

lbs. oz. dwt. grs.
12)36 8 14 16
3 0 14 $13\frac{1}{3}$

(22)

a. r. per.
6 3 12
7 2 0
9 0 13
5 2 36
29 21

(23)

5
7
9
21)294(14
21
84
84

(24)

lbs.	oz.	dwt.	grs.
5	9	8	0
3	2	16	16
4	6	17	0
1	8	19	22
15	4	1	14

(25)

£972 × 400 = $3888·00
11s. × 20 = 2·20
11¼d. = 45 far. × 5 ÷ 12 = ·18¾
$3890·38¾

(26)

lbs. oz. drs. scr. grs.
179 3 3 1 14
12
2151 oz.
8
17211 drs.
3
51634 scr.
20
1032694 grs.

(27)

56
25
280
112
1400
2
2800 sq. ft. in roof.
6
16800

(28)

cwt.	qr.	lbs.
6	2	11
5	3	16
8	0	7
3	1	17
24	0	1 = 2401 lbs.

·15
12005
2401
$360·15

(29)

29
57
203
145
1653
.15
8265
1653
$247.95

(30)

$
139468
98579
$238047

370129
238047
$132082

(31)

£ s. d. £ s. d.
9 19 $11\frac{3}{4}$ 1694 16 $0\frac{11}{42}$
20 20
$\frac{3}{4}=\frac{63}{84}$ $\frac{11}{42}=\frac{22}{84}$
199 33896
12 12
2399 406752
84 84
9659 1627030
19192 3254016
201579)34167190(169·49
201579
1400929
1209474
1914550
1814211
100339·0
80631·6
19707·40
18142·11
1565·29

(32)

£19 × 400 = $76·00
19s. × 20 = 3·80
$11\frac{3}{4}$d = 47far × 5 ÷ 12 = ·19$\frac{7}{12}$
$79·99$\frac{7}{12}$

(33)

cwt. qr. lbs. cwt. qr. lbs
3 2 11 12 0 0
4 1 15 8 0 1
lbs.
8 0 1 3 3 24 = 399
·15
1995
399
$59·85

(34)

cwt. qr. lbs.
2 0 17
3 2 15
2 1 20
5 3 17
lbs.
14 0 19 = 1419
·37$\frac{1}{2}$
9933
4257
709$\frac{1}{2}$
$532·12$\frac{1}{2}$

(36)

43·2 ÷ 76·8437 =
768437)432000·0(0·562
384218·5
47781·50
46106·22
1675·280
1536·874
138·406

(37)

123·4÷·00000066=
123400000000÷66
6)123400000000
11)20566666666·666
1869696969·$\dot{6}\dot{9}$

(38)

\$63·29
17
44303
6329
\$1075·93

\$2789·27
1075·93
\$1713·34

(39)

£29×400 =\$116·00
6s× 20 = 1·20
11$\frac{3}{4}$d.=47far.×5÷12= ·19$\frac{7}{8}$
117·39$\frac{7}{8}$

\$278·43
417·16
11·27
2110·40
723·15
117·39$\frac{7}{8}$
173) 3657·80$\frac{7}{8}$
12 12
2076)43893·67(\$21·1433
4152
2373
2076
297·6
207·6
90·07
83·04
7·030
6·228
·8020
·6228
·1792

(40)

2076)491544(236$\frac{402}{519}$
4152
7634
6228
14064
12456
1608

$\frac{1608}{2076}=\frac{402}{519}$

Exercise 24—Page 127.

(1)

2)11368
2)5684
2)2842
7)1421
7)203
29
$2^3\times7^2\times29$

(2)

2)2934
3)1467
3)489
163
$2\times3^2\times163$

(3)

3)1011
337
3×337

(4)

2)1000
2)500
2)250
5)125
5)25
5
$2^3\times{}^35$

(5)

2)1024
2)512
2)256
2)128
2)64
2)32
2)16
2)8
2)4
2
2^{10}

(6)

2)32320
2)16160
2)8080
2)4040
2)2020
2)1010
5)505
101
$2^6\times5\times101$

(7)

7)707
101
7×101

(8)

2)1118
13)559
43
2×13×43

EXERCISE 25—Page 128.

(1)

$100=2^2\times5^2$

1..2..4
1..5..25
1..2..4..5..10..20..25..50..100

(2)

$810=3^4\times2\times5.$

1..3..9..27..81
1..2
1..3..9..27..81..2..6..18..54..162
1..5
1..3..9..27..81..2..6..18..54..162..5..15..45..135..405..
10..30..90..270..810=
1..2..3..5..6..9..10..15..18..27..30..45..54..81..90..135..
162..270..405..810.

(3)

$920=2^3\times5\times23.$

1..2..4..8
1..5
1..2..4..8..5..10..20..40
1..23
1..2..4..8..5..10..20..40..23..46..92..184..115..230..460..920=
1..2..4..5..8..10..20..23..40..46..92..115..184..230..460..920.

(4)

$25000=5^5\times2^3$

1..5..25..125..625..3125
1..2..4..8
1..5..25..125..625..3125..2..10..50..250..1250..6250..4..20..100..
500..2500..12500..8..40..200..1000..5000..25000=
1..2..4..5..8..10..20..25..40..50..100..125..200..250..500..625..
1000..1250..2500..3125..5000..6250..12500..25000.

EXERCISE 26—Page 128.

(1)

$88200=2^3\times3^2\times5^2\times7^2$
$3+1=4$
$2+1=3$
$2+1=3$
$2+1=3$
$4\times3\times3\times3=108$

(2)

$3500=2^2\times5^3\times7$
$2+1=3$
$3+1=4$
$1+1=2$
$3\times4\times2=24$

(3)

$6336=2^6\times3^2\times11$
$6+1=7$
$2+1=3$
$1+1=2$
$7\times3\times2=42$

(4)

$824=2^3\times103$
$3+1=4$
$1+1=2$
$4\times2=8$

(5)

$49000=2^3\times5^3\times7^2$
$3+1=4$
$3+1=4$
$2+1=3$
$4\times4\times3=48$

(6)

$81000=2^3\times3^4\times5^3$
$3+1=4$
$4+1=5$
$3+1=4$
$4\times5\times4=80$

(7)

$75600=2^4\times3^3\times5^2\times7$
$4+1=5$
$3+1=4$
$2+1=3$
$1+1=2$
$5\times4\times3\times2=120$

(8)

$25600=2^{10}\times5^2$
$10+1=11$
$2+1=3$
$11\times3=33$

EXERCISE 27—Page 129.

(1)

$21=7\times3$
$18=2\times3\times3$
$27=3\times3\times3$
$36=4\times3\times3$
3 is common to all.

(2)

$21=3\times7$
$77=11\times7$
$42=2\times3\times7$
$35=5\times7$
7 is common to all.

(3)

$26=2\times13$
$52=2\times2\times13$
$91=7\times13$
$143=11\times13$
13 is common to all.

(4)

$82=41\times2$
$118=59\times2$
$146=73\times2$

2 is common to all.

EXERCISE 28—Page 130.

(1)

```
296)407(1
    296
    ---
    111)296(2
        222
        ---
         74)111(1
             74
             ---
             37)74(2
                74
G. C. M. = 37.
```

(2)

```
308)506(1
    308
    ---
    198)308(1
        198
        ---
        110)198(1
            110
            ---
             88)110(1
                 88
                 ---
                 22)88(4
                    88
G. C. M. = 22.
```

(3)

```
74)84(1
   74
   --
   10)74(7
      70
      --
       4)10(2
         8
         -
         2)4
           -
           2
G. C. M. = 2.
```

(4)

```
1825)2555(1
     1825
     ----
      730)1825(2
          1460
          ----
           365)730(2
               730
G. C. M. = 365.
```

(5)

```
556)672(1
    556
    ---
    116)556(4
        464
        ---
         92)116)1
             92
             --
             24)92(3
                72
                --
                20)24(1
                   20
                   --
                    4)20(5
                      20
G. C. M. = 4.
```

Exercise 29—Page 131.

(1)
110)140(1
110
30)110(3
90
20)30(1
20
10)680
68
10)20
2
Therefore 10 is their G. C. M.

(2)
1326)3094(2
2652
442)1326(3
1326
Also 4420 is divisible by 442;
therefore it is their G. C.M.

(3)
468)922(1
468
454)468(
454
14)454(32
42
34
28
6)14(2
12
2)6
3
375 is not divisible by 2,
and therefore their G. C. M. is 1.

(4)
204)1190(5
1020
170)204(1
170
34)170(5
170
34)1445(42
136
85
68
17)34(2
34
17)2006(118
17
30
17
136
136
G. C. M.=17.

Exercise 30—Page 132.

(2)
$56=2^3\times7$
$84=2^2\times3\times7$
$140=2^2\times5\times7$
$168=2^3\times3\times7$

The greatest factors which are common are 2^2 and 7; therefore the G. C. M.$=2^2\times7=28$.

(3)

$241920=2^8\times3^3\times5\times7$
$380160=2^8\times3^3\times5\times11$
$69120=2^9\times3^3\times5$
$103680=2^8\times3^4\times5$

The greatest factors which are common are 2^8, 3^3 and 5; therefore the G. C. M. $=2^8\times3^3\times5=34560$.

(4)

$10800=2^4\times3^3\times5^2$
$28040=2^3\times5\times701$
$2160=2^4\times3^3\times5$

The greatest factors which are common are 2^3 and 5; therefore the G. C. M. $=2^3\times5=40$.

EXERCISE 31—Page 133.

(2)

$6=2\times3$
$7=7$
$42=2\times3\times7$
$9=3^2$
$10=2\times5$
$630=2\times3^2\times5\times7$

$2\times3^2\times5\times7=630.$

(3)

$1=1$
$2=2$
$3=3$
$4=2^2$
$5=5$
$6=2\times3$
$7=7$
$8=2^3$
$9=3^2$
$3^2\times2^3\times5\times7=2520.$

(4)

$6=2\times3$
$9=3^2$
$12=2^2\times3$
$15=3\times5$
$18=2\times3^2$
$21=3\times7$
$30=2\times3\times5$

$2^2\times3^2\times5\times7=1260.$

(5)

$670=2\times5\times67$
$100=2^2\times5$
$335=5\times67$
$25=5^2$

$2^2\times5^2\times67=6700$

(6)

$8=2^3$
$10=2\times5$
$18=2\times3^2$
$27=3^3$
$36=2^2\times3^2$
$44=2^2\times11$
$396=2^2\times3^2\times11$
$2^3\times3^3\times5\times11=11880.$

EXERCISE 32—Page 134.

(1)
2)12..10..24
2) 6.. 5..12
3) 3.. 5.. 6
1.. 5.. 2
2×2×3×5×2=120

(2)
2)14..21..3..2 .63
3) 7..21..3..1..63
7) 7.. 7..1..1..21
1.. 1..1..1.. 3
2×3×7×3=126

(3)
2)18..12..39..
2) 9.. 6..39..
3) 9.. 3..39..
3) 3.. 1..13..
13) 1.. 1..13..
1.. 1.. 1..
2×2×3×3×13+

(4)
2)8..18..15..20..70
2)4.. 9..15..10..35
3)2.. 9..15.. 5..35
5)2.. 3.. 5.. 5..35
2.. 3.. 1.. 1.. 7
2×2×3×5×2×3×7=2520

(5)
2)24..16..18..20
2)12.. 8.. 9..10
2) 6.. 4.. 9.. 5
3) 3.. 2.. 9.. 5
1.. 2.. 3.. 5
2×2×2×3×2×3×5=720.

(6)
2)60..50..144..35..18
2)30..25.. 72..35.. 9
3)15..25.. 36..35.. 9
3) 5..25.. 12..35.. 3
5) 5..25.. 4..35.. 1
1.. 5.. 4.. 7.. 1
2×2×3×3×5×5×4×7=25200.

(7)
2)27..54..81..14..63
3)27..27..81.. 7..63
3) 9.. 9..27.. 7..21
3) 3.. 3.. 9.. 7.. 7
7) 1.. 1.. 3.. 7.. 7
1.. 1.. 3.. 1.. 1
2×3×3×3×7×3=1134.

EXERCISE 33—Page 136.

(1)
300|~~300~~..200..~~150~~..~~50~~..~~60~~..~~75~~..125
10| ~~2~~ ~~5~~
300×10=3000.

(2)
165|20..60..~~15~~..~~165~~..210..63..27
21| 4.. 4 14..~~21~~.. 9
12| ~~4~~..~~4~~ ~~2~~ ~~3~~
165×21×12=41580.

(8)

144| 12..132..144..60..96..1728
12| 11 5 2 12
55| 11 5
144×12×55=95040.

Exercise 34—Page 138.

(1)
12)592835
12)49402..e
12)4116..t
12)343..0
12)28..7
2..4
2470te

(2)
5)3700
5)740..0
5)148..0
5)29..3
5)5..4
1..0
104300.

(3)
11)10000
11)909..1
11)82..7
7..5
7571.

(4)
6)1000000
6)166666..4
6)27777..4
6)4629..3
6)771..3
6)128..3
6)21..2
3..3
33233344

(5)
8)10000
8)1250..0
8)156..2
8)19..4
2..3
23420

(6)
12)12345654321
12)1028804526..9
12)85733710..6
12)7144475..t
12)595372..e
12)49614..4
12)4134..6
12)344..6
12)28..8
2..4
248664et69

(7)
9)10000
9)1111..1
9)123..4
9)13..6
1..4
14641

(8)
2)300
2)150..0
2)75..0
2)37..1
2)18..1
2)9..0
2)4..1
2)2..0
1..0
100101100

EXERCISE 35—Page 139.

(1)	(2)		(3)
IX	V	V	IV
8)37704	7)444	7)4321	9)1212201
8)4311..5	7)32..5	7)313..5	9)23121..0
8)480..1	2..3	7)21..6	9)1101..0
	235.		
8)54..4		1..4	9)21..0
		1465.	
6..1			1..0
61415.			10000.

EXERCISE 36—Page 140.

(1)	(2)	(3)	(4)
IV	III	IX	VI
20212331	101202220	1522365	33233344
4	3	9	6
8	3	14	21
4	3	9	6
34	10	128	128
4	3	9	6
137	32	1154	771
4	3	9	6
550	96	10389	4629
4	3	9	6
2203	290	93507	27777
4	3	9	6
8815	872	841568	166666
4	3		6
35261	2618		1000000
	3		
	7854		

(6)

IX

3)132713
3)40834..0
3)13271..1
3)4083..1
3)1327..0
3)408..1
3)132..2
3)40..2
3)13..0
3)4..0
1..1

IX

12)132713
12)10207..9
12)682..*t*
12)51..8
3. *t*

IX

8)132713
8)14757..1
8)1652..0
8)184..6
8)21..5
2..3

IX III XII VIII

132713 = 11002210110 = 3*t*8*t*9 = 235601

IX

132713
9
12
9
110
9
997
9
8974
9
80769 denary.

III

3
4
3
12
3
36
3
110
3
332

332
3
997
3
2991
3
8974
3
26923
3
80769 denary.

XII

12
46
12
560
12
6730
12
80769 den.

VIII

8
19
8
157
8
1262
8
10096
8
80769 denary.

(7)

XII	XII	XII	XII
9) *t*2*t*290	6) *t*2*t*290	4) *t*2*t*290	2) *t*2*t*290
9)117978..0	6)185856..0	4)268683..0	2)515146..0
9)1624*t*..2	6)34*e*4*e*..0	4)78180..3	2)268683..0
9)2032..4	6)69 *t*9..5	4)1*e*050..0	2)134341..1
9)284..2	6)1179..3	4)5913..0	2)78180..1
9)37..1	6)233..3	4)1533..3	2)3*t*0 *t*0..0
4..7	6)46..3	4)439..3	2)1*e*050..0
	6)9..0	4)10*e*..1	2)*e*626..0
	1..3	4)32..3	2)5913..0
		4)9..2	2)2*t*67..1
		2..1	2)1533..1
			2)877..1
			2)439..1
			2)21*t*..1
			2)10*e*..0
			2)65..1
			2)32..1
			2)17..[illegible]
			2)9..1
			2)4..1
			2)2..0
			1..0

(Continued on next page)

(7 continued.)

```
       XII         IX         VI         IV                    II

t2t290=4712420=130333500=21211330030=1001101101111100001100

        12          9          6          4          2
       ---        ---          -          -          -
       122         43          9          9          2       1243
        12          9          6          4          2          2
      ----       ----       ----       ----          -       ----
      1474        388         54         38          4       2487
        12          9          6          4          2          2
      ----       ----       ----       ----          -       ----
     17690       3494        327        155          9       4975
        12          9          6          4          2          2
    ------     ------     ------     ------          -     ------
    212289      31450       1965        621         19       9951
        12          9          6          4          2          2
   -------    -------    -------    -------          -    -------
   2547468     283052      11793       2487         38      19902
                    9          6          4          2          2
              -------    -------    -------          -    -------
              2547468      70763       9951         77      39804
                               6          4          2          2
                         -------    -------          -    -------
                          424578      39804        155      79608
                               6          4          2          2
                         -------    -------          -    -------
                         2547468     159216        310     159216
                                          4          2          2
                                    -------          -    -------
                                     636867        621     318433
                                          4          2          2
                                    -------          -    -------
                                    2547468       1243     636867
                                                                2
                                                          -------
                                                          1273734
                                                                2
                                                          -------
                                                          2547468
```

EXERCISE 37—Page 142.

(1)
```
    VI
    252
    253
   ----
    544
  2224
  544
 ------
 122024
```

(2)
```
           XII
62te)32e75721(62te
      31556
     ------
      161e7
      1059t
      -----
       58192
       52512
      ------
        58801
        58801
```

(3)
```
   III
 201210
 102221
 ------
  21212
```

(4)
```
     VIII
    57264
      675
  -------
   354604
  513354
 434070
 --------
 51117344
```

(5)
```
     II
    101
   1001
   1111
   1011
   1000
   1111
  10101
 -------
 1010100
```

(6)
```
            VII
2143)142613(50·5254+
       14111
       -----
        1503·0
        1411·1
        ------
          61 60
          43·16
          -----
           15·410
           14·111
           ------
            1·2660
            1·1635
            ------
             ·1022
```

(7)
```
    VII
   65432
   43210
    1444
   65001
   54321
  ------
  326041
```

(8)
```
   XII
  7t348
  5e6t4
  -----
  1t864
```

(9)
```
     XII
     34t7
     6666
     ----
    18536
   18636
  18536
 18536
 -------
 1t36e296
```

(10)
```
                II
100101)1010100001(10010 111/100101
       100101
       ------
          101000
          100101
          ------
             111
```

EXERCISE 38—Page 146.

(1)
4 ft. 7′ 6″ 10‴
9 7 11 11
4 2 11'''' 3''''' 2''''''
4 2 11 3 2
2 8 4 11 10
41 8 1 6
44 9 1 8 0 5 2

(2)
19 ft. 10′ 3″
11 2 7
11 6 11‴ 9''''
3 3 8 0
218 4 9
222 8 0 5 9

(3)
9″ 7‴ 4''''
7 3 11'''''
8'''''' 9''''''' 8'''''''' 8'''''''''
2 4 10 0
5 7 3 4
5 10 4 11 8 8

(4)
9¾ in.=9′ 9″
9′ 9″
5 7 4‴
3 3'''' 0'''''
5 8 3
4 0 9
4 6 8 6

(5)
7 ft. 4′ 11″
3 2 2
1 2 9‴ 10''''
1 2 9 10
22 2 9
23 6 9 7 10

EXERCISE 39—Page 147.

(1)
15 ft.
1 2′
2 6
15
17 6
8
11 8 0″=
11⅔ cu. ft.=11 cu. ft. 1152 cu. in.

(2)
XII
45·6
t·3
1146
3870
398·46
2
——— cu. ft.
774·90=1096 9′
XII
774=1096 com. scale,

(3)
10 ft.
5
50 sq. ft.
7
—— c′rds. c. ft.
128)350(2 94
256
94 cub. ft.

(4)

4 ft.

$5\frac{1}{4}$

20

1

21 sq. ft.

70

128)1470(11$\frac{31}{64}$ cords

128

190

128

62

$\frac{62}{128}=\frac{31}{64}$.

(5)

XII

4·78

9·6

23t0

3590

38·0t

2·e

34492

7418

- cub.ft.

t8·652=128 6′ 5″ 2‴

t8 duoden.= 128 den.

(6)

25 ft.=300 in.

20 " =240 "

2 ft. 6 in.= 30 "

8

4

32

2

64=8×8

300

240

72000

30

8)2160000

8)270000

33750

EXERCISE 40—Page 149.

(1)

£93×400 =\$372·00

14s.×20 = 2·80

$7\frac{1}{2}$d.=30f.×5÷12 = ·$12\frac{1}{2}$

£93 14s. $7\frac{1}{2}$d. =\$374 $92\frac{1}{2}$

£275×400 =\$1100·00

4s ×20 = ·80

$11\frac{3}{4}$d.=47f.×5÷12= ·$19\frac{7}{12}$

£275 4s. $11\frac{3}{4}$d. =\$1100·$99\frac{7}{12}$

£276×400 =\$1104·00

19s.×20 = 3·80

$10\frac{1}{2}$d.=42f.×5÷12= ·$17\frac{1}{2}$

£276 19s. $10\frac{1}{2}$d. =\$1107·$97\frac{1}{2}$

\$729·18

710 50

166·78

374·$92\frac{1}{2}$

1107·$97\frac{1}{2}$

497 81

1100·$99\frac{7}{12}$

\$4688·$16\frac{7}{12}$

(2)

576=6+10×7+10×10×5

m.fur.per.yds.ft.in

47 6 17 4 2 7×6=

10

478 0 18 4 1 10×7=

10

4780 4 28 2 0 4×5 =

m.fur.per.yds.ft.in.

286 6 27 1 2 0

3346 3 11 4 2 4

23902 7 21 4 3 2

27536 1 21 0 1 6

(3)

$243000 = 2^3 \times 3^5 \times 5^3$

$3+1=4$

$4+1=5$

$3+1=4$

$4 \times 6 \times 4 = 96$

(4)

V

8)4234434

8)241110..4

8)13428..1

8)1024..1

8)32..3

2..1

VIII

5)713427

5)133721..2

5)22303..2

5)3532..1

5)570..2

5)113..1

5)17..0

3..0

VIII

713427

213114

500313

V

30012122

4234434

20222133

(5)

79·342÷·00006378=

6378)7934200000(1243994·98275

6378

15562

12756

28060

25512

25480

19134

63460

57402

60580

57402

31780

25512

6268·0

5740·2

527·80

510·24

17·560

12·756

4·8040

4·4646

·33940

·31890

·02050

(7)

```
40|5.7.9.11.15.18.20.21.22.24.28.30.33.35.36.40 42.44.45.48.50.
21|  7.9.11. 3. 9.   21.11. 3. 7. 3.33. 7. 9    21.11. 9. 6. 5.
33|    3.11.    3.      11.           11.    3        11. 3. 2. 5.
10|                                                          2. 5.
```

40×21×33×10=277200.

(9)

9999993000=10000000000—7000.

64276·3427×10000000000=642763427000000

64276·3427×7000 = 449934398·9

642762977065601·1

(10)

```
   IX                    IX
5)78263               11)78263
 5)15230..3            11)6430..3
  5)2760..0             11)526..6
   5)511..4              11)43..0
   5)102..0                  3..6
    5)17..3
       3..1

      IX                V                  XI
 7)78263     =     7)3130403     =     7)36063
 7)11160..3         7)214200..3          7)5640..3
  7)1407..5          7)13220..5           7)884..5
   7)177..3           7)1101..3           7)128..3
    7)23..4             7)41..4            7)18..4
       3..0                3..0                3..0
```

(12)

£672 × 400 = $2688·00
7s. × 20 = 1·40
7d. = 28 f. × 5 ÷ 12 = ·11⅔
£672 7s. 7d. = $2689·51⅔

(13)

891)243000(272
1782
6480
6237
2430
1782
648)891(1
648
243)648(2
486
162)243)1
162
81)162(2
162

(13 continued.)

81)37800(466
324
540
486
540
486
54)81(1
54
27)54(2
54

27)35100
1300

Therefore G. C. M. = 27.

(17)

£	s.	d.
178	16	4¾
97	15	11½
693	19	11¾
216	11	9¼
678	14	7½
197	13	11¾
117	6	5
91	1	1¾
2272	0	3¼

(18)

2)276000
2)138000
2)69000
2)34500
2)17250
3)8625
5)2875
5)575
5)115
23
$2^5 \times 3 \times 5^3 \times 23$

(19)

6 ft. 2′ 7″ 9‴ 10⁗
13 11 11 11 7
3 7 6′′′′′ 6′′′′′′ 8′′′′′′′ 10′′′′′′′′
5 8 5 2 0 2
5 8 5 2 0 2
5 8 5 2 0 2
80 10 5 7 10
87 1 1 3 0 10 8 10 10

(20)	(21)	(22)
XII	V	VIII
713t96)7te9·047(·011436	3333333	10000
713t·96	4	8
97t·2t7	15	8
713·t96	4	8
266·4110	63	64
245·3720	4	8
21·05e00	255	512
19·3e846	4	8
3·862760	1023	4096
3·67e490	4	
·1t3290	4095	
	4	
	16383	

(23)

74002702 ÷ 144 = 513907 ft. 94 in.
513907 ft. ÷ 9 = 57100 yards 7 ft.
57100 yds. ÷ 30¼ = 1887 per. 18¼ yds.
1887 per. 18 yds. 2 ft. 36 in.
Add 7 ft. 94 in.

40)1887 per. 19 yds. 0 ft. 130 in.

4)47 r. 7 per. 19 yds. 0 ft. 130 in.

11 a. 3 r. 7 per. 19 yds. 0 ft. 130 in.

(24)

1728 | 240 .. 780 .. 1620 .. 1728
65 | 5 .. 65 .. 15
3 | 3

1728 × 65 × 3 = 336960

(25)

6 children will have 6 children's shares
4 women will have 4 × 2 = 8 " "
3 men will have 3 × 5 × 2 = 30 " "

3 men 4 w'n & 6 chi'n will have 44 children's sha.

4)$7894·16

11)$1973·54

$179·41$\frac{3}{11}$ = child's share.
$179·41$\frac{3}{11}$ × 2 = $ 358·82$\frac{6}{11}$ = woman's share.
$358·82$\frac{6}{11}$ × 5 = $1794·12$\frac{8}{11}$ = man's share.

(26)

II	II
1111111111	1000000000
2	2
3	2
2	2
7	4
2	2
15	8
2	2
31	16
2	2
63	32
2	2
127	64
2	2
255	128
2	2
511	256
2	2
1023	512

(27)

yds. qrs. na. in. yds. qrs. na. in.
7 1 1 1) 729 3 3 1

4	4
29	2919
4	4
117	11679
2¼	2¼
235	23359
29¼	2919¾
264¼	26278¾
4	4
1057)	105115(99$\frac{472}{1057}$
	9513
	9985
	9513
	472

(28)

762·4978
63·423

22874934
15249956
30499912
22874934
45749868

48359·8979694

(29)

723426
·938·9126141

722487·0873859

(30)

lbs.	oz.	drs.	scr.
129	0	0	0
63	4	7	2
65	7	0	1

(31)

$1064 = 2^3 \times 7 \times 19.$

1..2..4..8
1..7

1..2..4..8..7..14..28..56
1..19

1..2..4..8..7..14..28..56..19..38..76..152..133..266..532..1064=
1..2..4..7..8..14..19..28..38..56..76..133..152..266..532..1064

(32)

30 ft. 6 in. = 366 in.
20 ft. 11 in. = 251 in.
2 ft. 7 in. = 31 in.

366
251

366
1830
732

in.
$31)91866(2963\frac{13}{31}$
62

298
279

196
186

106
93

13

$2963\frac{13}{31} \div 36 = 82\frac{59}{186}$ yds.

EXERCISE 46—Page 158

(1)

$$\frac{2}{5}, \frac{5}{7}, \frac{8}{9}, \frac{3}{5}, \frac{5}{18}, = \frac{2\times7\times9\times5\times18}{5\times7\times9\times5\times18}, \frac{5\times5\times9\times5\times18}{5\times7\times9\times5\times18}, \frac{8\times5\times7\times5\times18}{5\times7\times9\times5\times18}, \frac{3\times5\times7\times9\times18}{5\times7\times9\times5\times18}, \frac{5\times5\times7\times9\times5}{5\times7\times9\times5\times18} = \frac{11340}{28350}, \frac{20250}{28350}, \frac{25200}{28350}, \frac{17010}{28350}, \frac{7875}{28350}$$

(2)

$$\frac{8}{11}, \frac{12}{13}, \frac{5}{14} = \frac{8\times13\times14}{11\times13\times14}, \frac{12\times11\times14}{11\times13\times14}, \frac{5\times11\times13}{11\times13\times14} = \frac{1456}{2002}, \frac{1848}{2002}, \frac{715}{2002}$$

(3)

$$\frac{6}{7}, \frac{4}{11}, \frac{5}{13}, \frac{4}{7}, \frac{1}{2}, = \frac{6\times11\times13\times7\times2}{7\times11\times13\times7\times2}, \frac{4\times7\times13\times7\times2}{7\times11\times13\times7\times2}, \frac{5\times7\times11\times7\times2}{7\times11\times13\times7\times2}, \frac{4\times7\times11\times13\times2}{7\times11\times13\times7\times2}, \frac{1\times7\times11\times13\times7}{7\times11\times13\times7\times2} = \frac{12012}{14014}, \frac{5096}{14014}, \frac{5390}{14014}, \frac{8008}{14014}, \frac{7007}{14014},$$

(4)

$$\frac{6}{11}, \frac{4}{7}, \frac{8}{13}, = \frac{6\times7\times13}{11\times7\times13}, \frac{4\times11\times13}{11\times7\times13}, \frac{8\times11\times7}{11\times7\times13}, = \frac{546}{1001}, \frac{572}{1001}, \frac{616}{1001}$$

(5)

$$\frac{5}{6}, \frac{4}{7}, \frac{4}{5}, \frac{2}{11}, = \frac{5\times7\times3\times11}{6\times7\times5\times11}, \frac{4\times6\times5\times11}{6\times7\times5\times11}, \frac{4\times6\times7\times11}{6\times7\times5\times11}, \frac{2\times6\times7\times5}{6\times7\times5\times11} = \frac{1925}{2310}, \frac{1320}{2310}, \frac{1848}{2310}, \frac{420}{2310},$$

(6)

$$\frac{1}{2},\ \frac{2}{3},\ \frac{3}{5},\ \frac{2}{7},\ =\ \frac{1\times3\times5\times7}{2\times3\times5\times7},\ \frac{2\times2\times5\times7}{2\times3\times5\times7},\ \frac{3\times2\times3\times7}{2\times3\times5\times7},$$

$$\frac{2\times2\times3\times5}{2\times3\times5\times7}\ =\ \frac{105}{210},\ \frac{140}{210},\ \frac{126}{210},\ \frac{60}{210}.$$

EXERCISE 47—Page 159.

(1)

$\frac{4}{5}$, $\frac{3}{8}$, $\frac{4}{6}$, $\frac{3}{4}$,

The least common multiple of 5, 8, 6, 4, 15 is 120.

The multiplier for both terms of the first fraction is $\frac{120}{5}=24$; for the second $\frac{120}{8}=15$; for the third ${}^{0}=20$; for the fourth $\frac{120}{4}=30$; for the fifth $\frac{120}{15}=8$.

Multiplying by these numbers, we obtain $\frac{96}{120}$, $\frac{45}{120}$, $\frac{80}{120}$, $\frac{90}{120}$, and $\frac{56}{120}$.

(2)

$\frac{6}{11}$, $\frac{2}{3}$, $\frac{4}{7}$, $\frac{18}{77}$, $\frac{19}{33}$.

The least common multiple of 11, 3, 7, 77 and 33 is 231.

The multiplier for both terms of the first fraction is $\frac{231}{11}=21$; for the second, $\frac{231}{3}=77$; for the third, $\frac{231}{7}=33$; for the fourth, $\frac{231}{77}=3$; and for the fifth, ${}_{3}\ =7$.

Multiplying by these numbers, we obtain $\frac{126}{231}$, $\frac{154}{231}$, $\frac{132}{231}$, $\frac{54}{231}$, and $\frac{133}{231}$.

(3)

$\frac{1}{2}$, $\frac{2}{3}$, $\frac{3}{5}$, $\frac{5}{6}$, $\frac{7}{8}$, $\frac{9}{\ }$, $\frac{13}{15}$, $\frac{7}{16}$, $\frac{37}{80}$.

The least common multiple of 2, 3, 5, 6, 8, 10, 15, 16 and 80 is 240.

The multiplier for both terms of the first fraction is $\frac{240}{2}=120$; for the second, $\frac{240}{3}=80$; for the third, $\frac{240}{5}=48$; for the fourth, $\frac{240}{6}=40$; for the fifth, $\frac{240}{8}=30$; for the sixth, $\frac{240}{10}=24$; for the seventh, $\frac{240}{15}=16$; for the eighth, $\frac{240}{16}=15$; and for the ninth, $\frac{240}{80}=3$.

Multiplying by these numbers, we obtain $\frac{120}{240}$, $\frac{160}{240}$, $\frac{144}{240}$, $\frac{200}{240}$, $\frac{210}{240}$, $\frac{216}{240}$, $\frac{208}{240}$, $\frac{105}{240}$, and $\frac{111}{240}$.

(4)

$\frac{3}{5}, \frac{7}{10}, \frac{6}{25}, \frac{11}{30}, \frac{13}{45}, \frac{23}{60}$.

The least common multiple of 5, 10, 25, 30, 45, and 60 is 900.

The multiplier for both terms of the first fraction is $\frac{900}{5} = 180$; for the second, $\frac{900}{10} = 90$; for the third, $\frac{900}{25} = 36$; for the fourth, $\frac{900}{30} = 30$; for the fifth, $\frac{900}{45} = 20$; and for the sixth, $\frac{900}{60} = 15$.

Multiplying by these numbers, we obtain $\frac{540}{900}, \frac{630}{900}, \frac{216}{900}, \frac{330}{900}, \frac{260}{900}$, and $\frac{345}{900}$.

(5)

$\frac{19}{20}, \frac{7}{30}, \frac{11}{40}, \frac{1}{50}$.

The least common multiple of 20, 30, 40 and 50 is 600.

The multiplier for both terms of the first fraction is $\frac{600}{20} = 30$; for the second, $\frac{600}{30} = 20$; for the third, $\frac{600}{40} = 15$; and for the fourth, $\frac{600}{50} = 12$.

Multiplying by these numbers, we obtain $\frac{570}{600}, \frac{140}{600}, \frac{165}{600}$ and $\frac{12}{600}$.

(6)

$\frac{1}{2}, \frac{2}{3}, \frac{3}{4}, \frac{5}{6}, \frac{7}{8}, \frac{11}{12}, \frac{15}{16}, \frac{23}{24}$.

The least common multiple of 2, 3, 4, 6, 8, 12, 16, and 24 is 48.

The multiplier for both terms of the first fraction is $\frac{48}{2} = 24$; for the second, $\frac{48}{3} = 16$; for the third, $\frac{48}{4} = 12$; for the fourth, $\frac{48}{6} = 8$; for the fifth, $\frac{48}{8} = 6$; for the sixth, $\frac{48}{12} = 4$; for the seventh, $\frac{48}{16} = 3$; and for the eighth, $\frac{48}{24} = 2$.

Multiplying by these numbers, we obtain $\frac{24}{48}, \frac{32}{48}, \frac{36}{48}, \frac{40}{48}, \frac{42}{48}, \frac{44}{48}, \frac{45}{48}$, and $\frac{46}{48}$.

(7)

$\frac{5}{7}, \frac{11}{12}, \frac{2}{15}, \frac{8}{27}, \frac{9}{35}, \frac{17}{40}$.

The least common multiple of 7, 12, 15, 27, 35 and 40 is 7560.

The multiplier for both terms of the first fraction is $\frac{7560}{7} = 1080$; for the second, $\frac{7560}{12} = 630$; for the third, $\frac{7560}{15} = 504$; for the fourth, $\frac{7560}{27} = 280$; for the fifth, $\frac{7560}{35} = 216$; for the sixth, $\frac{7560}{40} = 189$.

Multiplying by these numbers, we obtain $\frac{5400}{7560}, \frac{6930}{7560}, \frac{1008}{7560}, \frac{2240}{7560}, \frac{1944}{7560}$, and $\frac{3213}{7560}$.

(8)

$\frac{14}{15}, \frac{7}{8}, \frac{4}{3}, \frac{11}{12}, \frac{6}{11}, \frac{19}{20}, \frac{6}{7}, \frac{29}{35}$.

The least common multiple of 15, 8, 3, 12, 11, 20, 7, and 35 is 9240.

The multiplier for both terms of the first fraction is $\frac{9240}{15} = 616$; for the second, $\frac{9240}{8} = 1155$; for the third, $\frac{9240}{3} = 3080$; for the fourth, $\frac{9240}{12} = 770$; for the fifth, $\frac{9240}{11} = 840$; for the sixth, $\frac{9240}{20} = 462$; for the seventh, $\frac{9240}{7} = 1320$; for the eighth, $\frac{9240}{35} = 264$.

Multiplying by these numbers, we obtain $\frac{8624}{9240}, \frac{8085}{9240}, \frac{12320}{9240}, \frac{8470}{9240}, \frac{5040}{9240}, \frac{8778}{9240}, \frac{7920}{9240}$, and $\frac{7656}{9240}$.

EXERCISE 48—Page 160.

(1)

$$\tfrac{4}{7} \text{ of } \tfrac{3}{5} \text{ of } \tfrac{6}{11} \text{ of } \tfrac{35}{72} = \frac{4\times3\times6\times35}{7\times5\times11\times72} = \frac{2520}{27720} = \tfrac{1}{11}.$$

(2)

$$\tfrac{2}{3} \text{ of } \tfrac{4}{9} \text{ of } \tfrac{6}{7} \text{ of } \tfrac{81}{100} \text{ of } \tfrac{25}{24} = \frac{2\times4\times6\times81\times25}{3\times9\times7\times100\times24} = \frac{97200}{453600} = \tfrac{3}{14}.$$

(3)

$$\tfrac{21}{35} \text{ of } \tfrac{6}{11} \text{ of } \tfrac{77}{36} = \frac{21\times6\times77}{35\times11\times36} = \tfrac{7}{10}.$$

(4)

$$\tfrac{2}{5} \text{ of } \tfrac{4}{7} \text{ of } \tfrac{3}{11} \text{ of } \tfrac{13}{17} = \frac{2\times4\times3\times13}{5\times7\times11\times17} = \tfrac{312}{6545}.$$

EXERCISE 49—Page 161.

(1)

$$\tfrac{5}{9} \text{ of } \tfrac{6}{7} \text{ of } \tfrac{2}{3} \text{ of } \tfrac{3}{16} = \frac{5\times6\times2\times\ 3}{9\times7\times3\times16} = \frac{5\times\overset{2}{\cancel{6}}\times\cancel{2}\times\cancel{3}}{\underset{3}{\cancel{9}}\times7\times\cancel{3}\times\underset{\underset{4}{\cancel{8}}}{\cancel{16}}} = \frac{5}{3\times7\times4} = \tfrac{5}{84}$$

(2)

$$\tfrac{2}{3} \text{ of } \tfrac{5}{9} \text{ of } \tfrac{18}{132} \text{ of } \tfrac{6}{11} \text{ of } \tfrac{11}{13} \text{ of } \tfrac{13}{17} = \frac{2\times5\times\ 18\times\ 6\times11\times13}{3\times9\times132\times11\times13\times17} =$$

$$\frac{2\times5\ \times\overset{2}{\cancel{18}}\times\ \overset{2}{\cancel{6}}\times\cancel{11}\times\cancel{13}}{\cancel{3}\times\cancel{9}\times\underset{33}{\cancel{132}}\times\cancel{11}\times\cancel{13}\times17} = \frac{2\times5}{33\times17} = \tfrac{10}{561}.$$

(3)

$$\tfrac{2}{7} \text{ of } \tfrac{4}{11} \text{ of } 5\tfrac{1}{2} = \frac{2\times\ 4\times11}{7\times11\times\ 2} = \frac{\cancel{2}\times\ 4\times\cancel{11}}{7\times\cancel{11}\times\ \cancel{2}} = \tfrac{4}{7}.$$

(4)

$$\tfrac{1}{9} \text{ of } \tfrac{8}{13} \text{ of } \tfrac{117}{200} \text{ of } \tfrac{50}{169} \text{ of } \tfrac{13}{17} \text{ of } \tfrac{13}{6} = \frac{1\times\ 8\times117\times\ 50\times13\times13}{9\times13\times200\times169\times17\times\ 6} =$$

$$\frac{1\times\ \overset{2}{\cancel{8}}\times\overset{9}{\cancel{117}}\times\ \cancel{50}\times\cancel{13}\times\cancel{13}}{\cancel{9}\times\cancel{13}\times\underset{4}{\cancel{200}}\times\underset{\cancel{13}}{\cancel{169}}\times17\times\underset{3}{\cancel{6}}} = \frac{1}{17\times3} = \tfrac{1}{51}.$$

(5)

$$\tfrac{3}{11} \text{ of } \tfrac{4}{7} \text{ of } \tfrac{9}{19} \text{ of } \tfrac{33}{47} \text{ of } \tfrac{38}{72} \text{ of } \tfrac{47}{7} = \frac{3\times4\times\ 9\times33\times38\times17}{11\times7\times19\times47\times72\times\ 7} =$$

$$\frac{3\times\cancel{4}\times\ \cancel{9}\times\overset{3}{\cancel{33}}\times\overset{2}{\cancel{38}}\times\cancel{47}}{\cancel{11}\times7\times\cancel{19}\times\cancel{47}\times\underset{\underset{2}{\cancel{18}}}{\cancel{72}}\times\ 7} = \frac{3\times3}{7\times7} = \tfrac{9}{49}.$$

(6)

$$\frac{4}{7}\text{ of }\frac{3}{11}\text{ of }\frac{154}{1}=\frac{4\times3\times154}{7\times11\times\ 1}=\frac{4\times3\times\overset{\overset{2}{\cancel{14}}}{\cancel{154}}}{\cancel{7}\times\cancel{11}\times\ 1}=\frac{2\times4\times3}{1}=24.$$

EXERCISE 50—Page 162.

(1)

$$\frac{\frac{14}{45}}{1\frac{17}{25}}=\frac{\frac{14}{45}}{\frac{42}{25}}=\frac{14\times25}{45\times42}=\frac{\cancel{14}\times\overset{5}{\cancel{25}}}{\underset{9}{\cancel{45}}\times\underset{3}{\cancel{42}}}=\frac{5}{3\times9}=\frac{5}{27}$$

(2)

$$\frac{\frac{11}{12}}{7\frac{17}{18}}=\frac{\frac{11}{12}}{\frac{143}{18}}=\frac{11\times18}{12\times143}=\frac{\cancel{11}\times\overset{3}{\cancel{18}}}{\underset{2}{\cancel{12}}\times\underset{13}{\cancel{143}}}=\frac{3}{2\times13}=\frac{3}{26}$$

(3)

$$\frac{15\frac{3}{5}}{7\frac{4}{5}}=\frac{\frac{78}{5}}{\frac{39}{5}}=\frac{78\times\ 5}{5\ \times39}=\frac{\overset{2}{\cancel{78}}\times\ \cancel{5}}{\cancel{5}\ \times\cancel{39}}=\ 2.$$

(4)

$$\frac{11\frac{2}{3}}{12\frac{3}{5}},\ \frac{3\frac{1}{4}}{9},\ \frac{\frac{2}{7}}{\frac{3}{5}}=\frac{\frac{35}{3}}{\frac{68}{5}},\ \frac{\frac{13}{4}}{\frac{9}{1}}\ \frac{\frac{2}{7}}{\frac{3}{5}}=\frac{35\times\ 5}{3\ \times68},\ \frac{13\times1}{9\ \times4},\ \frac{2\times5}{7\times3}=\frac{175}{204},\ \frac{13}{36},\ \frac{10}{21}.$$

(5)

$$\frac{\frac{7}{12}}{15\frac{3}{4}},\ \frac{5\frac{7}{8}}{\frac{3}{16}},\ \frac{2\frac{2}{5}}{3\frac{3}{7}}=\frac{\frac{7}{12}}{\frac{63}{4}},\ \frac{\frac{47}{8}}{\frac{3}{16}},\ \frac{\frac{12}{5}}{\frac{24}{7}}=\frac{\cancel{7}\times\cancel{4}}{\underset{3}{\cancel{12}}\times\underset{9}{\cancel{63}}},\ \frac{47\times\overset{2}{\cancel{16}}}{\cancel{8}\ \times\ 3},\ \frac{\cancel{12}\times\ 7}{5\ \times\underset{2}{\cancel{24}}}=$$

$$\frac{1}{3\times6},\ \frac{47\times2}{3},\ \frac{7}{5\times2},\ =\frac{1}{27},\ 31\tfrac{1}{3},\ \frac{7}{10}.$$

(6)

$$\frac{16\frac{2}{3}}{11\frac{2}{3}},\ \frac{6\frac{1}{5}}{13},\ \frac{17}{18\frac{1}{3}},\ \frac{21\frac{3}{5}}{10\frac{2}{7}},\ \frac{\frac{1}{2}}{4\frac{3}{5}} = \frac{\frac{50}{3}}{\frac{35}{3}},\ \frac{\frac{31}{5}}{\frac{13}{1}},\ \frac{\frac{17}{1}}{\frac{55}{3}},\ \frac{\frac{108}{5}}{\frac{72}{7}},\ \frac{\frac{1}{2}}{\frac{23}{5}} = \frac{\overset{10}{\cancel{50}}\times\cancel{3}}{\cancel{3}\times\underset{7}{\cancel{35}}},$$

$$\frac{31\times1}{5\times13},\ \frac{17\times3}{55\times1},\ \frac{\overset{3}{\cancel{108}}\times7}{\underset{2}{\cancel{72}}\times5},\ \frac{1\times5}{2\times23}, = \frac{10}{7},\ \frac{31}{65},\ \frac{51}{55},\ \frac{21}{10},\ \frac{5}{46} = 1\tfrac{3}{7}, \tfrac{31}{65}, \tfrac{51}{55}, 2\tfrac{1}{10}, \tfrac{5}{46}.$$

EXERCISE 51—Page 163.

(1)

$$\frac{\cancel{4}}{5} \text{ of } \frac{1}{\underset{4}{\cancel{16}}} = \tfrac{1}{20} \text{ of a lb.}$$

(2)

$$\frac{\cancel{2}}{\cancel{3}} \text{ of } \frac{\cancel{3}}{7} \text{ of } \frac{1}{\underset{6}{\cancel{12}}} \text{ of } \frac{1}{20} = \frac{1}{7\times6\times20} = £\tfrac{1}{840}.$$

(3)

$$\frac{\cancel{2}}{9} \text{ of } \frac{\overset{5}{\cancel{35}}}{\underset{2}{\cancel{4}}} \text{ of } \frac{1}{\cancel{7}} = \frac{5}{9\times2} = \tfrac{5}{18} \text{ wk.}$$

(4)

$$\frac{\cancel{5}}{11} \text{ of } \frac{81}{\cancel{5}} \text{ of } \frac{1}{4} \text{ of } \frac{1}{5} = \frac{81}{11\times4\times5} = \tfrac{81}{220} \text{ Eng. Ell.}$$

(5)

$$\frac{3}{7} \text{ of } \frac{4}{11} \text{ of } \frac{1}{5\frac{1}{2}} = \frac{3}{7} \text{ of } \frac{4}{11} \text{ of } \frac{2}{11} = \tfrac{24}{847} \text{ per.}$$

(6)

$$\frac{2}{3}\text{ of }\frac{4}{7}\text{ of }21\frac{1}{14}\text{ of }\frac{1}{8}=\frac{\cancel{2}\times\cancel{4}\times 295\times 1}{3\times 7\times 14\times\underset{2}{\cancel{8}}}=\frac{295}{294}=1\tfrac{1}{294}\text{ a.}$$

(7)

$$\frac{3}{19}\text{ of }\frac{4}{17}\text{ of }9\frac{1}{2}\text{ of }\frac{1}{40}\text{ of }\frac{1}{4}=\frac{3\times\cancel{4}\times\cancel{19}\times 1\times 1}{\cancel{19}\times 17\times 2\times 40\times\cancel{4}}=\frac{3}{17\times 2\times 40}=\tfrac{3}{1360}\text{ a.}$$

EXERCISE 52—Page 164.

(1)

$$\frac{14}{79}\text{ of }\frac{4}{1}\text{ of }\frac{2}{1}\text{ of }\frac{4}{1}=\tfrac{448}{79}\text{ qt.}$$

(2)

$$\frac{2}{\underset{3}{\cancel{9}}}\text{ of }\frac{4}{1}\times\frac{\cancel{2}}{1}\times\frac{4}{1}\times\frac{5}{1}\times\frac{\cancel{3}}{\cancel{2}}=\frac{2\times 4\times 4\times 5}{3}=\tfrac{160}{3}$$

(3)

$$\frac{7}{\underset{3}{\cancel{9}}}\times\frac{2}{1}\times\frac{2}{1}\times\frac{4}{1}\times\frac{2}{1}\times\frac{\cancel{2}}{1}\times\frac{\cancel{3}}{\cancel{2}}=\frac{7\times 2\times 2\times 4\times 2}{3}=\tfrac{224}{3}.$$

(4)

$$\frac{17}{\underset{11}{\cancel{22}}}\times\frac{\overset{6}{\cancel{12}}}{1}\times\frac{8}{1}\times\frac{3}{1}=\frac{17\times 6\times 8\times 3}{11}=\tfrac{2448}{11}\text{ scr.}$$

(5)

$$\frac{1}{\underset{625}{\cancel{5000}}}\times\frac{2}{\cancel{3}}\times\frac{\cancel{3}}{\cancel{4}}\times\frac{6}{\cancel{11}}\times\frac{\overset{2}{\cancel{22}}}{7}\times\frac{\overset{2}{\cancel{16}}}{1}\times\frac{\overset{4}{\cancel{16}}}{1}=\frac{2\times 6\times 2\times 2\times 4}{625\times 7}=\tfrac{192}{4375}\text{ dr.}$$

EXERCISE 53—Page 164.

(1)

bush. pk. gal. qt. pt.
11)3 0 0 0 0
——————
1 0 0 $1\frac{5}{11}$

lbs. oz. dr.
7)6 0 0
——————
13 $11\frac{3}{7}$

(2)
yds. qr. na. in,
13) 7 (2 0 $1\frac{5}{13}$
4
——
28 qrs
26
——
2
4
——
8 na.
$2\frac{1}{4}$
——
18
13
——
5

(4)
fur. per. yds. ft. in.
9)8 0 0 0 0
——————
35 3 0 2

£ s. d.
7)4 0 0
——————
11 $5\frac{1}{7}$

(3)

lbs. oz. dwt. grs.
9)8 0 0 0
——————
10 13 8

sq. m. a. r. pr. yds. ft. in.
113) 11 (62 1 8 4 2 $79\frac{1}{113}$
640
——
7040 a.
678
——
260
226
——
34
4
——
136 r.
113
——
23
40
——
920 per.
904
——
16
$30\frac{1}{4}$
——
480
4
——
484 yds.

484 yds
452
——
32
9
——
288 ft.
226
——
62
144
——
248
248
62
——
8928 in.
791
——
1018
1017
——
1

EXERCISE 54—Page 165.

(1)

6 bus. 1 pk. 1 gal. 1 qt. 1 pt. = 411 pts.
50 bush. = 3200 pts.
And the required fraction is $\frac{411}{3200}$.

(2)

35 per. 9 ft. 2 in. = 7040 in.
1 fur. = 7920 in.
The required fraction is $\frac{7040}{7920} = \frac{88}{99} = \frac{8}{9}$.

(3)

7 hrs. 12 min. = 432 min.
1 day = 1440 min.
Therefore the fraction is $\frac{432}{1440} = \frac{3}{10}$.

(4

2 sq. yds. 2 ft. 120 in. = 3000 in.
sq. per. 13¼ yds. 1 ft. 72 in. = 135000 in
And the fraction is $\frac{3000}{135000} = \frac{1}{45}$.

(5)

7 oz. 7 drs. 2 scr. 14 grs. = 3834 grs.
21 lbs. = 120960 grs.
The fraction is $\frac{3834}{120960} = \frac{426}{13440} = \frac{71}{2240}$.

(6)

9 min. 48 sec. = 588 sec.
1 day = 86400 sec.
The required fraction is $\frac{588}{86400} = \frac{49}{7200}$.

(7)

16 bush. 1 pk. 1 pt. = 1041 pts.
69 bush. = 4416 pts.
Therefore the fraction is $\frac{1041}{4416} = \frac{347}{1472}$.

(8)

3 qrs. $3\frac{1}{9}$ na. $= 15\frac{1}{9} = \frac{136}{9}$ na.
1 Eng. ell $=$ 20 na.

And the fraction is $\dfrac{\frac{136}{9}}{20} = \frac{136}{180} = \frac{34}{45}$.

(9)

13 dwt. 7 grs. $=$ 319 grs.
1 lb. Troy $=$ 5760 grs.
The required fraction is $\frac{319}{5760}$.

(10)

4800 cub. ft.
54 cords $=$ 6912 cub. ft.
Therefore the fraction is $\frac{4800}{6912} = \frac{400}{576} = \frac{50}{72} = \frac{25}{36}$.

EXERCISE 55—Page 167.

(1)

$$\frac{11}{13} + \frac{10}{13} + \frac{9}{13} = \frac{30}{13} = 2\frac{4}{13}.$$

(2)

$$\frac{1}{12} + \frac{6}{12} + \frac{7}{12} + \frac{9}{12} + \frac{11}{12} + \frac{5}{12} = \frac{39}{12} = 3\frac{3}{12} = 3\frac{1}{4}.$$

(3)

$$4\frac{3}{7} + 11\frac{1}{7} + 16\frac{2}{7} + 21\frac{3}{7} + 19\frac{6}{7} = 4 + 11 + 16 + 21 + 19 + \left(\frac{3}{7} + \frac{1}{7} + \frac{2}{7} + \frac{3}{7} + \frac{6}{7}\right) = 71 + \frac{15}{7} = 73\frac{1}{7}.$$

(4)

$$16\frac{21}{23} + 11\frac{17}{23} + 18\frac{4}{23} + 17\frac{19}{23} + 112\frac{22}{23} = 16 + 11 + 18 + 17 + 112 + \left(\frac{21}{23} + \frac{17}{23} + \frac{4}{23} + \frac{19}{23} + \frac{22}{23}\right) = 174 + \frac{83}{23} = 174 + 3\frac{14}{23} = 177\frac{14}{23}$$

(5)

$$4\frac{1}{4} + 1\frac{1}{3} + \frac{7}{11} = 4 + 1 + \left(\frac{1}{4} + \frac{1}{3} + \frac{7}{11}\right) = 5 + \left(\frac{33}{132} + \frac{44}{132} + \frac{84}{132}\right) = 5 + \frac{161}{132} = 6\frac{29}{132}.$$

(6)

$$\frac{1}{2} + \frac{2}{3} + \frac{3}{4} + \frac{4}{5} + \frac{5}{6} + \frac{6}{7} + \frac{7}{8} + \frac{8}{9}.$$

These fractions reduced to their least common denominator become $\frac{1260}{2520} + \frac{1680}{2520} + \frac{1890}{2520} + \frac{2016}{2520} + \frac{2160}{2520} + \frac{2160}{2520} + \frac{2205}{2520} + \frac{2240}{2520} = \frac{15551}{2520} = 6\frac{431}{2520}$.

(7)

$\frac{3}{4} + \frac{5}{6} + \frac{4}{5}$ when reduced to their least common denominator become $\frac{45}{60} + \frac{50}{60} + \frac{48}{60} = \frac{143}{60} = 2\frac{23}{60}$.

(8)

$\frac{4}{5} + \frac{5}{6} + \frac{6}{7} + \frac{7}{8} + \frac{8}{11}$.

These fractions when reduced to their least common denominator become $\frac{7392}{9240} + \frac{7700}{9240} + \frac{7920}{9240} + \frac{8465}{9240} + \frac{6720}{9240} = \frac{33187}{9240} = 3\frac{5467}{9240}$.

(9)

$\frac{1}{2} + \frac{1}{3} + \frac{1}{4} + \frac{1}{5} + \frac{1}{6} + \frac{1}{7}$.

These fractions when reduced to their least common denominator become $\frac{210}{420} + \frac{140}{420} + \frac{105}{420} + \frac{84}{420} + \frac{70}{420} + \frac{60}{420} = \frac{669}{420} = \frac{223}{140} = 1\frac{83}{140}$.

(10)

$16\frac{3}{11} + 47\frac{2}{9} + 21\frac{17}{33} + \frac{7}{18} + 19\frac{1}{2} = 16 + 47 + 21 + 19 \times (\frac{3}{11} + \frac{2}{9} + \frac{17}{33} + \frac{7}{18} + \frac{1}{2})$.

$16 + 47 + 21 + 19 = 103$.

$\frac{3}{11} + \frac{2}{9} + \frac{17}{33} + \frac{7}{18} + \frac{1}{2} = \frac{486}{1782} + \frac{396}{1782} + \frac{918}{1782} + \frac{693}{1782} + \frac{891}{1782} = \frac{3384}{1782} = \frac{376}{198} = \frac{188}{99} = 1\frac{89}{98}$.

$103 + 1\frac{89}{99} = 104\frac{89}{99}$.

(11)

$17\frac{1}{2} + 43\frac{3}{7} + 168\frac{4}{9} + 207\frac{8}{21} + 506\frac{125}{126} = 17 + 43 + 168 + 207 + 506 + (\frac{1}{2} + \frac{3}{7} + \frac{4}{9} + \frac{8}{21} + \frac{125}{126})$.

$17 + 43 + 168 + 207 + 506 = 941$.

$\frac{1}{2} + \frac{3}{7} + \frac{4}{9} + \frac{8}{21} + \frac{125}{126} = \frac{63}{126} + \frac{54}{126} + \frac{56}{126} + \frac{48}{126} + \frac{125}{126} = \frac{346}{126} = \frac{173}{63} = 2\frac{47}{63}$.

$941 + 2\frac{47}{63} = 943\frac{47}{63}$

(12)

$6\frac{3}{4} + 11\frac{4}{7} + \frac{9}{56} + 16\frac{7}{16} + \frac{1}{2} + \frac{5}{21} + 17\frac{11}{12} = 6 + 11 + 16 + 17 + (\frac{3}{4} + \frac{4}{7} + \frac{9}{56} + \frac{7}{16} + \frac{1}{2} + \frac{5}{21} + \frac{11}{12})$.

$6 + 11 + 16 + 17 = 50$.

$\frac{3}{4} + \frac{4}{7} + \frac{9}{56} + \frac{7}{16} + \frac{1}{2} + \frac{5}{21} + \frac{11}{12} = \frac{252}{336} + \frac{192}{336} + \frac{54}{336} + \frac{147}{336} + \frac{168}{336} + \frac{80}{336} + \frac{308}{336} = \frac{1201}{336} = 3\frac{193}{336}$.

$50 + 3\frac{193}{336} = 53\frac{193}{336}$.

(13)

$\frac{1}{5} + \frac{2}{3} + \frac{7}{9} + 68\frac{1}{4} = 68 + (\frac{1}{5} + \frac{2}{3} + \frac{7}{9} + \frac{1}{4})$.
$\frac{1}{5} + \frac{2}{3} + \frac{7}{9} + \frac{1}{4} = \frac{36}{180} + \frac{120}{180} + \frac{140}{180} + \frac{45}{180} = \frac{341}{180} = 1\frac{161}{180}$.
$68 + 1\frac{161}{180} = 69\frac{161}{180}$.

(14)

$173\frac{3}{16} + 8\frac{5}{7} + 91\frac{11}{12} = 173 + 8 + 91 + (\frac{3}{16} + \frac{5}{7} + \frac{11}{12})$.
$173 + 8 + 91 = 272$.
$\frac{3}{16} + \frac{5}{7} + \frac{11}{12} = \frac{63}{336} + \frac{240}{336} + \frac{308}{336} = \frac{611}{336} = 1\frac{275}{336}$.
$272 + 1\frac{275}{336} = 273\frac{275}{336}$.

(15)

$1\frac{15}{16} + 2\frac{23}{24} + 3\frac{24}{25} + 4\frac{29}{30} = 1 + 2 + 3 + 4 + (\frac{15}{16} + \frac{23}{24} +$
$\frac{24}{25} + \frac{29}{30})$.
$1 + 2 + 3 + 4 = 10$.
$\frac{15}{16} + \frac{23}{24} + \frac{24}{25} + \frac{29}{30} = \frac{6750}{7200} + \frac{6900}{7200} + \frac{6912}{7200} + \frac{6960}{7200} = \frac{27522}{7200} =$
$\frac{1529}{400} = 3\frac{329}{400}$.
$10 + 3\frac{329}{400} = 13\frac{329}{400}$.

(16)

$\frac{1}{8} + \frac{3}{12} + \frac{4}{48} + \frac{5}{24} + \frac{7}{16} + \frac{2}{3} + \frac{1}{2} + \frac{5}{6} = \frac{6}{48} + \frac{12}{48} + \frac{4}{48} +$
$\frac{10}{48} + \frac{21}{48} + \frac{32}{48} + \frac{24}{48} + \frac{40}{48} = \frac{149}{48} = 3\frac{5}{48}$.

(17)

$7 + 11\frac{1}{2} + 18 + 26\frac{3}{7} + 79\frac{4}{11} = 7 + 11 + 18 + 26 + 79 +$
$(\frac{1}{2} + \frac{3}{7} + \frac{4}{11})$.
$7 + 11 + 18 + 26 + 79 = 141$.
$\frac{1}{2} + \frac{3}{7} + \frac{4}{11} = \frac{77}{154} + \frac{66}{154} + \frac{56}{154} = \frac{199}{154} = 1\frac{45}{154}$.
$141 + 1\frac{45}{154} = 142\frac{45}{154}$.

(18)

$\frac{4}{5}$ of $\frac{3}{7}$ of $\frac{21}{2} = \frac{18}{5} = 3\frac{3}{5}$. $\frac{2}{3} + 7\frac{2}{11} + 3\frac{3}{5} = 10 + (\frac{2}{3} + \frac{2}{11} + \frac{3}{5})$.
$\frac{2}{3} + \frac{2}{11} + \frac{3}{5} = \frac{110}{165} + \frac{30}{165} + \frac{99}{165} = \frac{239}{165} = 1\frac{74}{165}$.
$10 + 1\frac{74}{165} = 11\frac{74}{165}$.

(19)

$$\frac{4\frac{1}{3}}{\frac{7}{18}} = \frac{\frac{13}{3}}{\frac{7}{18}} = \frac{13 \times 18}{3 \times 7} = \frac{78}{7} = 11\tfrac{1}{7}.$$

$$\tfrac{1}{2} \text{ of } \tfrac{36}{11} \text{ of } \tfrac{4}{15} \text{ of } \tfrac{11}{4} = \frac{1\times36\times4\times11}{2\times11\times15\times4} = \tfrac{18}{15} = 1\tfrac{3}{15}.$$

$$\frac{20\frac{3}{4}}{7\frac{6}{11}} = \frac{\frac{83}{4}}{\frac{83}{11}} = \frac{83 \times 11}{4 \times 83} = \tfrac{11}{4} = 2\tfrac{3}{4}.$$

$11\frac{1}{7} + 1\frac{3}{15} + 2\frac{3}{4} = 11 + 1 + 2 + (\frac{1}{7} + \frac{3}{15} + \frac{3}{4}) = 14 + (\frac{1}{7} +$
$\frac{3}{15} + \frac{3}{4})$.
$\frac{1}{7} + \frac{3}{15} + \frac{3}{4} = \frac{60}{420} + \frac{84}{420} + \frac{315}{420} = \frac{459}{420} = \frac{153}{140} = 1\frac{13}{140}$.
$14 + 1\frac{13}{140} = 15\frac{13}{140}$.

(20)

$3\frac{5}{8} + 11\frac{1}{6} + 14\frac{33}{48} = 3 + 11 + 14 + (\frac{5}{8} + \frac{1}{6} + \frac{33}{48}) = 28 +$
$(\frac{5}{8} + \frac{1}{6} + \frac{33}{48})$.
$\frac{5}{8} + \frac{1}{6} + \frac{33}{48} + \frac{30}{48} + \frac{8}{48} + \frac{33}{48} = \frac{71}{48} = 1\frac{23}{48}$. $28 + 1\frac{23}{48} = 29\frac{23}{48}$.

(21)

$\frac{1}{2}$ of $\frac{3}{4} = \frac{3}{8}$, $\frac{2}{3}$ of $\frac{6}{7} = \frac{4}{7}$, $\frac{3}{5}$ of $\frac{7}{9} = \frac{7}{15}$, $\frac{2}{9}$ of $\frac{27}{20} = \frac{3}{10}$, $\frac{9}{2}$ of $\frac{1}{3}$ of $\frac{1}{3}$
of $\frac{1}{3}$ of $\frac{1}{6}$ of $\frac{1}{3} = \frac{3}{80}$.
$\frac{3}{8} + \frac{4}{7} + \frac{7}{15} + \frac{3}{10} + \frac{3}{80} = \frac{630}{1680} + \frac{960}{1680} + \frac{784}{1680} + \frac{504}{1680} +$
$\frac{63}{1680} = \frac{2941}{1680} = 1\frac{1261}{1680}$.

(22)

$41\frac{1}{2} + 105\frac{2}{9} + 300\frac{3}{4} + 241\frac{3}{5} + 472\frac{1}{4} = 41 + 105 + 300 + 241$
$= 472 + (\frac{1}{2} + \frac{2}{9} + \frac{3}{4} + \frac{3}{5} + \frac{1}{4})$.
$41 + 105 + 300 + 241 + 472 = 1159$.
$\frac{1}{2} + \frac{2}{9} + \frac{3}{4} + \frac{3}{5} + \frac{1}{4} = \frac{90}{180} + \frac{40}{180} + \frac{135}{180} + \frac{108}{180} + \frac{45}{180} = \frac{418}{180} =$
$\frac{209}{90} = 2\frac{29}{90}$.
$1159 + 2\frac{29}{90} = 1161\frac{29}{90}$.

(23)

$92\frac{5}{14} + 37\frac{8}{19} + 7\frac{2}{3} = 92 + 37 + 7 + (\frac{5}{14} + \frac{8}{19} + \frac{2}{3}) = 136 +$
$(\frac{5}{14} + \frac{8}{19} + \frac{2}{3})$.
$\frac{5}{14} + \frac{8}{19} + \frac{2}{3} = \frac{285}{798} + \frac{336}{798} + \frac{532}{798} = \frac{1153}{798} = 1\frac{355}{798}$.
$136 + 1\frac{355}{798} = 137\frac{355}{798}$.

(24)

$$\frac{10\frac{3}{5}}{2\frac{2}{5}} = \frac{\frac{53}{5}}{\frac{12}{5}} = \frac{53\times 5}{5\times 12} = \frac{53}{12} = 4\frac{5}{12} \qquad \frac{2}{3} \text{ of } \frac{7}{8} = \frac{7}{12}.$$

$21\frac{1}{3} + 35\frac{1}{2} + 4\frac{5}{12} + \frac{7}{12} = 21 + 35 + 5 + (\frac{1}{3} + \frac{1}{2}) = 61\frac{5}{6}$.

(25)

$\frac{11}{4}$ of $\frac{11}{3} = \frac{121}{12} = 10\frac{1}{12}$. $\frac{14}{5}$ of $\frac{33}{8}$ of $\frac{11}{8} = \frac{2541}{160} = 15\frac{141}{160}$.
$\frac{101}{16} = 6\frac{5}{16}$. $\frac{14}{3}$ of $\frac{2}{15}$ of $\frac{17}{8}$ of $\frac{10}{7} = \frac{17}{9} = 1\frac{8}{9}$.
$10\frac{1}{12} + 6\frac{5}{16} + 15\frac{141}{160} + 1\frac{8}{9} = 10 + 6 + 15 + 1 + (\frac{1}{12} + \frac{5}{16} + \frac{141}{160} + \frac{8}{9}) = 32 + (\frac{1}{12} + \frac{5}{16} + \frac{141}{160} + \frac{8}{9})$.
$\frac{1}{12} + \frac{5}{16} + \frac{141}{160} + \frac{8}{9} = \frac{840}{10080} + \frac{3150}{10080} + \frac{8883}{10080} + \frac{8960}{10080} = \frac{21833}{10080} = \frac{3119}{1440} = 2\frac{239}{1440}$.
$32 + 2\frac{239}{1440} = 34\frac{239}{1440}$.

EXERCISE 56—Page 169.

(1)

	oz.	dr.	scr.	grs.
$\frac{4}{11}$ of a lb. =	4	2	2	$14\frac{6}{11}$
$\frac{3}{7}$ of an oz. =		3	1	$5\frac{5}{7}$
$\frac{4}{11}$ of a dr. =			1	$1\frac{9}{11}$
$\frac{5}{6}$ of a scr. =				$16\frac{2}{3}$
	4	6	2	$18\frac{172}{231}$

(2)

	qr.	na.	in.
$\frac{3}{5}$ of a yard =	2	1	$1\frac{7}{20}$
$\frac{1}{7}$ of an Eng. ell. =		2	$1\frac{13}{14}$
$\frac{6}{7}$ of a qr. =		3	$0\frac{27}{28}$
	3	3	$1\frac{139}{140}$

(3)

	in.
$\frac{1}{7}$ of a yd. =	$5\frac{1}{7}$
$\frac{1}{7}$ of a ft. =	$1\frac{5}{7}$
$\frac{1}{7}$ of an in. =	$\frac{1}{7}$
	7

(4)

	fur.	per.	yds.	ft.	in.
$\frac{7}{11}$ of a mile =	5	3	3	1	6
$\frac{4}{13}$ of a fur. =		12	1	2	$0\frac{12}{13}$
$\frac{9}{22}$ of a yd. =				1	$2\frac{8}{11}$
	5	16	0	0	$3\frac{93}{143}$

(5)

	day	hrs.	min.
$\frac{1}{4}$ of a week =	1	18	0
$\frac{1}{3}$ of a day =		8	0
$\frac{1}{5}$ of an hour =			12
	2	2	12

(6)

	s.	d.
$\frac{1}{7}$ of a £ =	2	$10\frac{2}{7}$
$\frac{2}{9}$ of a s. =		$2\frac{2}{3}$
$\frac{5}{12}$ d. =		$\frac{5}{12}$
	3	$1\frac{31}{84}$

(7)

		£	s.	d.
$\frac{5}{8}$ of 21s.	=		13	$1\frac{1}{2}$
$\frac{5}{8}$ of 5s.	=		3	$1\frac{1}{2}$
$\frac{5}{8}$ of £3 12s. 6d.	=	2	5	$3\frac{3}{4}$
$\frac{7}{13}$ of a £	=		10	$9\frac{3}{13}$
[illegible]	=			[illegible]
		3	12	4[illegible]

EXERCISE 57—Page 171.

(1)

$$\frac{3}{4}-\frac{7}{20}=\frac{15}{20}-\frac{7}{20}=\frac{8}{20}=\frac{2}{5}$$

(2)

$$\frac{7}{17} \text{ of } \frac{9}{14} \text{ of } \frac{32}{11} \text{ of} =\frac{3\times 48}{17\times 11}=\frac{144}{187}. \quad \frac{905}{1496}+\frac{144}{187}=\frac{905}{1496}+\frac{1152}{1496}=\frac{2057}{1496}=1\frac{561}{1496}=1\frac{3}{8}.$$

$$\frac{8\frac{3}{4}}{6\frac{4}{11}}=\frac{\frac{35}{4}}{\frac{70}{11}}=\frac{35\times 11}{4\times 70}=\frac{11}{4\times 2}=\frac{11}{8}=1\frac{3}{8},$$

$$1\frac{3}{8}-1\frac{3}{8}=0.$$

(3)

$$982\frac{17}{87}-29\frac{19}{20}=982\frac{340}{1740}-29\frac{1653}{1740}=981+1\frac{340}{1740}-29\frac{1653}{1740}=981\frac{2080}{1740}-29\frac{1653}{1740}=952\frac{427}{1740}.$$

(4)

$$69\frac{1}{51}-18\frac{86}{146}=69\frac{146}{7446}-68\frac{4386}{7446}=68+1\frac{146}{7446}-18\frac{4386}{7446}=68\frac{7592}{7446}-18\frac{4386}{7446}=50\frac{3206}{7446}=50\frac{1603}{3723}.$$

(5)

$$100\frac{1}{2}-9\frac{5}{8}=100\frac{4}{8}-9\frac{5}{8}=99+1\frac{4}{8}-9\frac{5}{8}=99\frac{12}{8}-9\frac{5}{8}=90\frac{7}{8}$$

(6)

$$\frac{1}{2} \text{ of } \frac{37}{4}=\frac{37}{8}=4\frac{5}{8}. \quad 6\frac{1}{4}-4\frac{5}{8}=6\frac{2}{8}-4\frac{5}{8}=5+1\frac{2}{8}-4\frac{5}{8}=5\frac{10}{8}-4\frac{5}{8}=1\frac{5}{8}.$$

(7)

$$611\frac{43}{191}-610\frac{198}{199}=611\frac{8557}{38009}-610\frac{37818}{38009}=610+1\frac{8557}{38009}-610\frac{37818}{38009}=610\frac{46566}{38009}-610\frac{37818}{38009}=\frac{8748}{38009}.$$

(8)

$\frac{5}{6}$ of $\frac{4}{3}=\frac{10}{9}$. $\frac{1}{5}+\frac{1}{9}=\frac{9}{45}+\frac{5}{45}=\frac{14}{45}$. $\frac{6}{7}$ of $\frac{14}{45}=\frac{4}{15}$.
$\frac{10}{9}-\frac{4}{15}=\frac{50}{45}-\frac{12}{45}=\frac{38}{45}$.

(9)

	oz.	dr.
$\frac{2}{3}$ of a lb. =	10	$10\frac{2}{3}$
$\frac{5}{6}$ of a dr. =		$\frac{5}{6}$
	10	$9\frac{5}{6}$

(10)

$24\frac{1}{24}-21\frac{1}{21}=24\frac{7}{168}-21\frac{8}{168}=$
$23+1\frac{7}{168}-21\frac{8}{168}=23\frac{175}{168}-$
$21\frac{8}{168}=2\frac{167}{168}$.

(11)

	fur.	per.	yds.	ft.	in.
$\frac{2}{9}$ of a mile =	1	31	0	1	10
$\frac{7}{11}$ of a fur. =		25	2	1	6
	1	5	3	1	10

(12)

$\frac{3}{5}$ of $\frac{25}{2}=\frac{15}{2}=7\frac{1}{2}$. $\frac{1}{16}$ of $\frac{57}{2}=\frac{57}{32}=1\frac{25}{32}$.
$7\frac{1}{2}-1\frac{25}{32}=7\frac{16}{32}-1\frac{25}{32}=6+1\frac{16}{32}-1\frac{25}{32}=6\frac{48}{32}-1\frac{25}{32}=5\frac{23}{32}$.

(13)

$$\frac{1}{2}\text{ of }\frac{3}{7}\text{ of }\frac{2}{9}\text{ of }\frac{33}{4}\text{ of }\frac{\frac{62}{5}}{\frac{33}{6}}=\frac{1\times3\times2\times33\times62\times5}{2\times7\times9\times\ 4\times33\times6}=\frac{155}{252}.$$

$12\frac{319}{1764}+\frac{155}{252}=12\frac{319}{1764}+\frac{1085}{1764}=12\frac{1404}{1764}=12\frac{351}{441}=12\frac{39}{49}$.

$$\frac{17\frac{9}{11}}{1\frac{23}{33}}=\frac{\frac{196}{11}}{\frac{56}{33}}=\frac{196\times33}{11\times56}=\frac{21}{2}=10\frac{1}{2}.$$

$12\frac{39}{49}-10\frac{1}{2}=12\frac{78}{98}-10\frac{49}{98}=2\frac{29}{98}$.

(14)

$3\frac{1}{12}+8\frac{1}{9}+5\frac{1}{5}+6\frac{1}{2}=3+8+5+6+(\frac{1}{12}+\frac{1}{9}+\frac{1}{5}+\frac{1}{2})=$
$22+(\frac{1}{12}+\frac{1}{9}+\frac{1}{5}+\frac{1}{2})$.
$\frac{1}{12}+\frac{1}{9}+\frac{1}{5}+\frac{1}{2}=\frac{15}{180}+\frac{20}{180}+\frac{36}{180}+\frac{90}{180}=\frac{161}{180}$.
$22+\frac{161}{180}=22\frac{161}{180}$.
$3\frac{3}{10}+2\frac{5}{6}+16\frac{1}{4}=3+2+16+(\frac{3}{10}+\frac{5}{6}+\frac{1}{4})=21+$
$(\frac{3}{10}+\frac{5}{6}+\frac{1}{4})$.
$\frac{3}{10}+\frac{5}{6}+\frac{1}{4}=\frac{18}{60}+\frac{50}{60}+\frac{15}{60}=\frac{83}{60}=1\frac{23}{60}$. $21+1\frac{23}{60}=22\frac{23}{60}$.
$22\frac{161}{180}-22\frac{23}{60}=22\frac{161}{180}-22\frac{69}{180}=\frac{92}{180}=\frac{23}{45}$.

(15)

		r.	per.	yds.	ft.	in.
$\frac{4}{11}$ of an acre	=	1	18	5	4	72
$\frac{1}{3}$ of a per.	=			13	4	
		1	17	22	2	108

(16)

$16\frac{1}{7} - 9\frac{14}{19} = 16\frac{19}{133} - 9\frac{98}{133} = 15 + 1\frac{19}{133} - 9\frac{98}{133} = 15\frac{152}{133} -$
$9\frac{98}{133} = 6\frac{54}{133}$.

$169\frac{17}{100} - 83\frac{17}{26} = 169\frac{221}{1300} - 83\frac{850}{1300} = 168 + 1\frac{221}{1300} -$
$83\frac{850}{1300} = 168\frac{1521}{1300} - 83\frac{850}{1300} = 85\frac{671}{1300}$.

EXERCISE 58—Page 173.

(1)

$$\tfrac{7}{12} \text{ of } \tfrac{5}{6} = \frac{7\times 5}{12\times 6} = \tfrac{35}{72}.$$

(2)

$$\frac{\cancel{5}}{\underset{2}{\cancel{8}}} \times \frac{\cancel{4}}{\cancel{5}} = \tfrac{1}{2}.$$

(3)

$$\frac{\cancel{4}}{\underset{3}{\cancel{15}}} \times \frac{\cancel{5}}{\underset{6}{\cancel{24}}} = \tfrac{1}{18}.$$

(4)

$$\tfrac{7}{8} \times \tfrac{5}{6} \times \tfrac{7}{16} = \tfrac{245}{768}.$$

(5)

$$\frac{14}{1} \times \frac{241}{\cancel{16}} \times \frac{\overset{2}{\cancel{32}}}{9} = \frac{14\times 241\times 2}{9} = \tfrac{6748}{9} = 749\tfrac{7}{9}$$

(6)

$$\frac{\overset{3}{\cancel{9}}}{\underset{2}{\cancel{10}}} \times \frac{\overset{7}{\cancel{35}}}{4} \times \frac{9}{\cancel{11}} \times \frac{\cancel{11}}{\underset{4}{\cancel{12}}} = \frac{3\times 7\times 9}{2\times 4\times 4} = \tfrac{189}{32} = 5\tfrac{29}{32}.$$

(7)

$$\frac{\cancel{4}}{\cancel{5}} \times \frac{\overset{3}{\cancel{6}}}{11} \times \frac{\cancel{2}}{17} \times \frac{182}{\underset{\underset{25}{\cancel{50}}}{\cancel{200}}} \times \frac{\cancel{5}}{\cancel{4}} = \frac{3\times 182}{11\times 17\times 25} = \tfrac{546}{4675}.$$

(8)

$$\frac{\overset{3}{\cancel{6}}}{\cancel{7}}\times\frac{11}{\underset{\underset{2}{\cancel{4}}}{\cancel{8}}}\times\frac{\overset{3}{\cancel{6}}}{\underset{\cancel{3}}{\cancel{33}}}\times\frac{\overset{3}{\cancel{21}}}{\cancel{1}}\times\frac{\cancel{3}}{\cancel{5}}\times\frac{\cancel{5}}{1}=\frac{3\times3\times3}{2}=\tfrac{27}{2}=13\tfrac{1}{2}.$$

(9)

$$\frac{\overset{2}{\cancel{6}}}{\underset{\cancel{3}}{\cancel{9}}}\times\frac{\cancel{3}}{5}\times\frac{6}{11}\times\frac{4}{\cancel{19}}\times\frac{\overset{11}{\cancel{209}}}{1}=\frac{2\times6\times4}{5}=\tfrac{48}{5}=9\tfrac{3}{5}.$$

(10)

$$\frac{\cancel{13}}{\cancel{2}}\times\frac{\cancel{30}}{\cancel{7}}\times\frac{\overset{\cancel{2}}{\cancel{180}}}{11}\times\frac{2}{\cancel{13}}\times\frac{\cancel{7}}{\cancel{30}}\times\frac{1}{\cancel{90}}=\tfrac{2}{11}.$$

(11)

$$\frac{\cancel{4}}{\cancel{7}}\times\frac{3}{11}\times\frac{9}{\underset{4}{\cancel{16}}}\times\frac{\overset{\cancel{7}}{\cancel{77}}}{1}\times\frac{\cancel{3}}{\cancel{7}}\times\frac{\cancel{8}}{\cancel{13}}\times\frac{\overset{\cancel{13}}{\cancel{91}}}{1}\times\frac{167}{\underset{\cancel{3}}{\cancel{24}}}=\frac{3\times9\times167}{4}=\tfrac{4509}{4}=1127\tfrac{1}{4}.$$

(12)

$$\frac{\frac{1}{7}}{\frac{8}{1}}\times\frac{\frac{8}{7}}{\frac{19}{2}}\times\frac{\frac{64}{9}}{\frac{8}{9}}\times\frac{\frac{19}{4}}{\frac{101}{14}}\times\tfrac{3}{27}\times\tfrac{9}{8}=$$

$$\frac{1}{\cancel{7}\times\cancel{8}}\times\frac{\cancel{8}\times\cancel{2}}{7\times\cancel{19}}\times\frac{\overset{\cancel{8}}{\cancel{64}}\times\cancel{9}}{\cancel{9}\times\cancel{8}}\times\frac{\cancel{19}\times\overset{\cancel{2}}{\cancel{14}}}{101\times\underset{\cancel{2}}{\cancel{4}}}\times\frac{1}{\cancel{9}}\times\frac{\cancel{9}}{\cancel{8}}=\frac{1}{7\times101}=\tfrac{1}{707}.$$

(13)

$$\frac{1}{\cancel{4}}\times\frac{\overset{2}{\cancel{8}}}{\cancel{1}}\times\frac{2}{7}\times\frac{19}{1}=\frac{2\times2\times19}{7}=\tfrac{76}{7}=10\tfrac{6}{7}.$$

(14)

$$\frac{9}{\underset{5}{\cancel{10}}}\times\frac{7}{1}\times\frac{\cancel{11}}{\cancel{15}}\times\frac{\overset{\overset{32}{\cancel{64}}}{\cancel{960}}}{\cancel{11}}=\frac{9\times7\times32}{5}=\frac{2016}{5}=403\tfrac{1}{5}.$$

(15)

$$\frac{27}{\cancel{4}}\times\frac{\cancel{7}}{\underset{2}{\cancel{8}}}\times\frac{\cancel{4}}{5}\times\frac{\cancel{4}}{\cancel{7}}=\frac{27}{10}=2\tfrac{7}{10}.$$

(16)

$$\frac{11}{\cancel{3}}\times\frac{\overset{13}{\cancel{39}}}{8}\times\frac{15}{1}=\frac{11\times13\times15}{8}=\frac{2145}{8}=268\tfrac{1}{8}.$$

(17)

$$\frac{1}{\cancel{3}}\times\frac{\overset{7}{\cancel{35}}}{\underset{2}{\cancel{4}}}\times\frac{\overset{\cancel{3}}{\cancel{6}}}{\cancel{10}}\times\frac{\cancel{10}}{\cancel{4}}\times\frac{\cancel{24}}{11}\times\frac{\overset{\cancel{2}}{\cancel{16}}}{17}\times\frac{49}{\cancel{8}}\times\frac{\overset{\cancel{2}}{\cancel{4}}}{\cancel{5}}\times\frac{27}{\cancel{31}}\times\frac{\cancel{31}}{\cancel{3}}\times\frac{191}{\underset{2}{\cancel{188}}}=$$

$$\frac{7\times49\times27\times191}{2\times11\times17}=\frac{1768851}{374}=4729\tfrac{205}{374}.$$

(18)

$$\frac{27}{37\frac{4}{5}}\times\frac{87\frac{2}{9}}{98\frac{1}{8}}\times\frac{\frac{7}{8}}{2\frac{1}{3}}\times\frac{81\frac{5}{11}}{128}=\frac{\frac{27}{1}}{\frac{189}{5}}\times\frac{\frac{785}{9}}{\frac{785}{8}}\times\frac{\frac{7}{8}}{\frac{7}{3}}\times\frac{\frac{896}{11}}{\frac{128}{1}}=$$

$$\frac{\overset{\cancel{9}}{\cancel{27}}\times5}{\underset{\underset{3}{\cancel{21}}}{\cancel{189}}}\times\frac{\cancel{785}\times\cancel{8}}{\underset{\cancel{3}}{\cancel{9}}\times\cancel{785}}\times\frac{\cancel{7}\times\cancel{3}}{\cancel{8}\times\cancel{7}}\times\frac{\overset{\cancel{7}}{\cancel{896}}\times1}{11\times\cancel{128}}=\frac{5}{3\times11}=\frac{5}{33}.$$

(19)

$$\$\frac{\overset{\cancel{5}}{\cancel{95}}}{11}\times\frac{1}{7}\times\frac{3}{\cancel{5}}\times\frac{17}{\cancel{19}}=\frac{3\times17}{11\times7}=\$\tfrac{51}{77}$$

(20)

$$\frac{75\frac{5}{8}}{6\frac{1}{11}} \times \frac{\frac{3}{7}\text{ of }8\frac{1}{2} \times \frac{1}{15}\text{ of }28}{\frac{2}{11}\text{ of }6\frac{3}{8} \times \frac{1}{17}\text{ of }24} \times \frac{7\frac{1}{5}}{15} \times \frac{\frac{3}{4}}{\frac{5}{7}} \times 14\frac{3}{7} \times \frac{100}{121} \times \frac{\frac{4}{5\frac{1}{3}}}{} \times \frac{\frac{5}{7}}{9} =$$

$$\frac{\frac{605}{8}}{\frac{67}{11}} \times \frac{\frac{3}{7} \times \frac{17}{2} \times \frac{1}{15} \times \frac{28}{1}}{\frac{2}{11} \times \frac{51}{8} \times \frac{1}{17} \times \frac{24}{1}} \times \frac{\frac{36}{5}}{\frac{15}{1}} \times \frac{\frac{3}{4}}{\frac{5}{7}} \times \frac{101}{7} \times \frac{100}{121} \times \frac{\frac{4}{1}}{\frac{16}{3}} \times \frac{\frac{5}{7}}{\frac{9}{1}} =$$

$$\frac{\overset{5}{\cancel{605}}\times 11}{\cancel{8} \times \cancel{67}} \times \frac{3\times\overset{11}{\cancel{17}}\times\overset{4}{\cancel{28}}\times 11\times\cancel{8}\times\cancel{17}}{\cancel{7}\times\cancel{2}\times\underset{5}{\cancel{15}}\times\cancel{2}\times\underset{3}{\cancel{51}}\times\underset{\underset{2}{\cancel{8}}}{\cancel{24}}} \times \frac{\overset{9}{\cancel{36}}}{\cancel{5}\times\underset{3}{\cancel{15}}} \times \frac{\cancel{3}\times\cancel{7}}{\cancel{4}\times\cancel{5}} \times$$

$$\frac{101}{7} \times \frac{\overset{\overset{\cancel{4}}{\cancel{20}}}{\cancel{100}}}{\underset{11}{\cancel{121}}} \times \frac{\overset{\cancel{3}}{\cancel{4}\times\cancel{3}}}{16} \times \frac{\cancel{5}}{\cancel{7}\times\cancel{9}} =$$

$$\frac{11\times 9\times 101}{5\times 7\times 16} = \frac{9999}{560} = 17\tfrac{479}{560}.$$

EXERCISE 59—Page 174.

(1)

$\frac{43}{36}$ of 4 days, 5 hours, $= \dfrac{180\text{ d. }23\text{ h.}}{36} =$ 5 d. 0 h. 38 min. 20 sec.

(2)

$\frac{13}{42}$ of £29 $= \dfrac{£29\times 13}{42} = \dfrac{£377}{42} =$ £8 19s. $6\frac{2}{7}$d.

(3)

$$\tfrac{7}{9}\text{ of 186 a. 3 r.}=\frac{\text{186 a. 3 r.}\times 7}{9}=\frac{\text{1307 a. 1 r.}}{9}=\text{145 a. 1 r.}$$

(4)

$$\tfrac{14}{47}\text{ of }\tfrac{3}{7}\text{ of }\tfrac{1}{30}\text{ of }\tfrac{47}{3}\text{ of 24 h. 30 m.}=\tfrac{1}{15}\text{ of 24 h. 30 m.}=\text{1 h. 38 m.}$$

(5)

$$\tfrac{3}{7}\text{ of }\tfrac{4}{9}\text{ of }\tfrac{21}{40}\text{ of }\tfrac{7}{9}\text{ of 33 bu. 2 p. 1 ga.}=\tfrac{7}{90}\text{ of 33 bu. 2 p. 1 ga.}=$$

$$\frac{\text{33 bu. 2 p. 1 ga.}\times 7}{90}=\frac{\text{235 b. 1 p. 1 g.}}{90}=\text{2 b. 2 p. 0 g. 3 q }1\tfrac{17}{45}\text{p}$$

EXERCISE 60—Page 175.

(1)

$$\tfrac{1}{2}\text{ of }\tfrac{3}{5}\div\tfrac{3}{4}\text{ of }\tfrac{35}{4}=\frac{1}{\cancel{2}}\times\frac{\cancel{3}}{5}\times\frac{\overset{2}{\cancel{4}}}{\cancel{3}}\times\frac{4}{35}=\frac{2\times 4}{5\times 35}=\tfrac{8}{175}.$$

(2)

$$\tfrac{15}{22}\div\tfrac{9}{5}\div\tfrac{5}{11}=\frac{\overset{5}{\cancel{15}}}{\underset{2}{\cancel{22}}}\times\frac{\cancel{5}}{\underset{3}{\cancel{9}}}\times\frac{\cancel{11}}{\cancel{5}}=\frac{5}{2\times 3}=\tfrac{5}{6}$$

(3)

$$82\tfrac{1}{17}\div 26\tfrac{5}{41}=\frac{\overset{155}{\cancel{1395}}}{17}\times\frac{41}{\underset{119}{\cancel{1071}}}=\frac{155\times 41}{17\times 119}=\frac{6355}{2023}=3\tfrac{286}{2023}.$$

(4)

$$2\tfrac{1}{2}\div\tfrac{3}{4}+\tfrac{5}{8}=\tfrac{5}{2}\div\tfrac{11}{8}=\frac{5}{\cancel{2}}\times\frac{\overset{4}{\cancel{8}}}{11}=\frac{5\times 4}{11}=\frac{20}{11}=1\tfrac{9}{11}$$

(5)

$$1\tfrac{3}{4} \div \tfrac{1}{7} \text{ of } 2\tfrac{3}{4} \text{ of } 16 \text{ of } \tfrac{5}{4} \text{ of } \tfrac{1}{10} = \frac{7}{\underset{2}{\cancel{4}}} \times \frac{7}{1} \times \frac{\cancel{4}}{11} \times \frac{1}{\underset{\cancel{4}}{\cancel{16}}} \times \frac{\cancel{4}}{\cancel{5}} \times$$

$$\frac{\overset{2}{\cancel{10}}}{1} = \frac{7\times7}{2\times11} = \frac{49}{22} = 2\tfrac{5}{22}.$$

(6)

$$2\tfrac{1}{3} \div \left(\tfrac{5}{9} \div \tfrac{6}{32} \text{ of } 9\right) = \tfrac{7}{3} \div \left(\tfrac{5}{9} \text{ of } \tfrac{32}{6} \text{ of } \tfrac{1}{9}\right) = \frac{7}{\cancel{3}} \times \frac{9}{5} \times \frac{\overset{3}{\cancel{6}}}{\underset{16}{\cancel{32}}} \times$$

$$\frac{\overset{3}{\cancel{9}}}{1} = \frac{7\times9\times3\times3}{5\times16} = \frac{567}{80} = 7\tfrac{7}{80}.$$

(7)

$$48\tfrac{1}{2} \div \tfrac{2}{9} + \tfrac{3}{8} \text{ of } 6 = \tfrac{97}{2} \div \tfrac{2}{9} + \tfrac{9}{4} = \tfrac{97}{2} \div \tfrac{89}{36} = \frac{97}{\cancel{2}} \times \frac{\overset{18}{\cancel{36}}}{89} =$$

$$\frac{97\times18}{89} = \frac{1746}{89} = 19\tfrac{55}{89}.$$

(8)

$$6\tfrac{1}{2} \div \tfrac{3}{5} \text{ of } \tfrac{9}{10} + \tfrac{8}{17} = \tfrac{13}{2} \div \tfrac{27}{50} + \tfrac{8}{17} = \tfrac{13}{2} \div \tfrac{859}{850} = \frac{13}{\cancel{2}} \times$$

$$\frac{\overset{425}{\cancel{850}}}{859} = \frac{13\times425}{859} = \frac{5525}{859} = 6\tfrac{371}{859}.$$

(9)

$$\tfrac{9}{2} \times \tfrac{10}{3} \div \tfrac{9}{4} \times \tfrac{4}{25} = \frac{\cancel{9}}{\cancel{2}} \times \frac{\overset{5}{\cancel{10}}}{3} \times \frac{4}{\cancel{9}} \times \frac{4}{\underset{5}{\cancel{25}}} = \frac{4\times4}{3\times5} = \frac{16}{15} = 1\tfrac{1}{15}.$$

(10)

$$\frac{\frac{67}{9}}{\frac{35}{3}} \div \frac{\frac{3}{7}}{\frac{33}{8}} = \frac{67\times 3}{9\times 35} \div \frac{3\times 8}{7\times 33} = \frac{67\times \cancel{3}}{\underset{3}{\cancel{9}}\times \underset{5}{\cancel{35}}} \times \frac{\cancel{7}\times \overset{11}{\cancel{33}}}{\cancel{3}\times 8} =$$

$$\frac{67\times 11}{3\times 5\times 8} = \frac{737}{120} = 6\tfrac{17}{120}.$$

(11)

$$\tfrac{5}{9} \text{ of } \tfrac{80}{11} \div \tfrac{4}{11} \text{ of } \tfrac{122}{7} = \tfrac{5}{9} \times \frac{\overset{\overset{10}{\cancel{20}}}{\cancel{80}}}{11} \times \frac{11}{\cancel{4}} \times \frac{7}{\underset{61}{\cancel{122}}} = \frac{5\times 10\times 7}{9\times 61} = \tfrac{350}{549}.$$

(12)

$$\tfrac{45}{28} \text{ of } \tfrac{10}{13} \text{ of } \tfrac{3}{4} \text{ of } \tfrac{7}{5} \div \tfrac{5}{6} \text{ of } \tfrac{3}{28} \text{ of } \tfrac{3}{4} \text{ of } \tfrac{5}{1} = \frac{\overset{\overset{3}{\cancel{9}}}{\cancel{45}}}{\cancel{28}} \times \frac{\overset{2}{\cancel{10}}}{13} \times \frac{\cancel{3}}{\cancel{4}} \times \frac{7}{\cancel{5}} \times \frac{6}{\cancel{5}} \times$$

$$\frac{\cancel{28}}{\cancel{3}} \times \frac{\cancel{4}}{\cancel{3}} \times \frac{1}{5} = \frac{3\times 2\times 7\times 6}{13\times 5} = \frac{252}{65} = 3\tfrac{57}{65}.$$

(13)

$$\frac{\frac{7}{4}}{\frac{9}{2}} \div \frac{\frac{7}{3}}{\frac{9}{4}} = \frac{7\times 2}{4\times 9} \div \frac{7\times 4}{3\times 9} = \frac{\cancel{7}\times \cancel{2}}{\underset{2}{\cancel{4}}\times \cancel{9}} \times \frac{3\times \cancel{9}}{\cancel{7}\times 4} = \frac{3}{2\times 4} = \tfrac{3}{8}.$$

(14)

$$\tfrac{3}{25} \div \frac{\frac{21}{5}}{\frac{35}{2}} = \frac{3}{25} \div \frac{21\times 2}{5\times 35} = \frac{\cancel{3}}{\underset{5}{\cancel{25}}} \times \frac{\cancel{5}\times \overset{7}{\cancel{35}}}{\underset{3}{\cancel{21}}\times 2} = \tfrac{1}{2}.$$

(15)

$$\tfrac{113}{8} \times \tfrac{1}{9} \div \tfrac{3}{7} \times \tfrac{107}{13} \times \frac{\frac{13}{2}}{\frac{136}{7}} = \frac{113}{\cancel{8}} \times \frac{1}{9} \times \frac{\cancel{7}}{3} \times \frac{\cancel{13}}{107} \times$$

$$\frac{2\times \overset{17}{\cancel{136}}}{\cancel{7}\times \cancel{13}} = \frac{113\times 2\times 17}{9\times 3\times 107} = \frac{3842}{2889} = 1\tfrac{953}{2889}.$$

(16)

$$\frac{31}{2}\times\frac{\frac{2}{7}}{\frac{7}{9}}\times\frac{\frac{7}{1}}{\frac{2}{3}}\times\frac{\frac{7}{10}}{\frac{3}{1}}\div\frac{\frac{41}{9}}{\frac{7}{1}}\times\frac{\frac{3}{1}}{\frac{19}{4}}\times\frac{\frac{7}{8}}{\frac{7}{2}}\times\frac{\frac{11}{4}}{\frac{4}{7}}=$$

$$\frac{31}{2}\times\frac{2\times9}{7\times7}\times\frac{7\times3}{2}\times\frac{7}{10\times3}\div\frac{41}{9\times7}\times\frac{3\times4}{19}\times\frac{7\times2}{8\times7}\times\frac{11\times7}{4\times4}=$$

$$\frac{31}{\cancel{2}}\times\frac{\overset{3}{\cancel{2}\times\cancel{9}}}{\cancel{7}\times\cancel{7}}\times\frac{\cancel{7}\times\cancel{3}}{\cancel{2}}\times\frac{\cancel{7}}{\underset{5}{\cancel{10}\times\cancel{3}}}\times\frac{9\times\cancel{7}}{41}\times\frac{19}{\cancel{3}\times\cancel{4}}\times\frac{\overset{4}{\cancel{8}\times\cancel{7}}}{\cancel{7}\times\cancel{2}}\times\frac{\overset{2}{\cancel{4}\times\cancel{4}}}{11\times\cancel{7}}=$$

$$\frac{31\times3\times9\times19\times4}{5\times41\times11}=\frac{63612}{2255}=28\tfrac{472}{2255}.$$

EXERCISE 61—Page 176.

(1)

$\frac{\frac{19}{11}}{\frac{5}{3}}=\frac{19\times3}{11\times5}=\frac{57}{55}$. £8 14s. 6¾d. ÷ $\frac{57}{55}$ = £8 14s. 6¾d. × $\frac{55}{57}$ =

$$\frac{\text{£8 14s. 6¾d.}\times55}{57}=\text{£8 8s. 5¼d.}$$

(2)

$\frac{23}{8}\times\frac{20}{11}=\frac{115}{22}$. 1 m. 5 fur. 91 yds. 2 ft. ÷ $\frac{115}{22}$ = 1 m. 5 fur. 91 yds. 2 ft. × $\frac{22}{115}$ =

$$\frac{\text{1 m. 5 fur. 91 yds. 2 ft.}\times\ 2}{115}=\text{2 fur. 124 yds. 2 ft}$$

(3)

3 a. 3 r. 3 per. ÷ $\frac{3}{5}$ = 3 a. 3 r. 3 p. × $\frac{5}{3}$ = $\frac{\text{3 a. 3 r. 3 p.}\times5}{3}$ =

6 a. 1 r. 5 per.

(4)

£7 16s. 2d. ÷ $\frac{4}{9}$ = £7 16s. 2d. × $\frac{9}{4}$ = $\frac{\text{£7 16s. 2d.}\times9}{4}$ =

£17 11s. 4½d.

EXERCISE 62—Page 178.

(1)

$$\dfrac{\dfrac{\dfrac{12\frac14}{7}}{3\frac14}}{\dfrac{\dfrac{\dfrac{\dfrac{9}{\frac23}}{\frac79}}{5}}{4\frac12}}=\dfrac{\dfrac{\dfrac{\frac{49}{4}}{\frac71}}{\frac{13}{4}}}{\dfrac{\dfrac{\dfrac{\dfrac{\frac91}{\frac23}}{\frac79}}{\frac51}}{\frac92}}=\dfrac{\dfrac{\frac74}{\frac{13}{4}}}{\dfrac{\dfrac{\dfrac{\frac{27}{2}}{\frac79}}{\frac51}}{\frac92}}=\dfrac{\frac{7}{13}}{\dfrac{\dfrac{\frac{243}{14}}{\frac51}}{\frac92}}=\dfrac{\frac{7}{13}}{\dfrac{\frac{243}{70}}{\frac92}}=\dfrac{\frac{7}{13}}{\frac{27}{35}}=\dfrac{7\times35}{13\times27}$$

$$\dfrac{\dfrac{\frac23\text{ of }32}{\frac73}}{\dfrac{\dfrac{9\frac13}{3\frac12}}{\frac72}}=\dfrac{\dfrac{\frac{64}{3}}{\frac73}}{\dfrac{\dfrac{\frac{28}{3}}{\frac72}}{\frac78}}=\dfrac{\frac{64}{7}}{\dfrac{\frac83}{\frac78}}=\dfrac{\frac{64}{7}}{\frac{64}{21}}=3.$$

$$\dfrac{7\times35}{13\times27}\times\dfrac{3}{1}=\dfrac{735}{351}=\tfrac{245}{117}=2\tfrac{11}{117}.$$

(2)

$$\dfrac{\dfrac{\dfrac{\frac13}{7}}{6\frac12}}{\dfrac{9\frac12}{\dfrac{3}{\frac12}}}=\dfrac{\dfrac{\dfrac{\frac13}{\frac71}}{\frac{13}{2}}}{\dfrac{\frac{19}{2}}{\dfrac{\frac31}{\frac12}}}=\dfrac{\dfrac{\frac{1}{21}}{\frac{13}{2}}}{\dfrac{\frac{19}{2}}{\frac61}}=\dfrac{\dfrac{2}{21\times13}}{\frac{19}{12}}=\dfrac{8}{7\times13\times19}$$

$$\dfrac{\frac56}{\frac71}=\dfrac{5}{42}.\quad\dfrac{8}{7\times13\times19}\div\dfrac{5}{42}=\dfrac{8}{7\times13\times19}\times\dfrac{42}{5}=\tfrac{48}{1235}.$$

(3)

$$\dfrac{\dfrac{12\frac{1}{2}}{5\frac{1}{4}}}{\dfrac{3\frac{3}{4}}{5\frac{1}{2}}} = \dfrac{\dfrac{\frac{25}{2}}{\frac{21}{4}}}{\dfrac{\frac{15}{4}}{\frac{11}{2}}} = \dfrac{\frac{50}{21}}{\frac{15}{22}} = \tfrac{220}{63}.$$

$$\dfrac{\dfrac{\frac{2\frac{1}{4}}{5}}{4\frac{1}{2}}}{\dfrac{3\frac{3}{4}}{\frac{16\frac{2}{3}}{\frac{1}{2}}}} = \dfrac{\dfrac{\frac{\frac{9}{4}}{\frac{5}{1}}}{\frac{9}{2}}}{\dfrac{\frac{15}{4}}{\frac{\frac{50}{3}}{\frac{1}{2}}}} = \dfrac{\dfrac{\frac{9}{20}}{\frac{9}{2}}}{\dfrac{\frac{15}{4}}{\frac{100}{3}}} = \dfrac{\frac{1}{10}}{\frac{9}{80}} = \tfrac{8}{9}.$$

$$\frac{220}{63} \div \frac{8}{9} = \frac{220}{63} \times \frac{9}{8} = \frac{55}{14} = 3\tfrac{13}{14}.$$

Exercise 63—Page 180.

(1)

$$\frac{800}{2000} = \tfrac{2}{5}. \quad \frac{420}{2000} = \tfrac{21}{100}. \quad \frac{100}{2000} = \tfrac{1}{20}. \quad \frac{160}{2000} = \tfrac{2}{25}.$$

$$\frac{35}{2000} = \tfrac{7}{400}.$$

(2)

$\frac{2}{3}$ of $\frac{3}{4}$ of $\frac{8}{5}$ of $\frac{1}{4}$ of $\frac{5}{2}$ of $\frac{4}{45} = \frac{2}{\cancel{3}} \times \frac{\cancel{3}}{\cancel{4}} \times \frac{\overset{4}{\cancel{8}}}{\cancel{5}} \times \frac{1}{\cancel{4}} \times \frac{\cancel{5}}{\cancel{2}} \times \frac{\cancel{4}}{45} = \frac{2}{45}.$

(3)

$6\frac{7}{8} \times 65\frac{3}{4}$ cts. $= \frac{55}{8} \times \frac{263}{4}$ cts. $= \frac{14465}{32}$ cts $= \$4.52\frac{1}{32}$.

(4)

$$\frac{3}{8}+\frac{4}{17}=\frac{51}{136}+\frac{32}{136}=\frac{83}{136}.$$

(5)

$$\frac{1}{3}+\frac{1}{10}+\frac{1}{8}+\frac{1}{6}=\frac{40}{120}+\frac{12}{120}+\frac{15}{120}+\frac{20}{120}=\frac{87}{120}=\frac{29}{40}$$

$$1 \text{ or } \frac{40}{40}-\frac{29}{40}=\frac{11}{40}.$$

(6)

$$\frac{5\frac{4}{5}-2\frac{1}{8}}{3\frac{3}{4}+\frac{9}{20}} \text{ of } \frac{4\frac{1}{2}+5\frac{19}{25}}{4\frac{1}{20}} \text{ of } \frac{2\frac{3}{5}+1\frac{2}{3}}{7\frac{19}{24}-2\frac{1}{4}}=\frac{5\frac{32}{40}-2\frac{5}{40}}{3\frac{15}{20}+\frac{9}{20}} \text{ of } \frac{4\frac{25}{50}+5\frac{38}{50}}{\frac{81}{20}} \text{ of}$$

$$\frac{2\frac{9}{15}+1\frac{10}{15}}{7\frac{19}{24}-2\frac{6}{24}}=\frac{3\frac{27}{40}}{4\frac{4}{20}} \text{ of } \frac{10\frac{13}{50}}{\frac{81}{20}} \text{ of } \frac{4\frac{4}{15}}{5\frac{13}{24}}=\frac{\frac{147}{40}}{\frac{84}{20}} \text{ of } \frac{\frac{513}{50}}{\frac{81}{20}} \text{ of } \frac{\frac{64}{15}}{\frac{133}{24}}=$$

$$\frac{7}{8}\times\frac{2\times\overset{3}{\cancel{57}}}{5\times\underset{3}{\cancel{9}}}\times\frac{\cancel{8}\times64}{5\times\underset{19}{\cancel{133}}}=\frac{2\times64}{5\times3\times5}=\frac{128}{75}=1\frac{53}{75}$$

(7)

$$1670\tfrac{7}{13}\times12\tfrac{3}{4}\text{ cts.}=\tfrac{21717}{13}\times\tfrac{51}{4}\text{ cts.}=\tfrac{1107567}{52}\text{ cts.}=\$212{\cdot}99\tfrac{19}{52}.$$

(8)

$\frac{2}{3}$ of the longer $=\frac{3}{4}$ of the shorter; therefore $\frac{1}{3}$ of the longer $=\frac{1}{2}$ of $\frac{3}{4}=\frac{3}{8}$ of the shorter.

Hence the longer $=\frac{3}{8}\times3=\frac{9}{8}$ of the shorter.

The whole tree = longer + shorter $=\frac{9}{8}+\frac{8}{8}$ of shorter $=\frac{17}{8}$ of the shorter.

If 136 ft. $=\frac{17}{8}$ of the shorter, $\frac{1}{17}$ of $136=8=\frac{1}{8}$ of the shorter.
Hence shorter $=8\times8=64$ ft.; and longer $=136-64=72$ ft.

(9)

$97\frac{1}{4}+127\frac{3}{8}+500\frac{3}{8}+333\frac{1}{3}=97\frac{30}{120}+127\frac{45}{120}+500\frac{45}{120}+333\frac{40}{120}=1057\frac{160}{120}=1058\frac{40}{120}.$

$\$1000+\$1375\frac{1}{2}+\$6831+\$4013\frac{3}{16}=\$1000+\$1375\frac{8}{16}+\$6831+\$4013\frac{3}{16}=\$13219\frac{11}{16}=\$13219{\cdot}68\frac{3}{4}.$

(10)

$12\frac{5}{6} + \frac{8}{15} = 13\frac{11}{30}$. $8\frac{1}{2} + 1\frac{1}{15} = 9\frac{17}{30}$. $13\frac{11}{30} - 9\frac{17}{30} = 3\frac{24}{30} = \frac{114}{30}$.
$7\frac{7}{12} - 6\frac{1}{2} = \frac{13}{12}$. $\frac{114}{30} \times$ [illegible] $\times \frac{13}{12} =$ [illegible] $= 14\frac{81}{180}$.
$\frac{5}{8} \div \frac{10}{7} = \frac{5}{8} \times \frac{7}{10} = \frac{7}{16}$. [illegible] $\frac{7}{16} -$ [illegible] $=$ [illegible].

(11)

$19\frac{7}{8} \times \$6\frac{3}{4} = \frac{159}{8} \times \$\frac{27}{4} = \$\frac{4293}{32} = \$134{\cdot}15\frac{5}{8}$.

(12)

$376\frac{11}{18} \times \$75\frac{3}{8} = \frac{6779}{18} \times \$\frac{603}{8} = \frac{4087737}{144} = \$28387{\cdot}06\frac{1}{4}$.

(13)

$147\frac{2}{3} + 320\frac{1}{5} = 147\frac{10}{15} + 320\frac{3}{15} = 467\frac{13}{15}$. $467\frac{13}{15} - 156\frac{1}{4} =$
$467\frac{52}{60} - 156\frac{15}{60} = 311\frac{37}{60}$.

(14)

$$\frac{7\left(1\frac{1}{2}\text{ of }\frac{3}{4}\right)}{\frac{1}{6}\left(\dfrac{3}{3\frac{1}{2}}\text{ of }7\right)} \div 7\frac{7}{8} = \frac{\frac{7}{1}\times\frac{3}{2}\times\frac{3}{4}}{\frac{1}{6}\times\dfrac{\frac{3}{1}}{\frac{7}{2}}\times\frac{7}{1}} \div \frac{63}{8} = \frac{\dfrac{7\times3\times3}{1\times2\times4}}{\frac{1}{6}\times\frac{6}{7}\times\frac{7}{1}} \times \frac{8}{63} =$$

$$\frac{7\times3\times3}{1\times2\times4} \times \frac{8}{63} = 1. \quad \frac{\frac{1}{2}+\frac{1}{3}+\frac{1}{4}}{\dfrac{1}{2\frac{1}{2}}+\dfrac{1}{3\frac{1}{4}}+\dfrac{1}{4\frac{1}{2}}} = \frac{\frac{6}{12}+\frac{4}{12}+\frac{3}{12}}{\dfrac{1}{\frac{5}{2}}+\dfrac{1}{\frac{13}{4}}+\dfrac{1}{\frac{9}{2}}} =$$

$$\frac{\frac{13}{12}}{\frac{2}{5}+\frac{4}{13}+\frac{2}{9}} = \frac{\frac{13}{12}}{\frac{234}{585}+\frac{180}{585}+\frac{130}{585}} = \frac{\frac{13}{12}}{\frac{544}{585}} = \frac{2535}{2176} = 1\frac{359}{2176}.$$

(15)

$17\frac{3}{7} \div 7\frac{1}{6} = \frac{122}{7} \div \frac{43}{6} =$ [illegible] $\times$ [illegible] $=$ [illegible] $= 2$ [illegible].

(16)

$3\frac{2}{3} + 4\frac{3}{4} + 4\frac{4}{5} = 3\frac{40}{60} + 4\frac{45}{60} + 4\frac{48}{60} = 13\frac{13}{60} = \frac{793}{60}$.
$7\frac{1}{6} - 5\frac{1}{7} = 7\frac{7}{42} - 5\frac{6}{42} = 2\frac{1}{42} = \frac{85}{42}$.
$94\frac{1}{8} + 93\frac{1}{9} = 94\frac{9}{72} + 93\frac{8}{72} = 187\frac{17}{72} = \frac{13481}{72}$.

$$\frac{793}{60} \times \frac{85}{42} + \frac{13481}{72} = \frac{\cancel{793}}{\underset{5}{\cancel{60}}} \times \frac{\overset{\cancel{5}}{\cancel{85}}}{\underset{7}{\cancel{42}}} \times \frac{\overset{6}{\cancel{72}}}{\underset{\cancel{793}}{\cancel{13481}}} = \frac{1}{7}.$$

(17)

$2\frac{4}{5} + \frac{2}{3} + 4 = 2\frac{12}{15} + \frac{10}{15} + 4 = 7\frac{7}{15} = \frac{112}{15}$.
$2 \div \frac{112}{15} = 2 \times \frac{15}{112} = \frac{15}{56}$. $1\frac{2}{3} - \frac{7}{9} = \frac{15}{9} - \frac{7}{9} = \frac{8}{9}$.
$\frac{15}{56} + \frac{8}{9} = \frac{583}{504}$. $5\frac{1}{5} - 4\frac{1}{2} = 4\frac{12}{10} - 4\frac{5}{10} = \frac{7}{10}$.

$$\frac{583}{\underset{72}{\cancel{504}}} \times \frac{\cancel{7}}{10} = \frac{583}{720}.$$

(18)

$\frac{1}{3} + \frac{1}{2} = \frac{5}{6}$. $1\frac{1}{3} + 2\frac{3}{4} = 4\frac{1}{12} = \frac{49}{12}$. $2\frac{1}{14} - 1\frac{1}{2} = \frac{8}{14} = \frac{4}{7}$.

$3\frac{1}{10} - \frac{3}{7} = 2\frac{47}{70} = \frac{187}{70}$. $$\frac{\cancel{5}}{\underset{3}{\cancel{6}}} \times \frac{\overset{\cancel{7}}{\cancel{49}}}{12} \times \frac{\overset{\cancel{2}}{\cancel{4}}}{\cancel{7}} \times \frac{187}{\underset{\underset{\cancel{2}}{\cancel{10}}}{\cancel{70}}} = \frac{187}{3 \times 12} = \frac{187}{36} = 5\frac{7}{36}.$$

$1\frac{3}{4} \div 2\frac{1}{2} = \frac{7}{4} \times \frac{2}{5} = \frac{7}{10}$. $5\frac{1}{2} \div 3\frac{1}{8} = \frac{11}{2} \times \frac{8}{25} = \frac{44}{25} = 1\frac{19}{25}$.
$\frac{7}{10} + 1\frac{19}{25} = 2\frac{23}{50}$.

(19)

$1 - (\frac{1}{3} + \frac{1}{2}) = \frac{1}{6}$. $\frac{3}{5}$ of $\frac{1}{3} = \frac{1}{5}$. $\frac{1}{3} - \frac{1}{5} = \frac{2}{15}$. $\frac{1}{6} + \frac{2}{15} = \frac{9}{30}$
$\frac{1}{3} - \frac{9}{30} = \frac{1}{30}$. $\frac{1}{30}$ of \$40000 = \$1333·33⅓.

EXERCISE 66—Page 183.

(1)

$\frac{1}{2} = 2)\underline{1}$
·5

$\frac{3}{8} = 8)\underline{3}$
·375

(2)

$\frac{9}{25} = 25)\underline{9}$
$·36 = \frac{36}{100}$

$\frac{1}{4} = 4)\underline{1}$
$·25 = \frac{25}{100}$

(3)

75)73 (·9733+
67.5
5·50
5.25
·250
·225
250
225
25

123)574(4·666+
492
82·0
73·8
8·20
7·38
·820
·738
82

34)15 (·44117+
13·6
1·40
1·36
40
34
60
34
260
238
22

(4)

7)6
·857142+

12)5
·4166+

9)4
·44444+

(5)

112)17 (·15178571428+
11·2
5·80
5·60
·200
·112
880
784
960
896
640
560
800

800
784
160
112
480
448
320
224
960
896
64

1296)718 (·554012+
648·0
70·00
64·80
5·200
5·184
1600
1296
3040
2592
448

EXERCISE 67—Page 184.

(1)

12)1·0 in.
3)2·083333 ft.
$5\frac{1}{2}$)3·694444 yd.
2 2
11) 7·388888
40)·671717 per.
·01679+ fur.

(2)

12)17·0 grs.
2)1·41666666
20)3·70833333 dwt.
12)·18541666 oz.
·01545138+ lb.

(3)

20)7·0 grs.
3)2.35 scr.
8)·7833333 dr.
12)·0979166 oz.
·0081597+lb.

(4)

12)9·0 in
3)2·75 ft.
$5\frac{1}{2}$)2·91666
2 2
11)5·83333
40)35·53030 per.
8)5.88825 fur.
·73603+ mile.

(5)

4)2·0 na.
4)3·5 qr.
·875 yd.

(6)

13s. 4d. = 160d.
5s. = 60d.
$\frac{60}{160} = \frac{3}{8} =$ ·375

(7)

60)21·0 sec.
60)55·35 min.
12)12·9225 hr.
2)1·076875
·5384375 day.

(8)

$\frac{2}{7}$ of $\frac{1}{2}$ of $6\frac{3}{4}$d. $= \frac{27}{28}$d. and £$\frac{1}{3}$=80d.
$\frac{27}{28}$d. $= \frac{1}{80}$ of $\frac{27}{28}$ of £$\frac{1}{3}$ $= \frac{27}{2240}$ of £$\frac{1}{4}$.
27 ÷ 2240 = 0·012053.

(9)

$\frac{2}{5}$ of $\frac{1}{2}$ of 1 mil. = 12672 in.
$3\frac{1}{2}$)12672
2 2
7)25344
3620·571428+

(10)

$\frac{1}{5}$ of $\frac{2}{3}$ of $\frac{13}{4}$ lbs. = $\frac{13}{30}$ lb. = $110\frac{14}{15}$ drs. = $\frac{1664}{15}$ drs.

$\frac{3}{4}$ of an oz. = 12 drs. $\frac{1664}{15} \div 12 = \frac{1664}{180}$

180)1664(9·2444+
1620
440
360
800
720
800
720
800
720
80

(11)

2)1·0 pts.
4)1·5 qt.
2)1·375 gal.
4)3·6875 pk.
·921875 bush.

EXERCISE 68—Page 186.

(1)

·3945
24
15780
7890
9·4680 hrs.
60
28·0800 min.
60
4·8000 sec.

(2)

·3965
8
3·1720 fur.
40
6·8800 per.
$5\frac{1}{2}$
44000
4400
4 [illegible] yds.
3
2·5200 ft.
12
6·2400 in.

(3)

·309153
20
6·183060 dwt.
24
732240
366120
4·393440 grs.

(4)

$22\cdot75 = 22\frac{75}{100} = 22\frac{3}{4}.$

£2 2s. 6d.$\times 22\frac{3}{4}$=£48 6s. $10\frac{1}{2}$d.

(5)

7 b. 1 p. 1 g. 1 qt. = 237 qts.

$11\cdot17825 \times 237 = 2649\cdot24525$ qt. =

82 b. 3 p. 0 g. 1 q. 0·4905 pts

(6)

$$\begin{array}{rl} \cdot2057 & \\ 12 & \\ \hline 2\cdot4684 & \text{oz.} \\ 20 & \\ \hline 9\cdot3680 & \text{dwt.} \\ 24 & \\ \hline 14720 & \\ 7360 & \\ \hline 8\cdot8320 & \text{grs.} \end{array}$$

(7)

1 f. 36 p. 2 y. 5 in. = 15125 in.

15125 × ·176 = 2662 in. =

13 per. 2 yds. 1 ft. 4 in.

(8)

$$\begin{array}{rl} \cdot625 & \\ 3 & \\ \hline 1.875 & \text{mil.} \\ 8 & \\ \hline 7\cdot000 & \text{fur.} \end{array}$$

(9)

$$\begin{array}{rl} \cdot015625 & \\ 4 & \\ \hline \cdot062500 & \text{pk.} \\ 2 & \\ \hline \cdot125000 & \text{gal.} \\ 4 & \\ \hline \cdot500000 & \text{qt.} \\ 2 & \\ \hline 1\cdot000000 & \text{pt.} \end{array}$$

(10)

$$\begin{array}{rl} \cdot9378 & \\ 4 & \\ \hline 3\cdot7512 & \text{r.} \\ 40 & \\ \hline 30\cdot0480 & \text{per.} \\ 30\frac{1}{4} & \\ \hline 14400 & \\ 120 & \\ \hline 1\cdot4520 & \text{yd.} \\ 9 & \\ \hline 4\cdot0680 & \text{ft.} \\ 144 & \\ \hline 2720 & \\ 2720 & \\ 680 & \\ \hline 9\cdot7920 & \text{in.} = 9\frac{99}{125} \text{ in.} \end{array}$$

(11)

1 sq. yd. 3 ft. 72 in.=1800 in.

·2775 × 1800 = 499·5 in. =

3 ft. $67\frac{1}{2}$ in.

EXERCISE 71—Page 191.

(1)

$\cdot\dot{8} = \frac{8}{9}$.

$\cdot\dot{0}\dot{5} = \frac{5}{99}$.

$\cdot\dot{3}4\dot{2} = \frac{342}{999} = \frac{38}{111}$.

$\cdot\dot{7}00\dot{4} = \frac{7004}{9999}$.

$\cdot\dot{0}0200\dot{3} = \frac{2003}{999999}$.

(2)

$\cdot\dot{1}\dot{9} = \frac{19}{99}$.

$\cdot\dot{1}06\dot{7} = \frac{1067}{9999} = \frac{97}{909}$.

$\cdot\dot{1}111\dot{5} = \frac{11115}{99999} = \frac{1235}{11111}$.

$\cdot\dot{7}0410\dot{3} = \frac{704103}{999999} = \frac{234701}{333333}$.

(3)

$\cdot\dot{1}0\dot{2} = \frac{102}{999} = \frac{34}{333}$.

$\cdot\dot{0}01\dot{3} = \frac{13}{9999}$.

$\cdot\dot{0}000710\dot{3} = \frac{7103}{99999999}$.

$\cdot\dot{0}102030\dot{4} = \frac{1020304}{99999999}$.

$\cdot\dot{9}8765432\dot{1} = \frac{987654321}{999999999} = \frac{109739369}{111111111}$.

EXERCISE 72—Page 192.

(1)

$$\begin{array}{r} \cdot 83\dot{2}\dot{5} \\ 83 \\ \hline \frac{8242}{9900} = \frac{4121}{4950} \end{array} \qquad \begin{array}{r} \cdot 147\dot{6}5\dot{8} \\ 147 \\ \hline \frac{147511}{999000} \end{array} \qquad \begin{array}{r} \cdot 432\dot{0}07\dot{5}. \\ 432 \\ \hline \frac{4319643}{9999000} = \frac{1439881}{3333000}. \end{array}$$

(2)

$$\begin{array}{r} 875 \cdot 49\dot{6}\dot{5} \\ 49 \\ \hline 875\frac{4916}{9900} = 875\frac{1229}{2475} \end{array} \qquad \begin{array}{r} 301 \cdot 82\dot{7}5\dot{6} \\ 82 \\ \hline 301\frac{82674}{99900} = 301\frac{9186}{11100} = 301\frac{1531}{1850}. \end{array}$$

(3)

$$\cdot 08\dot{3}$$
$$8$$
$$\frac{75}{900} = \frac{1}{12}$$

$$\frac{714285}{9999990} = \frac{79365}{1111110} = \frac{7215}{101010} = \frac{1443}{20202} = \frac{481}{6734} = \frac{1}{14}.$$

$$\cdot 123\dot{4}5\dot{6}$$
$$123$$
$$\frac{123333}{999000} = \frac{41111}{333000}$$

(4)

$$703\dot{4}$$
$$703$$
$$\frac{6331}{9000}$$

$$\cdot 96\dot{4}3\dot{2}$$
$$96$$
$$\frac{96336}{99900} = \frac{10704}{11100} = \frac{2676}{2775} = \frac{892}{925}$$

$$\cdot 002\dot{0}\dot{7}$$
$$2$$
$$\frac{205}{99000} = \frac{41}{19800}$$

$$\cdot 1432\dot{7}\dot{1}$$
$$1432$$
$$\frac{141839}{990000}$$

EXERCISE 73—Page 194.

(1)

Dissimilar.		Similar.		Similar and Coterminous.
$\cdot\dot{9}$	=	$\cdot 9999\dot{9}$	=	$\cdot 9999\dot{9}9999\dot{9}$
$6\cdot 3\dot{2}\dot{7}$	=	$6\cdot 3272\dot{7}\dot{2}$	=	$6\cdot 3272\dot{7}2727\dot{2}$
$19\cdot 43$	=	$19\cdot 4300\dot{0}$	=	$19\cdot 4300\dot{0}0000\dot{0}$
$27\cdot 02\dot{7}\dot{8}$	=	$27\cdot 0278\dot{7}\dot{8}$	=	$27\cdot 0278\dot{7}8787\dot{8}$
$\cdot 0347\dot{1}2\dot{3}$	=	$\cdot 0347\dot{1}2\dot{3}$	=	$\cdot 0347\dot{1}2312\dot{3}$
				2 carried,

Sum, $= 53\cdot 8198\dot{6}3827\dot{4}$

(2)

Dissimilar.		Similar.		Similar and Coterminous.
$7\cdot4\dot{2}\dot{7}$	=	$7\cdot427\dot{2}\dot{7}$	=	$7\cdot427\dot{2}72727272727\dot{7}$
$9\cdot1\dot{2}3\dot{4}$	=	$9\cdot123\dot{4}2\dot{3}$	=	$9\cdot123\dot{4}2342342342\dot{3}$
$17\cdot298\dot{7}64\dot{3}$	=	$17\cdot298\dot{7}64\dot{3}$	=	$17\cdot298\dot{7}6437643764\dot{3}$
$18\cdot\dot{6}\dot{7}$	=	$18\cdot676\dot{7}\dot{6}$	=	$18\cdot676\dot{7}6767676767\dot{6}$
				2 carried
		Sum,	=	$52\cdot526\dot{2}2820390147\dot{1}$

(3)

Dissimilar.		Similar.		Similar and Coterminous.
$4\cdot\dot{9}\dot{5}$	=	$4\cdot9595\dot{9}\dot{5}$	=	$4\cdot9595\dot{9}5959\dot{5}$
$7\cdot\dot{1}6\dot{4}$	=	$7\cdot1641\dot{6}4\dot{1}$	=	$7\cdot1641\dot{6}4164\dot{1}$
$4\cdot7\dot{1}2\dot{3}$	=	$4\cdot7123\dot{1}2\dot{3}$	=	$4\cdot7123\dot{1}2312\dot{3}$
$\cdot9731\dot{7}$	=	$\cdot9731\dot{7}$	=	$\cdot9731\dot{7}7777\dot{7}$
				2 carried.
		Sum,	=	$17\cdot3092\dot{5}021\dot{3}8$

(4)

Dissimilar.		Similar.		Similar and Coterminous
$1\cdot5$	=	$1\cdot500\dot{0}$	=	$1\cdot500\dot{0}0000\dot{0}$
$99\cdot083$	=	$99\cdot083\dot{0}$	=	$99\cdot083\dot{0}0000\dot{0}$
$\cdot\dot{1}6\dot{2}$	=	$\cdot162\dot{1}6\dot{2}$	=	$.162\dot{1}6216\dot{2}$
$\cdot\dot{8}1\dot{4}$	=	$\cdot814\dot{8}1\dot{4}$	=	$\cdot814\dot{8}1481\dot{4}$
$2\cdot\dot{9}\dot{3}$	=	$2.939\dot{3}\dot{9}$	=	$2.939\dot{3}9393\dot{9}$
$3\cdot\dot{7}6923\dot{0}$	=	$3\cdot769\dot{2}3076\dot{9}$	=	$3\cdot769\dot{2}3076\dot{9}$
$97\cdot2\dot{6}$	=	$97\cdot266\dot{6}$	=	$97\cdot266\dot{6}6666\dot{6}$
$134\cdot\dot{0}\dot{9}$	=	$134\cdot090\dot{9}\dot{0}$	=	$134\cdot090\dot{9}0909\dot{0}$
				3 carried,
		Sum,	=	$339\cdot626\dot{1}7744\dot{3}$

EXERCISE 74—Page 195.

(1)

Dissimilar.		Similar.		Similar and Coterminous.
$729\cdot3\dot{4}2\dot{7}$	=	$729\cdot342\dot{7}4\dot{2}$	=	$729\cdot342\dot{7}4\dot{2}$
$93\cdot126$	=	$93\cdot126\dot{0}$	=	$93\cdot126\dot{0}0\dot{0}$
				$636\cdot216\dot{7}4\dot{2}$

(2)

Dissimilar.		Similar.		Similar and Coterminous.
$1\cdot4\dot{3}729\dot{1}$	=	$1\cdot437\dot{2}913\dot{7}$	=	$1\cdot437\dot{2}91372913\dot{7}$
$\cdot007\dot{1}\dot{3}$	=	$\cdot007\dot{1}\dot{3}$	=	$\cdot007\dot{1}31313131\dot{3}$
				$1\cdot430\dot{1}60059782\dot{4}$

(3)

Dissimilar.		Similar.		Similar and Coterminous.
$1\cdot12\dot{7}5\dot{4}$	=	$1\cdot12\dot{7}5\dot{4}$	=	$1\cdot12\dot{7}5475475475\dot{4}$
$\cdot4\dot{7}38\dot{4}$	=	$\cdot47\dot{3}84\dot{7}$	=	$\cdot47\dot{3}8473847384\dot{7}$
				$\cdot65\dot{3}7001628090\dot{7}$

(4)

Dissimilar.		Similar.		Similar and Coterminous.
$42\cdot1876\dot{3}$	=	$42\cdot187633\dot{3}$	=	$42\cdot187633\dot{3}33\dot{3}$
$17\cdot000000\dot{8}43\dot{2}$	=	$17\cdot000000\dot{8}43\dot{2}$	=	$17\cdot000000\dot{8}43\dot{2}$
				$25\cdot187632\dot{4}90\dot{0}$

EXERCISE 75—Page 196.

(1)

$2\cdot\dot{9} = 2\frac{9}{9} = 3. \qquad 7\cdot25 \times 3 = 21\cdot75.$

(2)

$\cdot\dot{2}9\dot{7} = \frac{297}{999} = \frac{11}{37}$ and $7\cdot72 = 7\frac{72}{100} = 7\frac{18}{25} = \frac{193}{25}.$

$\frac{11}{37} \times \frac{193}{25} = \frac{2123}{925} = 2\cdot29\dot{5}1\dot{3}.$

(3)

$\cdot\dot{8}\dot{1}8 = \frac{810}{990} = \frac{9}{11}$ and $\cdot77 = \frac{77}{100}.\quad \frac{9}{11} \times \frac{77}{100} = \frac{63}{100} = \cdot63$

(4)

$1\cdot7\dot{3}\dot{5} = 1\frac{728}{990} = 1\frac{364}{495} = \frac{859}{495}$ and $\cdot4705\dot{3} = \frac{42348}{90000} = \frac{3529}{7500}.$

$\frac{859}{495} \times \frac{3529}{7500} = \frac{3031411}{3712500} = \cdot81654\dot{1}6835\dot{0}$

(5)

$4\cdot72\dot{2} = 4\frac{650}{900} = 4\frac{13}{18} = \frac{85}{18}$ and $\cdot\dot{1}9\dot{8} = \frac{198}{999} = \frac{22}{111}.$

$\frac{85}{18} \times \frac{22}{111} = \frac{935}{999} = \cdot\dot{9}3\dot{5}.$

Exercise 76—Page 196.

(1)

$\cdot\dot{0}8\dot{2} = \frac{82}{999}$ and $\cdot\dot{1}2\dot{3} = \frac{123}{999} = \frac{41}{333}.$

$\frac{82}{999} \div \frac{41}{333} = \frac{82}{999} \times \frac{333}{41} = \frac{2}{3} = \cdot\dot{6}.$

(2)

$389\cdot\dot{1}8\dot{5} = 389\frac{185}{999} = \frac{388796}{999}$ and $15\cdot\dot{7} = 15\frac{7}{9} = \frac{142}{9}.$

$\frac{388796}{999} \div \frac{142}{9} = \frac{388796}{999} \times \frac{9}{142} = \frac{2738}{111} = 24\cdot\dot{6}.$

(3)

$\cdot81654\dot{1}6835\dot{0} = \frac{81654086696}{99999900000} = \frac{10206760837}{12499987500}.$

$\cdot4705\dot{3} = \frac{42348}{90000} = \frac{10587}{22500}.$

$\frac{10206760837}{12499987500} \div \frac{10587}{22500} = \frac{10206760837}{12499987500} \times \frac{22500}{10587} = \frac{10206760837}{5881660785} =$

$1\cdot7\dot{3}\dot{5}.$

(4)

$$\cdot\dot{4}\dot{5} = \frac{45}{99} = \frac{5}{11} \text{ and } \cdot\dot{1}1888\dot{1} = \frac{118881}{999999} = \frac{17}{143}.$$

$$\frac{5}{11} \div \frac{17}{143} = \frac{5}{11} \times \frac{143}{17} = \frac{65}{17} = 3\cdot\dot{8}23529411764705\dot{8}.$$

EXERCISE 77.

(1)

$$\frac{1}{2} \text{ of } \frac{3}{7} \text{ of } \frac{4}{15} \text{ of } 14 = \frac{1}{2} \times \frac{3}{7} \times \frac{4}{15} \times \frac{14}{1} = \frac{4}{5} = \cdot 8.$$

(2)

$$\cdot 67 = \frac{61}{90} \text{ and } 2\cdot\dot{1}\dot{3} = 2\frac{13}{99} = \frac{211}{99}.$$

$$\frac{61}{90} \times \frac{211}{99} = \frac{12871}{8910} = 1\cdot 4445566778 +$$

(3)

wk.
·678125 = 4 days 17 hours 55 minutes 30 seconds.
7

4·746875 days.
24

2987500
1493750

17·925000 hours.
60

55·500000 minutes.
60

30·000000 seconds.

(4)

$\cdot 92\dot{4}3\dot{7}$
92

$$\cdot 92\dot{4}3\dot{7} = \frac{92345}{99900} = \frac{18469}{19980}.$$

(5)

Dissimilar.		Similar.		Similar and Coterminous.
$67·2\dot{3}\dot{4}$	=	$67·23434\dot{3}\dot{4}$	=	$67·23434\dot{3}4343\dot{4}$
$98·\dot{7}1\dot{3}$	=	$98·71371\dot{3}7\dot{1}$	=	$98·71371\dot{3}7137\dot{1}$
$91·03471\dot{2}3\dot{4}$	=	$91·03471\dot{2}3\dot{4}$	=	$91·03471\dot{2}3423\dot{4}$

Sum, = $256·98276\dot{9}4903\dot{9}$

Dissimilar.		Similar and Coterminous.
$256·98276\dot{9}4903\dot{9}$	=	$256·98276\dot{9}4903\dot{9}$
$100·123\dot{4}5678\dot{9}$	=	$100·12345\dot{6}7894\dot{5}$
Difference	=	$156·85931\dot{2}7009\dot{4}$

(6)

12) 9 in.

3) 2·75 ft.

5½) $2·91\dot{6}$ yds.
2 2

11) $5·8\dot{3}$

40) $36·53\dot{0}\dot{3}$ rds.

8) $5·9132\dot{5}\dot{7}$ fur.

$·7391571\dot{9}\dot{6}$ miles.

(7)

$17·\dot{4}2857\dot{1}$ sq. ft. $=17\frac{428571}{999999}$ sq. ft. $=17\frac{3}{7}$ sq. ft. $=17$ sq. ft. $61\frac{5}{7}$ in

$100·\dot{8}$ sq. in. $=$ $100\frac{8}{9}$

Difference, $=$ 16 sq. ft. $104\frac{52}{63}$ in.

(8)

$$\begin{array}{r} \cdot 917897\dot{7}\dot{2} \\ 917897 \\ \hline \end{array}$$

$\cdot 917897\dot{7}\dot{2}$ of 2 a. $= \frac{90871875}{99000000} \times \frac{2}{1}$ a. $= \frac{90871875}{49500000} = \frac{3231}{1760} =$ 1 a. 3 r. 13 per. 22 yds.

(9)

$$\begin{array}{r} 11\cdot 2\dot{8}\dot{7} \\ 2 \\ \hline \end{array}$$

$11\cdot 2\dot{8}\dot{7} = 11\frac{285}{990} = 11\frac{19}{66}$. $1\cdot 0\dot{4}28571\dot{1} = 1\frac{428571}{9999990} = 1\frac{3}{70}$.

(10)

$47\cdot 345 = \frac{47345}{1000}$ and $1\cdot\dot{7}\dot{6} = 1\frac{76}{99} = \frac{175}{99}$.

$\frac{47345}{1000} \div \frac{175}{99} = \frac{47345}{1000} \times \frac{99}{175} = \frac{937431}{35000} = 26\cdot 7837\dot{4}2857\dot{1}$.

(11)

Dissimilar.		Similar.		Similar and Coterminous.
$85\cdot\dot{6}\dot{2}$	=	$85\cdot 6\dot{2}\dot{6}$	=	$85.6\dot{2}62\dot{6}$
$13\cdot 7\dot{6}43\dot{2}$	=	$13\cdot 7\dot{6}43\dot{2}$	=	$13\cdot 7\dot{6}43\dot{2}$
		Difference,	=	$71\cdot 86\dot{1}9\dot{3}$

(12)

$\cdot 734$ of a lb. $= 11\cdot 744$ oz.
$\cdot 198$ of an oz. $= \ \ .198$ oz.

Difference, $= 11\cdot 546$ oz.

(13)

2 ft. $5\frac{1}{3}$ in. $= 29\frac{1}{3}$ in. $= \frac{88}{3}$ in.

$27\cdot\dot{3}$ ft. $= 27\frac{1}{3}$ ft. $= 328$ in

$20.1\dot{6}$ ft. $= 20\frac{1}{6}$ ft. $= 242$ in.

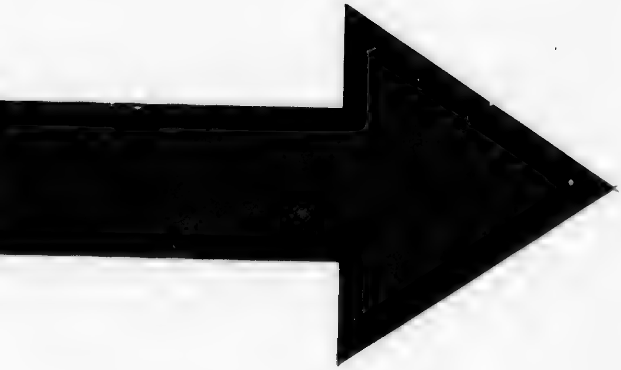
$328 \times 242 \div \frac{88}{3} = \frac{328}{1} \times \frac{242}{1} \times \frac{3}{88} = 2706$ in. $= 75\frac{1}{6}$ yds.

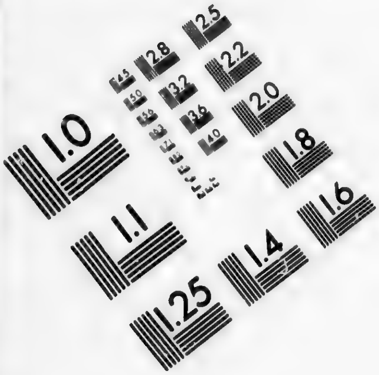
(14)

$3\cdot 1\dot{4}\dot{5} = 3\frac{144}{990} = 3\frac{8}{55} = \frac{173}{55}$ and $4\cdot\dot{2}9\dot{7} = 4\frac{297}{999} = 4\frac{11}{37} = \frac{159}{37}$.

$\frac{173}{55} \times \frac{159}{37} = \frac{27507}{2035} = 13\cdot 5\dot{1}695\dot{3}3$.

[redacted]





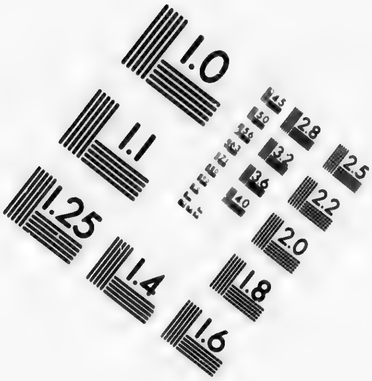
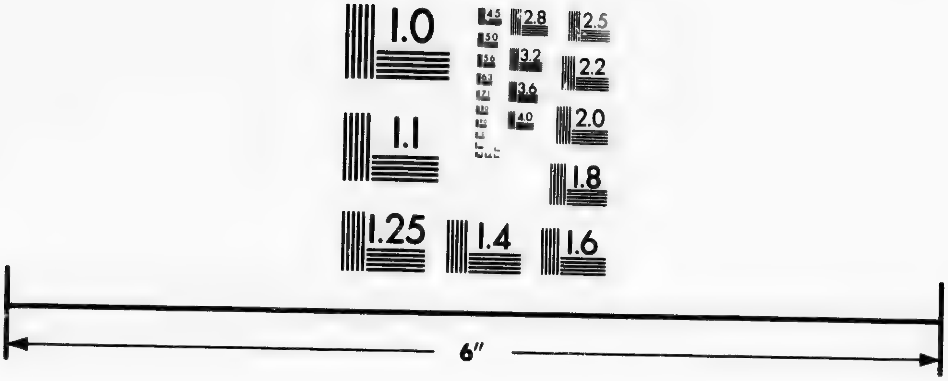
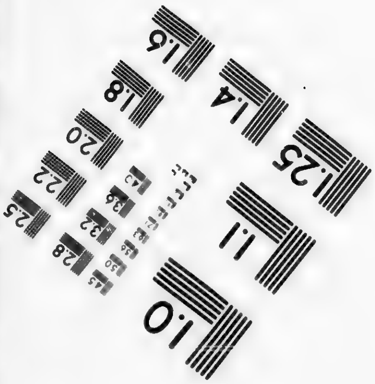
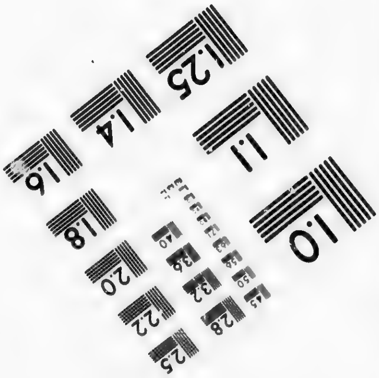


IMAGE EVALUATION
TEST TARGET (MT-3)







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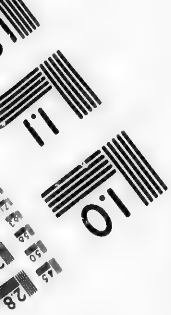
1.0
2.8
2.5
3.2
3.6
2.2
4.0
2.0
1.8
1.6



1.1
1.0

(15)

$\frac{3}{40}$. Here $40 = 2^3 \times 5$. Therefore the equivalent decimal will contain 3 places

$\frac{7}{24}$. " $24 = 2^3 \times 3$. " " " " 3 "

$\frac{8}{15}$. " $15 = 5 \times 3$. " " " " 1 "

$\frac{11}{144}$. " $144 = 2^4 \times 3^2$. " " " " 4 "

$\frac{6}{90} = \frac{1}{15}$ " $15 = 5 \times 3$. " " " " 1 "

$\frac{119}{3584}$. " $3584 = 2^9 \times 7$. " " " " 9 "

(16)

$81\tfrac{2}{3} = 81{\cdot}\dot{6}$ and $328\tfrac{23}{99} = 328{\cdot}\dot{2}\dot{3}$.

Dissimilar.		Similar		Similar and Coterminous
$81{\cdot}\dot{6}$	=	$81{\cdot}66\dot{6}$	=	$81{\cdot}66\dot{6}6666\dot{6}$
$61{\cdot}12\dot{6}$	=	$61{\cdot}12\dot{6}$	=	$61{\cdot}12\dot{6}6666\dot{6}$
$328{\cdot}\dot{2}\dot{3}$	=	$328{\cdot}23\dot{2}\dot{3}$	=	$328{\cdot}23\dot{2}3232\dot{3}$
$5{\cdot}\dot{6}2\dot{4}$	=	$5{\cdot}62\dot{4}6\dot{2}$	=	$5{\cdot}62\dot{4}6246\dot{2}$
				2 carried
		Sum,	=	$476{\cdot}65\dot{0}2811\dot{9}$

(17)

$$\left(\frac{4{\cdot}\dot{4}-2{\cdot}8\dot{3}}{1{\cdot}\dot{6}+2{\cdot}\dot{6}2\dot{9}} \times \frac{6{\cdot}8\times 3}{2{\cdot}25}\right) + \frac{2{\cdot}8\times 2{\cdot}\dot{2}\dot{7}}{1{\cdot}1\dot{3}\dot{6}}$$

$$= \left(\frac{1{\cdot}6\dot{1}}{4{\cdot}\dot{2}9\dot{6}} \times \frac{20{\cdot}4}{2{\cdot}25}\right) + \frac{2\frac{4}{5}\times 2\frac{27}{99}}{1\frac{135}{990}}$$

$$= \left(\frac{1\frac{55}{90}}{4\frac{296}{999}} \times \frac{20\frac{2}{5}}{2\frac{1}{4}}\right) + \frac{2\frac{4}{5}\times 2\frac{3}{11}}{1\frac{3}{22}}$$

$$= \left(\frac{1\frac{11}{18}}{4\frac{296}{999}} \times \frac{\frac{102}{5}}{\frac{9}{4}}\right) + \frac{\frac{14}{5}\times\frac{25}{11}}{\frac{25}{22}}$$

$$= \left(\frac{\frac{29}{18}}{\frac{4292}{999}} \times \frac{\frac{34}{5}}{\frac{3}{4}}\right) + \frac{\frac{14}{1}\times\frac{5}{11}}{\frac{25}{22}}$$

$$= \left(\frac{\frac{1}{2}}{\frac{148}{111}} \times \frac{\frac{34}{5}}{\frac{3}{4}}\right) + \frac{\frac{70}{11}}{\frac{25}{22}} = \left(\tfrac{111}{296} \times \tfrac{136}{15}\right) + \tfrac{28}{5}$$

$$= \left(\tfrac{3}{8} \times \tfrac{136}{15}\right) + \tfrac{28}{5} = \tfrac{17}{5} + \tfrac{28}{5} = \tfrac{45}{5} = 9.$$

Exercise 78—Page 198.

(1)

```
      ▼                     ▼                      ▼
  9)4312131             3)4312131             8)4312131
   ────────              ────────              ────────
   9)224322..8          3)1234023..2           8)242343..7
    ───────              ────────               ───────
     9)12043..5          3)224322..2             8)14022..2
      ──────               ───────                ─────
       9)344..7             3)41240..2             8)1032..1
        ───                  ──────                 ────
         9)21..0             3)12043..1              8)32..6
          ──                   ─────                  ──
           1..2                3)2144..1               2..1
                                ────
                                 3)344..2
                                  ───
                                  3)113..0
                                   ───
                                   3)21..0
                                    ──
                                    3)3..2
                                     ─
                                     1..0

    ▼            IX                III                        VIII
 4312131 =     120758 =       10200211222 =                  216127
   5              9            3                               8
  ──             ──            ─                              ──
  23             11            3           299                17
   5              9            3             3                 8
 ───             ──           ──           ───               ───
 116             99           11           898               142
   5              9            3             3                 8
 ───            ───           ──          ────              ────
 582            898           33          2695              1137
   5              9            3             3                 8
────           ────           ──          ────              ────
2911           8087           99          8087              9098
   5              9            3             3                 8
─────         ─────          ───         ─────             ─────
14558         72791 dec.     299         24263             72791 dec.
    5                                        3
─────                                    ─────
72791 decimal.                           72791 dec.
```

(3)

$$976{\cdot}432 \div {\cdot}00000096 = 97643200000 \div 96 \text{ and } 96 = 12 \times 8.$$

$$\begin{array}{r}12)\underline{97643200000}\\ 8)\underline{8136933333{\cdot}\dot{3}}\\ 1017116666{\cdot}\dot{6}\end{array}$$

(4)

$$\frac{\left(2\frac{7}{8} + {\cdot}5625 - 1{\cdot}5 + \frac{1}{16}\right) \div \frac{11}{8}}{\dfrac{\left(1\frac{8}{11} \times \frac{4}{9} \times 296 \times \frac{1}{101} \div \frac{11}{8}\right) \div {\cdot}9472947}{\frac{19}{6}}} =$$

$$\frac{\left(2\frac{7}{8} + \frac{9}{16} - 1\frac{1}{2} + \frac{1}{16}\right) \times \frac{8}{11}}{\dfrac{\left(\frac{19}{11} \times \frac{4}{9} \times \frac{296}{1} \times \frac{1}{101} \times \frac{8}{11}\right) \div \frac{9472}{9999}}{\frac{19}{6}}} = \frac{2 \times \frac{8}{11}}{\dfrac{\frac{19}{11} \times \frac{4}{9} \times \frac{296}{1} \times \frac{1}{101} \times \frac{8}{11} \times \frac{9999}{9472}}{\frac{19}{6}}} =$$

$$\frac{\frac{16}{11}}{\dfrac{\frac{19}{11}}{\frac{19}{6}}} = \frac{\frac{16}{11}}{\frac{6}{11}} = \frac{16}{6} = 2\frac{2}{3}.$$

(5)

$$\begin{array}{rrrrcrrrrr}
\text{lbs.} & \text{oz.} & \text{dr.} & \text{scr.} & & \text{lbs.} & \text{oz.} & \text{dr.} & \text{scr.} & \text{grs.}\\
9 & 7 & 7 & 2 &) & 97 & 3 & 4 & 1 & 17\\
\underline{12} & & & & & \underline{12} & & & & \\
115 & & & & & 1167 & & & & \\
\underline{8} & & & & & \underline{8} & & & & \\
927 & & & & & 9340 & & & & \\
\underline{3} & & & & & \underline{3} & & & & \\
2783 & & & & & 28021 & & & & \\
\underline{20} & & & & & \underline{20} & & & & \\
55660 & & & &) & 560437\left(10\frac{3837}{55660}.\right. & & & & \\
 & & & & & \underline{55660} & & & & \\
 & & & & & 3837 & & & &
\end{array}$$

(6)

15 yds. = 540 in. and 7 ft. = 84 in.

6 ft. = 72 in. and 4 ft. = 48 in.

$(540\times84\times13)-(72\times48\times13)=589680-44928=544752.$

$544752\div108=5044.$

(7)

9 ft. 6′ 4″ 7‴
11 7 9 11

8 8 10⁗ 2′′′′′ 5′′′′′′
7 1 9 5 3
5 6 8 8 1
104 10 2 5

111 0 9 7 4 5 5

(8)

$$\frac{4\frac{2}{7}+\frac{8}{9}-\frac{7}{12}}{\frac{2}{3}\text{ of }\frac{9}{13}+\frac{1}{6}\text{ of }\frac{5}{9}}=\frac{4\frac{74}{63}-\frac{7}{12}}{\frac{6}{13}+\frac{5}{54}}=\frac{\frac{1157}{252}}{\frac{389}{702}}=\frac{\frac{1157}{14}}{\frac{389}{39}}=\frac{45123}{5446}=8\frac{1555}{5446}.$$

(9)

pts.
2)782436
4)391218..0 pt.
2)97804..2 qt.
4)48902..0 gal.
12225..2 pks.

12225 bush. 2 pks. 0 gal. 2 qts.

(10)

$$\begin{array}{r|l}
77 & \cancel{77}..42..27..21..33..14..\cancel{7}..\cancel{11}..63..30 \\
27 & 6..\cancel{27}..\cancel{3}..\cancel{3}..2 \quad \cancel{9}..30 \\
10 & \cancel{2} \quad \cancel{2} \quad \cancel{10}
\end{array}$$

$77\times27\times10=20790=$ l. c. m.

(11)

xii

28e4)36t87942(1375t·12
28e4
9e47
82t0
18679
17274
14054
11888
23882
23554
32t·0
28e·4
5t·80
55·t8
4·94

ix

3762814
9
34
9
312
9
2810
9
25298
9
227683
9
2049151

(12)

150528 = $2^{10}\times3\times7^2$.

10+1 = 11.
1+1 = 2
2+1 = 3
11×3×2 = 66.

(13)

2 wks. 2 dys. = 16 dys.
·1234625
16
7407750
1234625
1·9754000 dys.
24
39016000
19508000
23·4096000 hrs.
60
24·5760000 min.
60
34·5600000 = 34 $\frac{14}{25}$ sec.

(14)

728½ = 8½ + 2 × 10 + 7 × 10 × 10.

lbs.	oz.	dr.				lbs.	oz.	dr.
27	4	3	× 8½	=		231	11	9½
		10						
272	9	14	× 2	=		545	3	12
		10						
2726	2	12	× 7	=		19083	3	4
						19860	2	9½

(15)

£16 3s. 8¾d. = \$64·74$\frac{7}{12}$ and £67 17s. 7¼d. = \$271.52$\frac{1}{12}$.
\$98·17 + \$42·20 + \$64·74$\frac{7}{12}$ + \$97·19 + \$127·87½ = \$430·27$\frac{1}{12}$
\$430·27$\frac{1}{12}$ − \$271·52$\frac{1}{12}$ = \$158·75.

(16)

$$\cdot\dot{8} = \tfrac{8}{9}. \quad \cdot\dot{7}\dot{6} = \tfrac{76}{99}. \quad \cdot 91\dot{2}\dot{3} = \frac{9123-91}{9900} = \tfrac{9032}{9900} = \tfrac{2258}{2475}.$$

$$\cdot 00\dot{3}32\dot{7} = \frac{3327-3}{999000} = \tfrac{3324}{999000} = \tfrac{277}{83250}.$$

(18)

$$\frac{\left[\left\{\left(2\tfrac{1}{3} \times \cdot 5 \text{ of } 1\tfrac{5}{7}\right) + 9\tfrac{1}{2}\right\} + \cdot\dot{0}\dot{9} + \tfrac{23}{231}\right\} - 11\tfrac{6}{17}\Big] \div \left(\tfrac{1}{5} \text{ of } \cdot 1\dot{6}\right)}{\dfrac{\left[\left(\cdot 763\dot{2}76\dot{3} \times 11\right) \times \tfrac{1}{8} \text{ of } \tfrac{101}{66}\right] \times \left(\tfrac{1}{3} \text{ of } \cdot 2 \text{ of } \cdot\dot{3} \text{ of } \cdot 25 \text{ of } 96\right) \div \cdot\dot{2}}{\tfrac{1}{4} \text{ of } \cdot 6\dot{7}3246\dot{7} \div \tfrac{1}{5}}} =$$

$$\frac{\left[\left\{\left(\tfrac{7}{3} \times \tfrac{1}{2} \times \tfrac{12}{7}\right) + 9\tfrac{1}{2}\right\} + \tfrac{1}{11} + \tfrac{23}{231}\right\} - 11\tfrac{6}{17}\Big] \div \left(\tfrac{1}{6} \text{ of } \tfrac{1}{6}\right)}{\dfrac{\left(\tfrac{7633}{9999} \times \tfrac{11}{1} \times \tfrac{1}{8} \times \tfrac{101}{66} \times \tfrac{1}{3} \times \tfrac{1}{5} \times \tfrac{1}{3} \times \tfrac{1}{4} \times \tfrac{96}{1}\right) \div \tfrac{2}{9}}{\tfrac{1}{4} \times \tfrac{67324}{99999} \div \tfrac{1}{5}}} =$$

$$\frac{\left\{\left(2 + 9\tfrac{1}{2} + \tfrac{1}{11} + \tfrac{23}{231}\right) - 11\tfrac{6}{17}\right\} \div \left(\tfrac{1}{6} \times \tfrac{1}{6}\right)}{\dfrac{\tfrac{7633}{9999} \times \tfrac{11}{1} \times \tfrac{1}{8} \times \tfrac{101}{66} \times \tfrac{1}{3} \times \tfrac{1}{5} \times \tfrac{1}{3} \times \tfrac{1}{4} \times \tfrac{96}{1} \times \tfrac{9}{2}}{\tfrac{1}{4} \times \tfrac{67324}{99999} \times \tfrac{5}{1}}} =$$

$$\frac{\dfrac{\left(12 - 11\tfrac{6}{17}\right) \div \left(\tfrac{1}{6} \times \tfrac{1}{6}\right)}{\tfrac{18}{5}}}{[illegible]} = \frac{\dfrac{\tfrac{11}{17} \times \tfrac{6}{1} \times \tfrac{6}{1}}{\tfrac{18}{5}}}{[illegible]} = \frac{\dfrac{[illegible]}{\tfrac{18}{5}}}{[illegible]} = \frac{[illegible]}{[illegible]} =$$

$$[illegible] = 3\tfrac{[illegible]}{[illegible]}.$$

(19)

8 children will have 8 children's shares.

One woman will have 3 children's shares ∴ 6 women will have 6 × 3 = 18 children's shares.

One man will have 6 children's shares ∴ 4 men will have 4 × 6 = 24 children's shares.

4 men, 6 women, and 8 child. will therefore have 50 child. shares

£550 3s. 1½d. ÷ 50 = £11 0s. 0¾d. = child's share.

£ 11, 0s. 0¾d. × 3 = £33 0s. 2¼d. = woman's share.

£ 33 0s. 2¼d. × 2 = £66 0s. 4½d. = man's share.

(20)

$$16\tfrac{7}{11} + 19\tfrac{4}{5} + 23\tfrac{7}{8} + 129\tfrac{6}{7} = 16 + 19 + 23 + 129 + \left(\tfrac{7}{11} + \tfrac{4}{5} + \tfrac{7}{8} + \tfrac{6}{7}\right) = 187 + 3\tfrac{519}{3080} = 190\tfrac{519}{3080}.$$

(21)

$$8100 = 2^2 \times 3^4 \times 5^2.$$

1..3..9..27..81
1..2..4

1..3..9..27..81..2..6..18..54..162..4..12..36..108..324
1..5..25

1..3..9..27..81..2..6..18..54..162..4..12..36..108..324..5..15..45..135..405..10..30..90..270..810..20..60..180..540..1620..25..75..225..675..2025..50..150..450..1350..4050..100..300..900..2700..8100.

Therefore the divisors of 8100 are 1, 2, 3, 4, 5, 6, 9, 10, 12, 15, 18, 20, 25, 27, 30, 36, 45, 50, 54, 60, 75, 81, 90, 100, 108, 135, 150, 162, 180, 225, 270, 300, 324, 405, 450, 540, 675, 810, 900, 1350, 1620, 2025, 2700, 4050, 8100.

(22)

2691)11817(4
10764

1053)2691(2
2106

585)1053(1
585

468)585(1
468

117)468(4
468

9828 is divisible by 117 ∴ 117 is the G. C. M.

(23)

sec.
60)2551443

60)42524 . . 3

24)708 . . 44

29 . . 12

29d., 12 h., 44 m., 3 sec.

sec.
60)31556928

60)525948 . . 48

24)8765 . . 48

365 . . 5

365 d., 5 h., 48 m., 48 sec.

(24)

14 ft. 11 in. = 179 in.
38 miles = 2407680 in.
2407680 ÷ 179 = 13450$\frac{130}{179}$.

(25)

11 ft. × 13 ft. × 15 ft. = 2145 cub. ft.
One cubic foot weighs 62½ lbs. 2145×62½ = 134062½ = weight of 2145 cub. ft.
One gallon weighs 10 lbs. 134062½ ÷ 10 = 13406¼ = gals. in 134062½ lbs.

(26)

£73 × 400 = \$292·00
17s. × 20 = 3·40
11¾d. = 47 far. × 5 ÷ 12 = ·19$\frac{7}{12}$

£73 17s. 11¾d. = \$295·59$\frac{7}{12}$

(27)

$93\frac{4}{11} - 76\frac{17}{23} = 92\frac{15}{11} - 76\frac{17}{23} = 16\frac{158}{253} = \frac{4206}{253}$

$\frac{4206}{253} \div \frac{17}{253} = \frac{4206}{253} \times \frac{253}{17} = \frac{4206}{17} = 247\frac{7}{17}$

(28)

$$\frac{5\frac{5}{8}\div\frac{1}{2}}{1\frac{1}{3}\text{ of }\frac{2}{3}+10\frac{1}{3}}\times\frac{3}{5}\text{ of }\frac{1\frac{1}{2}\text{ of }4\frac{1}{2}}{13\frac{1}{2}\text{ of }5\frac{1}{3}}=\frac{\frac{45}{8}\times\frac{2}{1}}{\frac{4}{3}\times\frac{2}{3}\times\frac{31}{3}}\times\frac{3}{5}\times\frac{\frac{3}{2}\times\frac{9}{2}}{\frac{27}{2}\times\frac{16}{3}}=$$

$$\frac{\frac{45}{16}\times2}{\frac{2}{3}\times\frac{1}{31}}\times\frac{3}{5}\times\frac{\frac{3}{2}\times\frac{27}{2}}{\frac{27}{1}\times2}=$$

$$\frac{45\times3\times31}{16\times2}\times\frac{3}{5}\times\frac{27}{2\times3\times27\times2}=\frac{3\times9\times31}{16\times2\times2\times2}=$$

$$\frac{837}{128}=6\tfrac{69}{128}.$$

(29)

```
      XI
  5)91342
  5)19074..4
   5)4015..1
    5)891..0
    5)184..3
     5)39..3
      5)8..2
        1..3
```

```
      XI
  12)91342
   12)83t4..9
    12)773..1
     12)70..3
         6..5
```

```
      XI
  2)91342
  2)46176..1
  2)23093..0
  2)115t1..1
   2)62t6..0
   2)3153..0
   2)1627..0
    2)869..0
    2)434..1
    2)217..1
    2)109..0
     2) 5t..0
     2)2t..1
     2)15..0
      2)8..0
      2)4..0
      2)2..0
        1..0
```

(29 continued.)

XI

91342
11
100
11
1103
11
12137
11
133509 dec.

V

13233014
5
8
5
42
5
213
5
1068
5
5340
5
26701
5
133509 dec.

XII

65319
12
77
12
927
12
11125
12
133509 dec.

II

100000100110000101
2
2
2
4
2
8
2
16
2
32
2
65
2
130
2
260

260
2
521
2
1043
2
2086
2
4172
2
8344
2
16688
2
33377
2
66754
2
133509 dec.

(30)

$2\overline{)7680} = 2^9\times3\times5$

$2\overline{)3840}$

$2\overline{)1920}$

$2\overline{)960}$

$2\overline{)480}$

$2\overline{)240}$

$2\overline{)120}$

$2\overline{)60}$

$2\overline{)30}$

$3\overline{)15}$

5

(31)

m.	f.	p.	y.	ft.	in.
72	3	7	2	1	7

8

579 fur.

40

23167 per.

$5\frac{1}{2}$

115837

$11583\frac{1}{2}$

$127420\frac{1}{2}$ yds.

3

$382262\frac{1}{2}$ ft.

12

4587157 in.

12

55045884 lines

(32)

$·47 × 97 = $45·59.

(33)

$(73\times4\times11)\div128=25\frac{3}{32}$.

$\$3·62\frac{1}{2}\times25\frac{3}{32}=\$90·96\frac{31}{64}$.

(34)

$93·7\dot{2}\dot{3} = 93\frac{716}{990} = \frac{92786}{990}$ and $29·4\dot{1}7\dot{3} = 29\frac{4169}{9990} = \frac{293879}{9990}$.

$$\frac{92786}{990} \div \frac{293879}{9990} = \frac{92786}{\underset{11}{\cancel{990}}} \times \frac{\overset{111}{\cancel{9990}}}{293879} = \frac{92786\times111}{11\times293879} =$$

$$\frac{10299246}{3232669} = 3·185988 +$$

(35)

One bushel of oats weighs 34 lbs. ∴ in 73429 lbs. there are $73429 \div 34 = 2159\frac{23}{34}$ bushels.

(36)

In 719630 lbs. of wheat there are $719630 \div 60 = 11993\frac{5}{6}$ bus,

$\$1·80 \times 11993\frac{5}{6} = \$21588·90$,

Or $1·80 per bushel = 3 cents per lb.

719630 × 3 = 2158890 cents. = $21588·90.

(37)

$72·14 + $93·76 = $165.90

$165·90 × 9·47 = $1571·0730

$1571·0730 ÷ 11 = $142·8248+

(38)

```
21389)180781(8
      171112
      ------
        9669)21389(2
             19338
             -----
              2051)9669(4
                   8204
                   ----
                   1465)2051(1
                        1465
                        ----
                         586)1465(2
                             1172
                             ----
                              293)586(2
```

Last divisor 293 = G. C. M.

(39)

$\frac{7}{11}, \frac{4}{5}, \frac{9}{7}, \frac{8}{33}, \frac{11}{14}, \frac{7}{10}, \frac{1}{2}$.

The least common multiple of 11, 5, 7, 33, 14, 10 and 2 is 2310.

The multiplier for both terms of the first fraction is $\frac{2310}{11} = 210$; for the second, $\frac{2310}{5} = 462$; for the third, $\frac{2310}{7} = 330$; for the fourth, $\frac{2310}{33} = 70$; for the fifth, $\frac{2310}{14} = 165$; for the sixth, $\frac{2310}{10} = 231$; for the seventh, $\frac{2310}{2} = 1155$.

Multiplying by these numbers, we obtain $\frac{1470}{2310}$, $\frac{1848}{2310}$, $\frac{2970}{2310}$, $\frac{560}{2310}$, $\frac{1815}{2310}$, $\frac{1617}{2310}$, and $\frac{1155}{2310}$ for the required fractions.

(40)

$·11 × 17 = $1·87. $·37½ × 19 = $7·12½. $2·17 × 14½ = $31·46½. $·27 × 67 = $18·09. $1·37½ × 15 = $20·62½. $1·87 + $7·12½ + $31·46½ + $4·75 + $11·50 + $18·09 + $20·62½ + $7·93 = $103·35½.

EXERCISE 84—Page 210.

(1)

Baskets.

$$11 : 87 :: \$13{\cdot}42 : \frac{\overset{1\cdot22}{\cancel{\$13\cdot42}}\times 87}{\cancel{11}} = \$106{\cdot}14.$$

(2)

Cords.

$$28 : 25 :: \$266 : \frac{\overset{19}{\cancel{\$266}}\times 25}{\underset{2}{\cancel{28}}} = \$237{\cdot}50.$$

(3)

$$\$29{\cdot}20 : \$83{\cdot}60 :: 16 : \frac{\text{days } \overset{4}{\cancel{16}}\times 83{\cdot}60}{\underset{7\cdot3}{\cancel{29\cdot20}}} = 45\tfrac{59}{73} \text{ days.}$$

(4)

Bags.

$$16 : 156 :: \$12{\cdot}80 : \frac{\overset{\cdot8}{\cancel{\$12\cdot80}}\times 156}{\cancel{16}} = \$124{\cdot}80.$$

(5)

Feet. ft.

$$5 : 112 :: 7 : \frac{7\times 112}{5} = 156\tfrac{4}{5} \text{ ft.}$$

(6)

Cows. days.

$$55 : 27 :: 99 : \frac{\overset{9}{\cancel{99}}\times 27}{\underset{5}{\cancel{55}}} = 48\tfrac{3}{5} \text{ days.}$$

(7)

Acres. bus.

$$5 : 48 :: 9 : \frac{9\times 48}{5} = 86\tfrac{2}{5} \text{ bush.}$$

(8)

Perches. days.

$$73 : 803 :: 2 : \frac{2\times\overset{11}{\cancel{803}}}{\cancel{73}} = 22 \text{ days.}$$

(9)

Pails. lbs.

$$176 : 1128 :: 100 : \frac{100\times\overset{141}{\cancel{1128}}}{\underset{22}{\cancel{176}}} = 640\tfrac{10}{11} \text{ lbs.}$$

(10)

$$108 : 465 :: \$20{\cdot}88 : \frac{\overset{\cdot 58}{\cancel{\$20{\cdot}88}}\times\overset{155}{\cancel{465}}}{\underset{\cancel{36}}{\cancel{108}}} = \$\,89{\cdot}90.$$

(11)

\$ \$ brls.

$$16 : 1278 :: 72 : \frac{\overset{9}{\cancel{72}}\times\overset{639}{\cancel{1278}}}{\underset{\cancel{2}}{\cancel{16}}} = 5751 \text{ barrels.}$$

(12)

Men. Acres

$$11 : 3 :: 165 : \frac{\overset{15}{\cancel{165}}\times 3}{11} = 45 \text{ acres.}$$

(13)

Barrels. loaves

$$4 : 67 :: 250 : \frac{\overset{125}{\cancel{250}}\times 67}{\underset{2}{\cancel{4}}} = 4187\tfrac{1}{2} \text{ loaves.}$$

(14)

Bushels. brls.

$$190 : 38 :: 16 : \frac{16\times\cancel{38}}{\underset{5}{\cancel{190}}} = 3\tfrac{1}{5} \text{ barrels.}$$

(15)

Days. men

$$15 : 12 :: 90 : \frac{\overset{6}{\cancel{90}} \times 12}{\cancel{15}} = 72 \text{ men}$$

(16)

D'. work. brls.

$$17 : 279 :: 2 : \frac{2 \times 279}{17} = 32\tfrac{14}{17} \text{ barrels.}$$

(17)

Hours. miles.

$$1 : 24 :: 27 : 27 \times 24 = 648 \text{ miles.}$$

(18)

Cows. lbs.

$$7 : 23 :: 30 : \frac{30 \times 23}{7} = 98\tfrac{4}{7} \text{ lbs.}$$

EXERCISE 85—Page 211.

(1)

$$\tfrac{3}{16} : \tfrac{21}{26} :: \$9750 : \frac{\overset{375}{\cancel{9750}}}{1} \times \frac{\overset{7}{\cancel{21}}}{\cancel{26}} \times \frac{16}{\cancel{3}} = \$42000.$$

(2)

Yard. s. s.

$$\tfrac{7}{8} : \tfrac{1}{4} :: \tfrac{5}{6} : \frac{5}{\underset{3}{\cancel{6}}} \times \frac{1}{\cancel{4}} \times \frac{\overset{2}{\cancel{8}}}{7} = \tfrac{5}{21} = 2\tfrac{6}{7}\text{d.}$$

(3)

Tons.

$$\tfrac{7}{9} : 8\tfrac{1}{3} :: \$7{\cdot}49 : \frac{\$7{\cdot}49 \times 8\tfrac{1}{3}}{\tfrac{7}{9}} = \frac{\overset{1\cdot07}{\cancel{7\cdot49}}}{1} \times \frac{25}{\cancel{3}} \times \frac{\overset{3}{\cancel{9}}}{\cancel{7}} = \$80{\cdot}25.$$

(4)

Yards.

$$5\tfrac{3}{5} : \tfrac{4}{7} :: \$28{\cdot}42 : \frac{\overset{\overset{\cdot14}{\cancel{4\cdot06}}}{\cancel{28\cdot42}}}{\cancel{1}} \times \frac{4}{\cancel{7}} \times \frac{5}{\cancel{28}} = \$2{\cdot}80.$$

(5)

$$\overset{\text{Dollar.}}{\tfrac{12}{25}} : \overset{\text{bag}}{\tfrac{7}{20}} :: \tfrac45 : \frac{\cancel{4}}{\cancel{5}} \times \frac{7}{\underset{\cancel{4}}{\cancel{20}}} \times \frac{\overset{\cancel{5}}{\cancel{25}}}{12} = \tfrac{7}{12} \text{ of a bag.}$$

(6)

$$\overset{\$}{100} : \overset{\$}{472\tfrac{11}{25}} :: \overset{\$}{98\tfrac78} : \frac{98\frac78 \times 472\frac{11}{25}}{100} = \frac{98\cdot875 \times 472\cdot44}{100} = \$467\cdot12\tfrac12.$$

(7)

$$\overset{\text{Tons.}}{17\tfrac35} : \overset{\text{days.}}{11\tfrac{11}{17}} :: 107\tfrac{3}{11} : \frac{107\frac{3}{11}\times 11\frac{11}{17}}{17\frac35} = \frac{\overset{295}{\cancel{1180}}}{11} \times \frac{\overset{9}{\cancel{198}}}{17} \times \frac{5}{\underset{\cancel{4}}{\cancel{88}}} = 70\tfrac{185}{187}\text{ dys.}$$

(8)

$$\overset{\text{Tons.}}{15\tfrac{7}{13}} : \overset{\text{cords.}}{11\tfrac{9}{26}} :: 22\tfrac49 : \frac{22\frac49\times11\frac{9}{26}}{15\frac{7}{13}} = \frac{\cancel{202}}{9}\times\frac{295}{\underset{2}{\cancel{26}}}\times\frac{\cancel{13}}{\cancel{202}} = 16\tfrac{7}{18}\text{ cords.}$$

(9)

$$\overset{\text{Yds.}}{\tfrac12 \text{ of } \tfrac35 \text{ of } 3\tfrac13} : \overset{\text{yds}}{\tfrac38 \text{ of } \tfrac12 \text{ of } \tfrac{55}{56}} :: \overset{\$}{\tfrac27 \text{ of } \tfrac{3}{11} \text{ of } 4\tfrac23} : \frac{\frac27 \text{ of } \frac{3}{11}\text{ of } 4\frac23 \times \frac38 \text{ of } \frac12 \text{ of } \frac{55}{56}}{\frac12 \text{ of } \frac35 \text{ of } 3\frac13} =$$

$$\frac{\cancel{4}}{\cancel{11}} \times \frac{\overset{15}{\cancel{165}}}{\underset{224}{\cancel{896}}} = \$\tfrac{15}{224}.$$

EXERCISE 86—Page 212.

(1)

37 sq. yds. 4 ft. 120 in. = 48648 in., and 9 sq. yds. 2 ft. = 11952 in.

$$\overset{\text{Inches.}}{11952 : 48648} :: \$3\cdot50 : \frac{3\cdot50 \times \overset{\overset{2027}{\cancel{6081}}}{\cancel{48648}}}{\underset{\underset{498}{\cancel{1494}}}{\cancel{11952}}} = \$14\cdot245 +$$

(2)

12 lbs. 10 oz. = 154 oz.

Ounces.

1 : 154 :: $1·25 : 1·25 × 154 = $192·50.

(3)

10 yds. = 40 qrs., and 3 yds. 2 qrs. = 14 qrs.

Quarters.

$$40 : 14 :: \$3{\cdot}40 : \frac{\overset{\cdot 17}{\cancel{3{\cdot}40}} \times \overset{7}{\cancel{14}}}{\underset{\underset{\cancel{20}}{}}{\cancel{40}}} = \$1{\cdot}19.$$

(4)

15 oz. 12 dwt. 16 grs. = 7504 grs., and 13 oz. 14 grs. = 6254 grs.

Grains.

$$7504 : 6254 :: \$3{\cdot}80 : \frac{\overset{\cdot 95}{\cancel{3{\cdot}80}} \times \overset{3127}{\cancel{6254}}}{\begin{array}{c}\cancel{7504}\\ \cancel{1876}\\ 938\end{array}} = \$3{\cdot}167 +$$

(5)

3 lbs. 1 oz. 11 dwt. = 751 dwt. and 12 lbs. 6 oz. 4 dwt. = 3004 dwt.

Dwt. $

$$3004 : 751 :: 600 : \frac{\overset{150}{\cancel{600}} \times \cancel{751}}{\begin{array}{c}\cancel{3004}\\ \cancel{4}\end{array}} = \$150.$$

(6)

Barrels. h. m. s.

$$54 : 24 :: 2\ 46\ 30 : \frac{2 \text{ h. } 46 \text{ m. } 39 \text{ s.} \times \overset{4}{\cancel{24}}}{\begin{array}{c}\cancel{54}\\ 9\end{array}} = 1 \text{ hr. } 14 \text{ min.}$$

(7)

3 yds. 3 qrs. 2 na. 1 in $=2660\frac{1}{2}$ in. 3 Fl. e. 2 qrs. 1 na. $=101\frac{1}{4}$ in.
And £4 17s. $8\frac{1}{4}$d. $=1172\frac{1}{4}$d.

$$\begin{array}{l}\text{Inches.}\qquad\qquad\text{d.}\\ 01\frac{1}{4} : 2660\frac{1}{2} :: 1172\frac{1}{4} : \dfrac{1172\frac{1}{4}\times 2660\frac{1}{2}}{101\frac{1}{4}} = \dfrac{\overset{521}{\cancel{4689}}}{\cancel{4}}\times\dfrac{5321}{2}\times\dfrac{\cancel{4}}{\underset{45}{\cancel{405}}} =\end{array}$$

$\frac{2772241}{90}$ d. $=$ £128 6s. $10\frac{61}{90}$d.

(8)

$8\frac{13}{24}$ lbs. $=136\frac{2}{3}$ oz.

$$\begin{array}{l}\text{nces.}\qquad\quad\text{s.}\\ : 136\frac{2}{3} :: 8\frac{31}{32} : \dfrac{\overset{7}{\cancel{287}}}{\underset{16}{\cancel{32}}}\times\dfrac{\overset{205}{\cancel{410}}}{\cancel{3}}\times\dfrac{\overset{3}{\cancel{9}}}{\cancel{41}} = \overset{\text{s.}}{\frac{4305}{16}} = \text{£}13\ 9\text{s. }0\tfrac{3}{4}\text{d.}\end{array}$$

(9)

$$\begin{array}{l}\text{Pages.}\\ 27 : 400 :: 156 : \dfrac{\overset{52}{\cancel{156}}\times 400}{\underset{109}{\cancel{327}}} = 190\tfrac{90}{109},\ \text{i. e. on the 191st p.}\end{array}$$

(10)

6 a., 3 r., 14 p. $=7494$ p., and 35 a., 2 r., 10 p. $=5690$ p.

$$\begin{array}{l}\text{Perches.}\qquad\quad\text{£}\\ 7494 : 5690 :: 100 : \dfrac{\overset{50}{\cancel{100}}\times 5690}{\underset{3747}{\cancel{7494}}} = \text{£}75\ 18\text{s. }6\tfrac{3}{4}\tfrac{1446}{3747}\text{d.}\end{array}$$

(11)

$$\begin{array}{l}\text{Days.}\qquad\quad\text{miles.}\\ 48 : 68 :: 12 : \dfrac{\cancel{12}\times\overset{17}{\cancel{68}}}{\underset{\cancel{4}}{\cancel{48}}} = 17\ \text{miles per day.}\end{array}$$

(12)

Shillings. lbs.

$$21\tfrac{1}{3} : 32\tfrac{2}{7} :: 16\tfrac{1}{7} : \frac{113}{7} \times \frac{\overset{113}{\cancel{226}}}{7} \times \frac{3}{\underset{32}{\cancel{64}}} = \frac{38307}{1568} = 24\tfrac{675}{1568} \text{ lbs}$$

(13)

17493 × 1000 × 5 cub. ft. = 87465000 cub. ft.

192724 × 1000 × 4 cub. ft. = 770896000 cub. ft.

87465000 + 770896000 = 858361000 cub. ft.

Cubic feet. ton.

$$9000 : 858361000 :: 1 : \frac{858361\cancel{000}}{9\cancel{000}} = 95373\tfrac{4}{9} \text{ tons.}$$

(14)

$50\cancel{000} \times 9\cancel{000} = 450000000 =$ cub. ft. of gas in 50000 tons of coal

Cubic feet. hour.

$$4 : 450000000 :: 1 : \frac{450000000}{4} = 112500000 \text{ h.} = 12842 \text{ y. } 170 \text{ d.}$$

(15)

lbs. lbs. lbs. lb. lb.

$4 + 3 + 2 + 1 + \tfrac{1}{2} = 10\tfrac{1}{2}$ lbs.

lbs.

$$10\tfrac{1}{2} : 11270 :: 1 : \frac{11270}{10\tfrac{1}{2}} = 1073, \text{ and } 3\tfrac{1}{2} \text{ lbs. remaining.}$$

(16)

180 miles = 180 × 1760 = 316800 yards.

Yards. day.

$$100 : 316800 :: 1 : \frac{3168\cancel{00} \times 1}{1\cancel{00}} = 3168 \text{ dys. or about } 8\tfrac{2}{3} \text{ yrs.}$$

Exercise 87—Page 216.

(1)

120 : 90 bush.
6 : 14 horses. } $:: 56 \text{ days} : \dfrac{\overset{7}{\cancel{56}}\times\overset{15}{\cancel{90}}\times 14}{\underset{\cancel{8}}{\cancel{120}}\times\cancel{6}} = 7\times14 = 98$ days.

(2)

8 : 32 ft. high.
8 : 15 days. } $:: 63 \text{ men} : \dfrac{\overset{9}{\cancel{63}}\times\overset{4}{\cancel{32}}\times 15}{\cancel{8}\times\underset{\cancel{7}}{\cancel{28}}} = 9\times15 = 135$ men.

(3)

3 : 45 length.
: 1 width. } $::1 \text{ lb.} : \dfrac{45}{3\times1\frac{1}{4}} = \dfrac{45}{\frac{15}{4}} = \dfrac{\overset{3}{\cancel{45}}\times4}{\cancel{15}} = 3\times4 = 12$ lbs.

(4)

: 100 length.
: 1¼ width. } $:: 3 \text{ lbs.} : \dfrac{3\times1\frac{1}{4}\times100}{1\frac{1}{2}\times10} = 2\times1\frac{1}{4}\times10 = 25$ lbs.

(5)

: 132 tons.
: 5 days. } $:: 12 \text{ horses} : \dfrac{\overset{2}{\cancel{12}}\times5\times\overset{3}{\cancel{132}}}{\cancel{44}\times\underset{\cancel{3}}{\cancel{18}}} = 2\times5 = 10$ horses.

(6)

14 men.
10 days. } $:: 27\text{s.} : \dfrac{27\times\overset{2}{\cancel{14}}\times\overset{5}{\cancel{10}}}{\underset{\cancel{2}}{\cancel{4}}\times\cancel{7}} = 27\times5 = 135\text{s.} = £6\ 15\text{s.}$

(7)

3 : 5 masters.
8 : 10 apprentices.
5 : 8 weeks
6 : 5½ days per wk.

$$:: \$144 : \frac{\overset{\overset{8}{\cancel{24}}}{\cancel{144}}\times 5\frac{1}{2}\times\cancel{8}\times 10\times\cancel{5}}{\cancel{3}\times\cancel{8}\times\cancel{5}\times\cancel{6}} = \$440.$$

(8)

6 : 18 s.mak.
4 : 5 weeks.

$$:: \text{36 pairs of men's shoes} : \frac{\overset{9}{\cancel{36}}\times\overset{3}{\cancel{18}}\times 5}{\cancel{6}\times\cancel{4}} =$$

135 pairs men's and the women's $= \frac{2\frac{1}{2}}{3\frac{3}{4}} = \frac{2}{3}$ of 135 = 90 pairs.

(9)

9 : 18 feet high.
4 : 6 days.

$$:: \text{12 men} : \frac{\overset{3}{\cancel{12}}\times\overset{2}{\cancel{18}}\times 6}{\cancel{9}\times\cancel{4}} = 3\times 2\times 6 = \text{36 men.}$$

(10)

130 : 390 miles.
7 : 14 hours.

$$:: \text{3 days} : \frac{3\times\overset{2}{\cancel{14}}\times\overset{3}{\cancel{390}}}{\cancel{130}\times\cancel{7}} = 3\times 2\times 3 = \text{18 days.}$$

(11)

10 : 60 oz.
22½ : 30 d.

$$:: \text{1 d.} : \frac{60\times 30}{10\times 22\frac{1}{2}} = \frac{\overset{4}{\cancel{60}}}{1}\times\frac{\overset{\cancel{3}}{\cancel{30}}}{1}\times\frac{1}{\cancel{10}}\times\frac{2}{\underset{\cancel{15}}{\cancel{45}}} = 4\times 2 = \text{8d.}$$

(12)

10 : 5 compositors
7 : 14 hours.
20 : 40 sheets.
24 : 16 pages.
50 : 60 lines.
40 : 50 letters.

$$:: \text{16 days} : \frac{\overset{2}{\cancel{10}}\times\cancel{5}\times\overset{2}{\cancel{14}}\times\cancel{40}\times 16\times\overset{\cancel{3}}{\cancel{60}}\times\cancel{50}}{\underset{\cancel{5}}{\cancel{10}}\times\cancel{7}\times\cancel{20}\times\underset{\cancel{8}}{\cancel{24}}\times\cancel{50}\times\cancel{40}} =$$

2 × 16 = 32 days.

(13)

336 : 240 men.
5 : 9 days.
10 : 12 hours.
6 : 5 degrees.
5 : 3 yards wide
3 : 2 yards deep

$$:: 70 \text{ yards} : \frac{\overset{\cancel{7}}{\cancel{70}}\times\overset{\cancel{5}}{\cancel{240}}\times 9\times\overset{2}{\cancel{12}}\times\cancel{5}\times\cancel{3}\times 2}{\underset{\cancel{48}}{\cancel{336}}\times\cancel{5}\times\cancel{10}\times\cancel{6}\times\cancel{5}\times\cancel{3}} =$$

$9 \times 2 \times 2 = 36$ yards.

(14)

6 : 12 horses.
4 : 9 months.

$$:: 16 \text{ acres} : \frac{\overset{4}{\cancel{16}}\times\overset{2}{\cancel{12}}\times 9}{\cancel{6}\times\cancel{4}} = 4 \times 2 \times 9 = 72 \text{ acres.}$$

(15)

25 . 139 persons
1 : 7 years.

$$:: 300 \text{ bush.} : \frac{\overset{12}{\cancel{300}}\times 139\times 7}{\cancel{25}} = 11676 \text{ bushels.}$$

(16)

48 : 32 men.
36 : 864 feet long.
8 : 5 feet high.
4 : 3 feet wide.

$$:: 4 \text{ days} : \frac{\cancel{4}\times\overset{2}{\cancel{32}}\times\overset{\overset{3}{\cancel{108}}}{\cancel{864}}\times 5\times\cancel{3}}{\underset{\cancel{16}}{\cancel{48}}\times\cancel{36}\times\cancel{8}\times\cancel{4}} = 30 \text{ days.}$$

(17)

679 : 22407 sold's.
336 : 112 days.

$$:: 702 \text{ bushels} : \frac{\overset{234}{\cancel{702}}\times\overset{33}{\cancel{22407}}\times\cancel{112}}{\cancel{679}\times\underset{\cancel{3}}{\cancel{336}}} =$$

$234 \times 33 = 7722$ bushels.

(18)

13 : 494 suits.
19 : 27 days.

$$:: 12 \text{ tailors} : \frac{12\times\overset{\overset{2}{\cancel{26}}}{\cancel{494}}\times 27}{\cancel{13}\times\cancel{19}} = 648 \text{ tailors.}$$

(19)

$$\left.\begin{array}{l}17:40 \text{ head of cattle}\\ 30:51 \text{ days.}\end{array}\right\} :: 5\text{ a. }2\text{ r. }10\text{ p.} : \frac{5\text{ a. }2\text{ r. }10\text{ p.}\times\overset{4}{\cancel{40}}\times\overset{3}{\cancel{51}}}{\cancel{17}\times\underset{\cancel{10}}{\cancel{30}}} =$$

$$5\text{ a. }2\text{ r. }10\text{ p.}\times 4 = 22\text{ a. }1\text{ r.}$$

(20)

$$\left.\begin{array}{r}20 : 100 \text{ ft. long}\\ 6 : 4 \text{ feet wide.}\end{array}\right\} :: 180 \text{ bricks} : \frac{\overset{30}{\cancel{180}}\times\overset{5}{\cancel{100}}\times 4}{\cancel{20}\times\cancel{6}} =$$

$$30\times 5\times 4 = 600 \text{ bricks.}$$

Exercise 88.—Page 21

(1)

$$\left.\begin{array}{ll}17 \text{ cords} & = 116 \text{ lbs.}\\ 87 \text{ lbs.} & = 23 \text{ barrels}\\ 19 \text{ barrels} & = 34 \text{ days' work}\\ 92 \text{ days' work} & = 57 \text{ baskets peaches}\\ 31 \text{ baskets peaches} & = 24 \text{ dollars}\\ 12 \text{ dollars} & = 2 \text{ tons}\\ 35 \text{ tons} & = x \text{ cords}\end{array}\right\} =$$

$$\frac{\cancel{17}\times\overset{3}{\cancel{87}}\times\cancel{19}\times\overset{23}{\cancel{92}}\times 31\times\cancel{12}\times 35}{\underset{4}{\cancel{116}}\times\cancel{23}\times\underset{2}{\cancel{34}}\times\underset{3}{\cancel{57}}\times\underset{2}{\cancel{24}}\times 2} = \frac{31\times 35}{2\times 2\times 2} = \tfrac{1085}{8} = 135\tfrac{5}{8}.$$

(2)

$$\left.\begin{array}{ll}6 \text{ lbs. tea} & = 29 \text{ lbs. sugar}\\ 17 \text{ lbs. sugar} & = 1 \text{ bushel}\\ 27 \text{ bushels} & = 4 \text{ tons}\\ 34 \text{ tons} & = 15 \text{ cows}\\ 29 \text{ cows} & = 1160 \text{ dollars}\\ 20 \text{ dollars} & = x \text{ lbs. tea.}\end{array}\right\} =$$

$$\frac{\overset{8}{\cancel{6}}\times 17\times\overset{9}{\cancel{27}}\times\overset{17}{\cancel{34}}\times\cancel{29}\times\cancel{20}}{\cancel{29}\times 1\times\underset{2}{\cancel{4}}\times\underset{5}{\cancel{15}}\times\underset{58}{\cancel{1160}}} = \frac{17\times 17\times 27}{5\times 58} = \tfrac{7803}{290} = 26\tfrac{263}{290}$$

(3)

11 bush. barley = 21 bush. potatoes
19 " potatoes = 29 " oats
115 " oats = 44 " wheat
14½ " wheat = 38 " peas
60 " peas = 55 " rye
75 " rye = 11½ " clover sd.
36 " clover sd. = x " barley

$$\left.\right\} =$$

$$\frac{\cancel{11}\times\cancel{19}\times\overset{\overset{\cancel{2}}{\cancel{23}}}{\cancel{115}}\times\cancel{14\tfrac{1}{2}}\times\overset{\overset{5}{\cancel{20}}}{\cancel{60}}\times75\times\overset{18}{\cancel{36}}}{\underset{7}{\cancel{21}}\times\underset{\cancel{2}}{\cancel{29}}\times\underset{11}{\cancel{44}}\times\underset{\cancel{2}}{\cancel{38}}\times\underset{\cancel{5}}{\cancel{55}}\times\cancel{11\tfrac{1}{2}}}=\frac{5\times75\times18}{7\times11}=\frac{6750}{77}=87\tfrac{51}{77}$$

(4)

16 baskets pears = 29 turkeys
17 turkeys = 7 days' work
7¼ days' work = 187 loaves
3½ loaves = 4 lbs. veal
1 lb. veal = 11 cents
792 cents = 63 lbs. sugar
x lbs. sugar = 21 baskets pears

$$\left.\right\} =$$

$$\frac{\overset{\cancel{4}}{\cancel{29}}\times7\times\overset{11}{\cancel{187}}\times\cancel{4}\times\cancel{11}\times\overset{\overset{\cancel{2}}{\cancel{7}}}{\cancel{63}}\times21}{\underset{4}{\cancel{16}}\times\cancel{17}\times\cancel{7\tfrac{1}{4}}\times\cancel{3\tfrac{1}{2}}\times1\times\underset{\underset{\underset{\cancel{2}}{\cancel{8}}}{\cancel{72}}}{\cancel{792}}}=\frac{11\times7\times21}{4}=\frac{1617}{4}=404\tfrac{1}{4}.$$

(5)

7 A = 11 B
5 B = 8 C
15 C = 21 D
11 D = 5 E
42 E = x A

$$\left.\right\}=\frac{7\times\cancel{5}\times15\times\cancel{11}\times\overset{\cancel{2}}{\cancel{42}}}{\cancel{11}\times\underset{4}{\cancel{8}}\times\cancel{21}\times\cancel{5}}=\frac{7\times15}{4}=\frac{105}{4}=26\tfrac{1}{4}$$

(6)

$$\left.\begin{array}{ll} 7 \text{ barrels flour} & = 23 \text{ cords} \\ 6 \text{ cords} & = 11 \text{ cwt.} \\ 46 \text{ cwt.} & = £28 \\ £77 & = 9 \text{ sheep} \\ 5 \text{ sheep} & = 8 \text{ tons} \\ 9 \text{ tons} & = x \text{ barrels flour} \end{array}\right\} =$$

$$\frac{\cancel{7}\times\overset{3}{\cancel{6}}\times\overset{2}{\cancel{46}}\times\overset{7}{\cancel{77}}\times5\times\cancel{9}}{\cancel{23}\times\cancel{11}\times\underset{\underset{\cancel{2}}{\cancel{4}}}{\cancel{28}}\times\cancel{9}\times8} = \frac{3\times7\times5}{8} = \frac{105}{8} = 13\tfrac{1}{8}.$$

(7)

$$\left.\begin{array}{ll} 15 \text{ N. England} & = 20 \text{ New York} \\ 24 \text{ New York} & = 22\tfrac{1}{2} \text{ N. Jersey} \\ 30 \text{ New Jersey} & = 20 \text{ Canada} \\ 4807\tfrac{1}{2} \text{ Canada} & = x \text{ N. England} \end{array}\right\} =$$

$$\frac{\overset{\cancel{3}}{\cancel{15}}\times\overset{6}{\cancel{24}}\times\overset{\cancel{3}}{\cancel{30}}\times\overset{961\frac{1}{2}}{\cancel{4807\tfrac{1}{2}}}}{\underset{\cancel{5}}{\cancel{20}}\times\underset{\cancel{3}}{\cancel{22\tfrac{1}{2}}}\times\underset{\cancel{2}}{\cancel{20}}} = 961\tfrac{1}{2}\times6 = 5769\text{ s.} = £288\ 9\text{s.}$$

EXERCISE 89.—Page 222.

(1)

$$\frac{\cancel{7}}{\cancel{8}}\times\frac{\cancel{17}}{\cancel{11}}\times\frac{\cancel{28}}{\cancel{29}}\times\frac{\overset{\cancel{11}}{\cancel{319}}}{\underset{\cancel{17}}{\cancel{119}}}\times\frac{\overset{2}{\cancel{16}}}{\underset{3}{\cancel{69}}} = \frac{2}{3} = \text{[illegible]}$$

(2)

£119 × 400 = $476·00
16s. × 20 = 3·20
£[illegible] = 26 far. × 5 ÷ 12 = ·10⅚

£119 16s. 6½d. = $479·30⅚

(4)

$$\left.\begin{array}{rcl} 9 : 13 = 9 \div 13 &=& \cdot 692 \\ 21 : 27 = 21 \div 27 &=& \cdot 777 \\ 7 : 10 = 7 \div 10 &=& \cdot 7 \\ 11 : 15 = 11 \div 15 &=& \cdot 733 \end{array}\right\}$$

Hence 21 : 27 is the greatest, and 9 : 13 the least.

(5)

Dissimilar.		Similar.		Similar and Coterminous.
$76 \cdot 23\dot{4}7\dot{8}$	=	$76 \cdot 234\dot{7}8\dot{4}$	=	$76 \cdot 234\dot{7}8478478478\dot{4}$
$19 \cdot 134\dot{2}29\dot{1}$	=	$19 \cdot 134\dot{2}29\dot{1}$	=	$19 \cdot 134\dot{2}2912291229\dot{1}$
		Difference,	=	$57 \cdot 100\dot{5}5566187249\dot{3}$

(6)

713244 undenary = 1146287 denary, 23421 quinary = 1736 denary, and *t4e7* duodenary = 17995 denary.

1146287 × 1736 = 1989954232 ÷ 17995 = $110583\frac{13147}{17995}$.
$110583\frac{13147}{17995}$ denary* = $53ee3\frac{7737}{t4e7}$ duodenary, $12014313\frac{410042}{1033440}$ quinary, and $760t0\frac{9972}{1257t}$ undenary.

(7)

$$\left.\begin{array}{r} 5\cdot 63 : 7\cdot 9 \text{ cubic inches.} \\ 1 : 1\cdot 220 \text{ spec. grav.} \end{array}\right\} :: \overset{\text{oz.}}{3\cdot 254} : \frac{3.254 \times 7\cdot 9 \times 1\cdot 220}{5\cdot 63} =$$

$$\frac{31\cdot 362052}{5\cdot 63} = 5\cdot 57052 \text{ oz. Ans.}$$

---

* To reduce the fractional part, reduce both numerator and denominator separately.

(8)

yds. qrs. na. in. yds. qrs. na. in.
17)63 3 2 1 (3 3 0 $0\frac{11}{34}$
51
—
12
4
—
51
51
—
0
4
—
2
$2\frac{1}{4}$
—
$5\frac{1}{2}=\frac{11}{2}\div17=\frac{11}{34}$.

(9)

·913625 of an acre = ·913625×4840=
4421·945 sq. yds.
4421·945×\$.67=\$2962·70+

(10)

$\frac{1}{2}$ of $\frac{3}{5}$ of $\frac{7}{8}$ of 20 bush. $\times\cdot5\times\cdot\dot{6}\times\frac{7}{8}=$
$\frac{1}{2}\times\frac{3}{5}\times\frac{7}{8}\times\frac{20}{1}\times\frac{1}{2}\times\frac{2}{3}\times\frac{7}{8}=$
$\frac{49}{32}$ bush. = 1 bush. 2 pks. 0 gal. 1 qt.

(12)

Whole amount of increase = 2571437—1842265 = 729172.

$$1842265 : 100 :: 729172 : \frac{729172\times100}{1842265}=39 \text{ per cent.}$$

(13)

$\frac{1}{2}$ of $\frac{2}{3}$ of $\frac{18}{29}-\frac{1}{8}$ of $\frac{2}{3}$ of $\frac{5}{7}=\frac{6}{29}-\frac{5}{84}=\frac{359}{2436}$.

(14)

$$100 : 7 :: 11 : \overset{\text{ft.}}{} \frac{11\times7}{100}=\frac{77}{100}.\quad 11-\frac{77}{100}=10\frac{23}{100}.$$

(15)

79 × 16 × £·00163 = £2·06032 = £2 1s. $2\frac{298}{625}$d.

(16)

$$\left.\begin{array}{r}4:3 \text{ men}\\ 10:12 \text{ hours}\\ 20:35 \text{ acres}\end{array}\right\} :: 2\tfrac{1}{2} \text{ days} : \frac{2\frac{1}{2}\times3\times\overset{3}{\cancel{12}}\times\overset{7}{\cancel{35}}}{\cancel{4}\times\underset{4}{\cancel{10}}\times\underset{4}{\cancel{20}}}=\frac{63}{16}=3\tfrac{15}{16} \text{ days.}$$

(17)

$$\left(\tfrac{4}{5}\text{ of }\tfrac{9}{11}\times\cdot02\times\cdot\dot{4}5\dot{6}\right)\div\left(\tfrac{16}{17}\text{ of }\tfrac{2}{3}\text{ of }\tfrac{1}{3}\text{ of }51\right)=$$

$$\frac{\overset{2}{\cancel{4}}}{5}\times\frac{\overset{3}{\cancel{9}}}{11}\times\frac{1}{\underset{25}{\cancel{50}}}\times\frac{\overset{\overset{38}{\cancel{76}}}{\cancel{152}}}{\underset{\underset{37}{\cancel{111}}}{\cancel{333}}}\times\frac{\cancel{17}}{\underset{\cancel{8}}{\cancel{16}}}\times\frac{\cancel{3}}{\cancel{2}}\times\frac{\cancel{3}}{1}\times\frac{1}{\underset{\cancel{3}}{\cancel{51}}}=\frac{2\times38}{5\times11\times25\times37}=\frac{76}{50875}.$$

(18)

$$\frac{\cancel{2}}{1}\times\frac{4}{\cancel{7}}\times\frac{13}{\cancel{5}}\times\frac{\cancel{7}}{\cancel{2}}\times\frac{\cancel{5}}{}=4\times13=52.$$

(19)

50 barrels = 125 yards
80 yards = 6 bales,
13 bales = $3\frac{1}{2}$ hogsheads
x hogsheads = 1000 barrels
=

$$\frac{\overset{5}{\cancel{125}}\times\overset{3}{\cancel{6}}\times3\tfrac{1}{2}\times\overset{125}{\cancel{1000}}}{\underset{2}{\cancel{50}}\times\underset{\underset{2}{\cancel{10}}}{\cancel{80}}\times13}=\frac{125\times3\times3\frac{1}{2}}{2\times13}=50\tfrac{25}{52}.$$

(20)

$$73\cdot47\times\cdot0063\div17\cdot2\dot{3}4\dot{5}=\tfrac{7347}{100}\times\tfrac{63}{10000}\div\tfrac{57391}{3330}=$$

$$\frac{7347}{100}\times\frac{63}{100\cancel{0}0}\times\frac{333\cancel{0}}{57391}=\tfrac{154132713}{5739100000}=\cdot026856599989+$$

(21)

2 roods 7 per. 4 yds. 3 ft. 117 in. = 3416481 in. and 7 acres = 43908480 inches.

3416481 ÷ 43908480 = ·0778+

(22

$\frac{4}{5}$ of $\frac{2}{7}$ of $\frac{1}{3}$ of 70 miles = $\frac{16}{3}$ miles = 5·33333+miles.
·73 of 11 fur. = 8·03 fur. = 1·00375 mile.
5·33333 — 1·00375 = 4·32958 miles.

(23)

274312 nonary = 167195 denary, 1101011010 = 858 denary, and ·5555 septenary = 2000 denary.

167195 — 858 = 166337×2000 = 332674000.
332674000 denary = 764876837 nonary.
= 10011110101000011001111010000 binary,
= 11146453021 septenary.

(24)

$$\begin{array}{r|l} 275 & \cancel{44}\,..\,\cancel{275}\,..\,18\,..\,\cancel{190}\,..\,\cancel{209}\,..\,\cancel{225} \\ 38 & \cancel{4}\qquad\quad \cancel{18}\,..\,\cancel{38}\,..\,\cancel{19}\,..\quad 9 \\ 18 & \cancel{2}\qquad\quad \cancel{9}\qquad\qquad\qquad\quad \cancel{9} \end{array}$$

275×38×18 = 188100 = l. c. m.

(25)

$$\left.\begin{array}{r} 10:6 \text{ weeks} \\ 6:5 \text{ days} \\ 11:10 \text{ hours} \\ 2400:8742 \text{ feet long.} \\ 18:20 \text{ feet wide} \\ 11:8 \text{ feet high} \end{array}\right| :: 60: \text{men } \frac{\overset{6}{\cancel{60}}\times\cancel{6}\times5\times\cancel{10}\times\overset{2914}{\cancel{8742}}\times\cancel{20}\times\overset{2}{\cancel{8}}}{\cancel{10}\times\cancel{6}\times11\times\underset{\underset{\underset{3}{\cancel{12}}}{\cancel{240}}}{\cancel{2400}}\times\underset{3}{\cancel{18}}\times11} =$$

$$\frac{5\times2914\times2}{11\times3\times11} = \frac{29140}{363} = 80\frac{100}{363}.$$

(26)

172000 = $2^5 \times 5^3 \times 43$. Increasing each exponent by 1 and multiplying them together we obtain 6×4×2 = 48.

(27)

$42\dot{7} = 42\frac{7}{9} = \frac{385}{9}$ and $9\cdot7\dot{1}2\dot{3} = 9\frac{7116}{9990} = 9\frac{1186}{1665} = \frac{16171}{1665}$.

$\frac{385}{9} \times \frac{16171}{1665} = \frac{6225835}{14985} = 415\cdot\dot{4}71137804\dot{4}$.

(28)

$$100 : 27 :: \$73\cdot42 : \frac{73\cdot42 \times 27}{100} = \$19\cdot8234.$$

$$\$73\cdot42 - \$19\cdot8234 = \$53\cdot5966.$$

(29)

$$6300 = 2^2 \times 3^2 \times 5^2 \times 7.$$

1..5..25

1..2.. 4

1..5..25..2..10..50..4..20..100

1..3.. 9

1..5..25..2..10..50..4..20..100..3..15..75..6..30..150..
12..60..300..9..45..225..18..90..450..36..180..900

1..7

1..5..25..2..10..50..4..20..100..3..15..75..6..30..150..
12..60..300..9..45..225..18..90..450..36..180..900..7..
35..175..14..70..350..28..140..700..21..105..525..42..
210..1050..84..420..2100..63..315..1575..126..630..3150
..252..1260..6300.

Therefore the divisors of 6300 are 1, 2, 3, 4, 5, 6, 7, 9, 10, 12, 14, 15, 18, 20, 21, 25, 28, 30, 35, 36, 42, 45, 50, 60, 63, 70, 75, 84, 90, 100, 105, 126, 140, 150, 175, 180, 210, 225, 252, 300, 315, 350, 420, 450, 525, 630, 700, 900, 1050, 1260, 1575, 2100, 3150, 6300.

(30)

$\frac{2}{7}$ of $\frac{3}{8}$ of $3\frac{1}{2}$ lbs. $= \frac{3}{8}$ lbs., $\frac{9}{7}$ of $\frac{3}{5}$ of $\frac{20}{27}$ of $\frac{1}{2}$ of $\$1 = \$\frac{2}{7}$,
and $\frac{3}{5}$ of $\frac{7}{9}$ of $\frac{6}{10}$ of $\frac{21}{80}$ of 90 lbs. $= \frac{1323}{200}$ lbs.

$$\frac{3}{8} : \overset{\text{lbs.}}{\frac{1323}{200}} :: \frac{2}{7} : \frac{\overset{\$}{\frac{2}{7}} \times \frac{1323}{200}}{\frac{3}{8}} = \frac{2}{\cancel{7}} \times \frac{\overset{63}{\overset{\cancel{441}}{\cancel{1323}}}}{\underset{25}{\cancel{200}}} \times \frac{\cancel{8}}{\cancel{3}} = \overset{\$}{\frac{126}{25}} = \$5\cdot04.$$

(31)

7 men will have 7 men's shares.

One woman has $\frac{3}{11}$ of a man's share; ∴ 2 women will have $2 \times \frac{3}{11} = \frac{6}{11}$ of a man's share.

One child has $\frac{2}{7}$ of $\frac{3}{11} = \frac{6}{77}$ of a man's share; ∴ 11 children will have $11 \times \frac{6}{77} = \frac{6}{7}$ of a man's share.

7 men, 2 women, and 11 children will have $7 + \frac{6}{11} + \frac{6}{7} = 8\frac{31}{77}$ men's shares.

$\$2739{\cdot}18 \div 8\frac{31}{77} = \$325{\cdot}99\frac{133}{647}$ = a man's share.
$\frac{3}{11}$ of $\$325{\cdot}99\frac{133}{647} = \$88{\cdot}90\frac{248}{647}$ = a woman's share.
$\frac{2}{7}$ of $\$88{\cdot}90\frac{248}{647} = \$25{\cdot}40\frac{128}{647}$ = a child's share.

(33)

	yds.	ft.	in.
$\frac{4}{9}$ of $6\frac{1}{2}$ yds. $= \frac{26}{9}$ yds. =	2	2	8
$\frac{3}{5}$ of $\frac{4}{7}$ of $8\frac{3}{4}$ ft. =	1	0	0
$\frac{2}{7}$ of $\frac{3}{11}$ of $7\frac{7}{10}$ in. =			$\frac{3}{5}$
Sum =	3	2	$8\frac{3}{5}$

(34)

$$\left.\begin{array}{rr} \cancel{2}\ \cancel{23} : & \cancel{7}\ \ \cancel{2} \\ 4 : & 11 \\ 2\ \ \cancel{6} : & 5 \\ 13 : & 11\frac{1}{2} \\ \cancel{8\frac{1}{2}}\ \cancel{88\frac{1}{2}} : & \cancel{8} \end{array}\right\} = 104 : 5.$$

(35)

23 bush. 2 pks. 1 gal. 1 qt. 1 pt. = 1515 pts.
$1515 \times 9000 \times \frac{1}{3} = 4545000$ in. = 71 miles 5 fur. 34 per. 3 yds.

(36)

$$\frac{4158}{10395} = \frac{462}{1155} = \frac{66}{165} = \frac{22}{55} = \frac{2}{5}.$$

(37

VIII.

$\frac{1}{2}, \frac{2}{3}, \frac{4}{5}, \frac{2}{7}$. Here the common denominator is $2 \times 3 \times 5 \times 7 = 322$. The numerators of the fractions are, for the first, $1 \times 3 \times 5 \times 7 = 151$; for the second, $2 \times 2 \times 5 \times 7 = 214$; for the third, $4 \times 2 \times 3 \times 7 = 250$; for the fourth, $2 \times 2 \times 3 \times 5 = 74$; and the equivalent fractions are, $\frac{151}{322}$, $\frac{214}{322}$, $\frac{250}{322}$, and $\frac{74}{322}$, which when added together $= \frac{731}{322} = 2\frac{65}{322}$, the numbers all through being in the *octenary* scale.

(38)

$$\left.\begin{array}{lcl}17 \text{ sheep} & = & 6 \text{ cows}\\ 26 \text{ cows} & = & 27\frac{1}{2} \text{ acres}\\ 12 \text{ acres} & = & 13 \text{ horses}\\ 11 \text{ horses} & = & 28 \text{ goats}\\ x \text{ goats} & = & 68 \text{ sheep}\end{array}\right\} = \frac{\overset{\cancel{3}}{\cancel{6}}}{\cancel{17}} \times \frac{\overset{2\frac{1}{2}}{\cancel{27\frac{1}{2}}}}{\underset{\cancel{2}}{\cancel{26}}} \times \frac{\cancel{13}}{\underset{\cancel{4}}{\cancel{12}}} \times \frac{28}{\cancel{11}} \times \frac{\overset{\cancel{17}}{\cancel{68}}}{1} =$$

$$2\frac{1}{2} \times 28 = 70 \text{ goats.}$$

(39)

$$\left.\begin{array}{l}27 : 54 \text{ days}\\ 24 : 18 \text{ cel.}\\ 36 : 43 \text{ ft. l.}\\ 21 : 28 \text{ ft. w.}\\ 10 : 9 \text{ ft. d.}\\ 3 : 5 \text{ hrs.}\end{array}\right\} :: 50 \text{ men} : \frac{50 \times \overset{\cancel{2}}{\cancel{54}} \times \overset{\cancel{6}}{\cancel{18}} \times \overset{\cancel{8}}{\cancel{48}} \times \overset{4}{\cancel{28}} \times \overset{\cancel{3}}{\cancel{9}} \times \cancel{5}}{\cancel{27} \times \underset{\cancel{3}}{\cancel{24}} \times \underset{\cancel{6}}{\cancel{36}} \times \underset{\cancel{7}}{\cancel{21}} \times \underset{\cancel{2}}{\cancel{10}} \times \cancel{3}} = 200 \text{ men.}$$

Exercise 90.—Page 226.

(1)

$\$\cdot35 \times 92647 = \$32426\cdot45.$

(2)

		£	s	d.	
4d.	$\frac{1}{3}$	4746	17	0	= cost of 94937 pails at 1s.
1d.	$\frac{1}{4}$	1582	5	8	= " " " at 4d.
		395	11	5	= " " " at 1d.
		£6724	14	1	= cost of 94937 pails at 1s. 5d.

(3)

$\$\cdot07\frac{1}{2} \times 95972 = \$7197\cdot90$

(4)

$\$28\cdot80 \times 62 = \$1785\cdot60.$

(5)

$\$\cdot32\frac{1}{2} \times 2310 = \$750.75.$

(6)

$\$\cdot37\frac{1}{2} \times 2117 = \$793\cdot87\frac{1}{2}.$

(7)

		£	s	d.	
6d.	$\frac{1}{2}$	375	6	0	= price of 7506 pairs at 1s.
3d.	$\frac{1}{2}$	187	13	0	= " " at 6d.
$\frac{3}{4}$d.	$\frac{1}{4}$	93	16	6	= " " at 3d.
		23	9	$1\frac{1}{2}$	= " " at $\frac{3}{4}$d.
		£680	4	$7\frac{1}{2}$	= price of 7506 pairs at 1s. $9\frac{3}{4}$d.

(8)

$\$\cdot17\frac{1}{2} \times 1217 = \$212.97\frac{1}{2}$.

(9)

$\$3\cdot07\frac{1}{2} \times 2108 = \$6466\cdot72\frac{1}{2}$.

(10)

10s.	$\frac{1}{6}$	2096			
		3			
		£6288	0	0	= cost of 2096 oz. at £3.
5s.	$\frac{1}{2}$	1048	0	0	= " " at 0 10s.
2s. 6d.	$\frac{1}{2}$	524	0	0	= " " at 0 5s.
1s. 3d.	$\frac{1}{2}$	262	0	0	= " " at 0 2s. 6d.
1½d.	$\frac{1}{10}$	131	0	0	= " " at 0 1s. 3d.
		13	2	0	= " " at 0 0 1½d.
		£8266	2	0	= " " at £3 18s. 10½d.

(11)

10 dwt.	$\frac{1}{2}$	$1·55	
		6	
		$9·30	= cost of 6 oz.
5 dwt.	$\frac{1}{2}$	$\cdot77\frac{1}{2}$	= " 10 dwt.
2 dwt. 12 grs.	$\frac{1}{2}$	$\cdot38\frac{3}{4}$	= " 5 dwt.
1 dwt. 6 grs.	$\frac{1}{2}$	$\cdot19\frac{3}{8}$	= " 2 dwt. 12 grs.
2 grs.	$\frac{1}{15}$	$\cdot09\frac{11}{16}$	= " 1 dwt. 6 grs.
		$\cdot00\frac{31}{48}$	= " 2 grs.
		$\$10\cdot75\frac{23}{24}$	= cost of 6 oz. 18 dwt. 20 grs.

(12)

10s.	$\frac{1}{2}$	£98	0	0	= cost of 98 yards at £1.
5s.	$\frac{1}{2}$	49	0	0	= " " 0 10s.
		24	10	0	= " " 0 5s.
		£171	10	0	= cost of 98 yards at £1 15s.
2 qrs.	$\frac{1}{2}$	£1	15		
1 qr.	$\frac{1}{2}$		17	6	= cost of 2 qrs.
1 na.	$\frac{1}{4}$		8	9	= " 1 qr.
			2	2¼	= " 1 na.
		£1	8	5¼	= cost of 3 qrs. 1 na.

Then £171	10	0	= cost of 98 yards at £1 15s.
1	8	5¼	= cost of 3 qrs. 1 na. at £1 15s. per yard.
£172	18	5¼	= cost of 98 yds. 3 qrs. 1 na. at £1 15s. per yd

(13)

1s.	$\frac{1}{20}$	344			
		4			
		£1376	0	0	= rent of 344 acres at £4.
1d.	$\frac{1}{12}$	17	4	0	= " " at 0 1s.
		1	8	8	= " " at 0 0 1d.
		£1394	12	8	= rent of 344 acres at £4 1s. 1d

2 r.	$\frac{1}{2}$	£4	1	1	
1 r.	$\frac{1}{2}$	2	0	$6\frac{1}{2}$	= rent of 2 roods.
10 per.	$\frac{1}{4}$	1	0	$3\frac{1}{4}$	= " 1 rood.
5 per.	$\frac{1}{2}$		5	$0\frac{13}{16}$	= " 10 perches.
			2	$6\frac{13}{32}$	= " 5 perches.
		£3	8	$4\frac{31}{32}$	= " 3 roods 15 perches.

£1394	12	8	= rent of 344 acres at £4 1s. 1d.
3	8	$4\frac{31}{32}$	= " 3 roods 15 per. at £4 1s. 1d. per ac.
£1398	1	$0\frac{31}{32}$	= " 344 a. 3 r. 15 per. at £4 1s. 1d.

(14)

5 dwt.	$\frac{1}{4}$		5	10	
				5	
		£1	9	2	= price of 5 oz. at 5s. 10d. per oz
1 dwt.	$\frac{1}{5}$		1	$5\frac{1}{2}$	= " 5 dwt. " "
12 grs.	$\frac{1}{2}$			$3\frac{1}{2}$	= " 1 dwt. " "
4 grs.	$\frac{1}{3}$			$1\frac{3}{4}$	= " 12 grs. " "
1 gr.	$\frac{1}{4}$			$0\frac{7}{12}$	= " 4 grs. " "
				$0\frac{7}{48}$	= " 1 gr. " "
		£1	11	$1\frac{23}{48}$	= " 5 oz. 6 dwt. 17 grs. at 5s. 10d. per oz.

(15)

2 qrs.	½	£1	2	4	
				4	
		£4	9	4	= price of 4 yards at £1 2 4 per yard
2 na.	¼		11	2	= " 2 qrs. " "
1 na.	½		2	9½	= " 2 na. " "
			1	4¾	= " 1 na. " "
		£5	4	8¼	= price of 4 yds. 2 qrs. 3 na. " "

(16)

1 rood.	¼	£1	16		
			32		
		£57	12	0	= price of 32 acres at £1 16s.
10 per.	¼		9	0	= " 1 rood. "
2 per.	⅕		2	3	= " 10 per. "
2 per.	⅕			5⅖	= " 2 per. "
				5⅖	= " 2 per. "
		£58	4	1⅘	= price of 32 acres 1 rood 14 per

(17)

4 pts.	½		7	6	
				3	
		£1	2	6	= price of 3 gals. at 7s. 6d per gal.
1 pt.	¼		3	9	= " 4 pts. "
				11¼	= " 1 pt. "
		£1	7	2¼	= price of 3 gals. 5 pts.

(18)

$1·67½ × 724 = $1212·70.

(19)

$1·93¾ × 721 = $1396·93¾.

(20)

10s.	$\frac{1}{2}$	4514			
		2			
		£9028	0	0	= cost of 4514 rods at £2.
6s. 8d.	$\frac{1}{3}$	2257	0	0	= " " at 0 10
10d.	$\frac{1}{8}$	1504	13	4	= " " at 0 6 8
1d.	$\frac{1}{10}$	188	1	8	= " " at 0 0 10
½d.	$\frac{1}{2}$	18	16	2	= " " at 0 0 1
		9	8	1	= " " at 0 0 0½
		£13005	19	3	= " " at £2 17 7½

(21)

10s.	$\frac{1}{2}$	£3749	7	6	
				3	
		£11248	2	6	= price of $3749\frac{3}{8}$ acres at £3
5s.	$\frac{1}{2}$	1874	13	9	= " " at 0 10
6d.	$\frac{1}{10}$	937	6	10½	= " " at 0 5
		93	14	8¼	= " " at 0 0 6
		£14153	17	9¾	= price of $3749\frac{3}{8}$ acres at £3 15 6

(22)

4s.	$\frac{1}{5}$	£17	0	0	= cost of 17 cwt. at £1
8d.	$\frac{1}{6}$	3	8	0	= " " at 0 4
1d.	$\frac{1}{8}$		11	4	= " " at 0 0 8
			1	5	= " " at 0 0 1
		£21	0	9	= cost of 17 cwt. at £1 4 9

1 qr.	$\frac{1}{4}$	£1	4	9	
16 lbs.	$\frac{1}{7}$		6	2¼	= cost of 1 qr.
1 lb.	$\frac{1}{16}$		3	$6\frac{3}{7}$	= " 16 lbs.
			0	$2\frac{73}{112}$	= " 1 lb.
			9	$11\frac{37}{112}$	= " 1 qr. 17 lbs.

£21	0	9	= cost of 17 cwt. at £1 4s. 9d. per cwt.
	9	$11\frac{37}{112}$	= " 1 qr. 17 lbs " "
£21	10	$8\frac{37}{112}$	= " 17 cwt. 1 qr. 17 lbs. " "

(23)

2 qrs.	½	$11·55	
		78	
		9240	
		8085	
		$900·90	= cost of 78 cwt. at $11·55 per cwt.
1 qr.	½	$5.77\frac{1}{2}$	= " 2 qrs. " "
7 lbs.	¼	$2·88\frac{3}{4}$	= " 1 qr. " "
4 lbs.	$\frac{1}{7}$	$·72\frac{3}{16}$	= " 7 lbs. " "
1 lb.	¼	$·41\frac{1}{4}$	= " 4 lbs. " "
		$·10\frac{5}{16}$	= " 1 lb. " "
		$910·80	= cost of 78 cwt. 3 qrs. 12 lbs.

(24)

£10 10
20

£210 0 = price of 20 tons at £10 10s.

19 cwt. 3 qrs. $27\frac{1}{2}$ lbs. = 1 ton. — ½ lb. The price of 1 ton is £10 10s., and the price of ½ lb. = $\frac{1}{4480}$ of £10 10s. = $\frac{63}{112}$ d. ∴ the price of 19 cwt. 3 qrs. $27\frac{1}{2}$ lbs. = £10 10s. — $\frac{63}{112}$ d. = £10 9s. $11\frac{49}{112}$ d.

£210 0 0		= price of 20 tons at £10 10s.
10 9 $11\frac{49}{112}$		= " 19 cwt. 3 qrs. $27\frac{1}{2}$ lbs.
£220 9 $11\frac{49}{112}$		= " 20 tons 19 cwt. 3 qrs. $27\frac{1}{2}$ lbs. at £10 10s. per ton.

(25)

10 cwt.	½	$45·50		
		219		
		40950		
		4550		
		9100		
		$9964·50	=	price of 219 tons at $45·50 per ton.
5 cwt.	½	22·75	=	" 10 cwt. " "
1 cwt.	⅕	11·37½	=	" 5 cwt. " "
2 qrs.	½	2·27½	=	" 1 cwt. " "
1 qr.	½	1·13¾	=	" 2 qrs. " "
		56⅞	=	" 1 qr. " "
		$10002·60⅝	=	price of 219 tons 16 cwt. 3 qrs.

Exercise 91—Page 228.

Bills of Parcels.

(No. 2.)

	s.	d.		£	s.	d.
9 pair of worsted stockings, at.....	4	6	per pair	2	0	6
6 pair of silk ditto, at.............	15	9	"	4	14	6
17 pair of thread ditto, at..........	5	4	"	4	10	8
23 pair of cotton ditto, at..........	4	10	"	5	11	2
14 pair of yarn ditto, at............	2	4	"	1	12	8
18 pair of women's silk gloves, at...	4	2	"	3	15	0
19 yards of flannel, at	1	7½	per yard	1	10	10½
			Ans.	£23	15	4½

(No. 3.)

75½ lbs. of sugar, at	7¾	cents per lb.	$5·85⅛
63 lbs. of tea, at...................	93	"	58·59
126 lbs. of butter, at...............	13	"	16·38
35⅘ lbs. of raisins, at	18¾	"	6·71¼
17 lbs. of sago, at	15	"	2·55
23 lbs. of rice, at..................	9	"	2·07
58½ lbs. of starch, at...............	22	"	12·87
		Ans.	$105·02⅜

(No. 4.)

198	Sangster's National Arithmetic, at	$0·60	$118·80
197	Robertson's Philosophy of Grammar, at	0·50	98·50
83	Hodgins' Geography, at	1·00	83·00
57	Sangster's Algebraic Formula, at	0·12½	7·12½
217	Strachan's Canadian Penmanship, at	0·37½	81 37½
143	Hodgins' Geography of British Provinces, at	0·45	64 35
227	Sangster's First Arithmetic, at	0·30	68·10
		Ans.	$521·25

(No. 5.)

		s.	d.		£	s.	d.
9½	yards of silk, at	12	9	per yard	6	1	1½
13	yards of flowered ditto, at	15	6	"	10	1	6
11¾	yards of lustring, at	6	10	"	4	0	3½
14	yards of brocade, at	11	3	"	7	17	6
12¼	yards of satin, at	10	8	"	6	10	8
11⅜	yards of velvet, at	18	0	"	10	4	9
				Ans.	£44	15	10

(No. 6.)

14 oz.	ipecacuanha, at	$0.67	$9·38
23 "	laudanum, at	0·89	20·47
17 "	emetic tartar, at	1·25	21·25
25 "	cantharides, at	2·17	54·25
27 "	gum mastic, at	0·61	16·47
56 "	gum camphor, at	0·27	15·12
		Ans.	$136·94

(No. 7.)

		s.	d.		£	s.	d.
15½	lbs. of currants, at	0	4	per lb.		5	2
17¼	lbs. of Mulaga raisins, at	0	5½	"		7	10⅞
19¾	lbs. of sun raisins, at	0	6	"		9	10½
17	lbs. of rice, at	0	3½	"		4	11½
8½	lbs. of pepper, at	1	6	"		12	9
3	loaves of sugar, weight 32½ lbs. at	0	8½	"	1	3	0¼
13	oz. of cloves, at	0	9	per oz.		9	9
				Ans.	£3	13	5⅛

EXERCISE 92—Page 231.

(2)

$427 \cdot 1 \div \cdot 0000637 = 4271000000 \div 637 = 6704866 \cdot 561 +.$

(3)

		£	s.	d.						£	s.	d.
10s.	$\frac{1}{2}$	£19										
		19										
		171										
		19										
		£361	0	0	=	cost of 19	tons	at		£19		
5s.	$\frac{1}{2}$	9	10	0	=	"	"	at		0	10	
4s.	$\frac{1}{5}$	4	15	0	=	"	"	at		0	5	
6d.	$\frac{1}{8}$	3	16	0	=	"	"	at		0	4	
3d	$\frac{1}{2}$		9	6	=	"	"	at		0	0	6
2d.	$\frac{1}{3}$		4	9	=	"	"	at		0	0	3
$\frac{1}{2}$d.	$\frac{1}{4}$		3	2	=	"	"	at		0	0	2
$\frac{1}{4}$d.	$\frac{1}{2}$			$9\frac{1}{2}$	=	"	"	at		0	0	$0\frac{1}{2}$
				$4\frac{3}{4}$	=	"	"	at		0	0	$0\frac{1}{4}$

£379 19 $7\frac{1}{4}$ = cost of 19 tons at £19 19 $11\frac{3}{4}$

19 cwt. 3 qrs. $27\frac{1}{2}$ lbs. = 1 ton — $\frac{1}{2}$ lb. The price of 1 ton is 19 19s. $11\frac{3}{4}$d., and the cost of $\frac{1}{2}$ lb. = $\frac{1}{4480}$ of £19 19s. $11\frac{3}{4}$d. = $1\frac{1279}{17920}$d.; ∴ the cost of 19 cwt. 3 qrs. $27\frac{1}{2}$ lbs. = £19 19s. $11\frac{3}{4}$d. — $1\frac{1279}{17920}$d. = £19 19s. $10\frac{12161}{17920}$d.

£379 19 $7\frac{1}{4}$ = cost of 19 tons.
19 19 $10\frac{12161}{17920}$ = " 19 cwt. 3 qrs. $27\frac{1}{2}$ lbs.

£399 19 $5\frac{16641}{17920}$ = " 19 tons 19 cwt. 3 qrs. $27\frac{1}{2}$ lbs.

(4)

Dissimilar.		Similar.		Similar and Coterminous.
$73\cdot\dot{7}2\dot{3}$	=	$73\cdot723\dot{7}2\dot{3}$	=	$73\cdot723\dot{7}2372\dot{3}$
$11\cdot34\dot{2}$	=	$11\cdot342\dot{2}$	=	$11\cdot342\dot{2}2222\dot{2}$
$16\cdot713$	=	$16\cdot713\dot{0}$	=	$16\cdot713\dot{0}0000\dot{0}$
$19\cdot\dot{0}3\dot{4}$	=	$19\cdot034\dot{0}3\dot{4}$	=	$19\cdot034\dot{0}3403\dot{4}$
$713\cdot213\dot{4}3\dot{7}$	=	$713\cdot213\dot{4}3\dot{7}$	=	$713\cdot213\dot{4}3743\dot{7}$
$12\cdot\dot{3}4567\dot{8}$	=	$12\cdot345\dot{6}7834\dot{5}$	=	$12.345\dot{6}7834\dot{5}$
				2 carried.
		Sum	=	$846\cdot372\dot{0}9576\dot{3}$

(5)

$5 : 7 = 5 \div 7 = \cdot714+$
$9 : 13 = 9 \div 13 = \cdot692+$
$12 : 17 = 12 \div 17 = \cdot705+$
$7 : 10 = 7 \div 10 = \cdot7$

Hence 5 : 7 is the greatest, and 9 : 13 least.

$$\frac{\cancel{5}}{\cancel{7}} \times \frac{9}{13} \times \frac{\overset{6}{\cancel{12}}}{17} \times \frac{\cancel{7}}{\underset{2}{\cancel{10}}} = \frac{54}{221} = 54 : 221.$$

(6)

1 acre = 160 rods, and 25 acres 2 roods 35 rods = 4115 rods.

$$\overset{\text{rods.}}{160 : 4115} :: \$80\cdot50 : \frac{\overset{40\cdot25}{\cancel{80\cdot50}} \times \overset{823}{\cancel{4115}}}{\underset{\underset{16}{\cancel{80}}}{\cancel{160}}} = \$2070\cdot3593.$$

(8)

$\$3\cdot68\tfrac{3}{4} \times 7439 = \$27431\cdot31\tfrac{1}{4}.$

(9)

$\frac{135795}{222210}$. The G. C. M. of 135795 and 222210 is 12345; when both terms of the fraction are divided by 12345, it becomes $\frac{11}{18}$.

$\frac{714235}{999999}$. Here 714235 and 999999 have no G. C. M.; ∴ the fraction cannot be reduced.

$\frac{103375}{100000}$. The G. C. M. of 103375 and 100000 is 3125; when both terms of the fraction are divided by 3125, it becomes reduced to $\frac{33}{32}$.

$\frac{20301}{33633}$. The G. C. M. of 20301 and 33633 is 303; when both terms of the fraction are divided by 303, it is reduced to its lowest terms, viz., $\frac{67}{111}$.

(10)

$$\left.\begin{array}{lll} 34\frac{1}{2} \text{ bushels turnips} & = & 17 \text{ bushels potatoes} \\ 9 \text{ " potatoes} & = & 59\frac{1}{2} \text{ lbs. tea} \\ 6 \text{ lbs. tea} & = & 11\frac{1}{2} \text{ stone flour} \\ 13 \text{ stone flour} & = & 360 \text{ cents} \\ 38 \text{ cents} & = & 12 \text{ lbs. bread} \\ 119 \text{ lbs bread} & = & x \text{ bushels turnips} \end{array}\right\} =$$

$$\frac{\overset{3}{\cancel{34\frac{1}{2}}}}{\cancel{17}} \times \frac{\cancel{9}}{\underset{8\frac{1}{2}}{\cancel{59\frac{1}{2}}}} \times \frac{\cancel{6}}{\cancel{11\frac{1}{2}}} \times \frac{13}{\underset{40}{\cancel{360}}} \times \frac{\overset{19}{\cancel{38}}}{\underset{\cancel{2}}{\cancel{12}}} \times \frac{\overset{\cancel{7}}{\cancel{119}}}{1} = \frac{3 \times 13 \times 19}{8\frac{1}{2} \times 40} = 2\frac{61}{340}.$$

(11)

$$\left.\begin{array}{l} 54 : 27 \text{ men} \\ 11 : 8 \text{ hours} \\ 42 : 77 \text{ floors} \\ 20 : 24 \text{ feet long} \\ 16 : 23 \text{ feet wide} \\ 3 : 5 \text{ coats paint} \end{array}\right\} :: 7 \text{ days} : \frac{7 \times \cancel{27} \times \overset{\cancel{4}}{\cancel{8}} \times \overset{\cancel{7}}{\cancel{77}} \times \overset{\overset{\cancel{2}}{\cancel{4}}}{\cancel{24}} \times \overset{11}{\cancel{22}} \times \cancel{5}}{\underset{\cancel{2}}{\cancel{54}} \times \cancel{11} \times \underset{\cancel{6}}{\cancel{42}} \times \underset{\underset{2}{\cancel{4}}}{\cancel{20}} \times \underset{\underset{\cancel{2}}{\cancel{4}}}{\cancel{16}} \times 3}$$

$$= \frac{7 \times 11}{2 \times 3} = 12\frac{5}{6} \text{ days.}$$

(13)

IX.
12)72342
12)5403..2
12)407..0
12)30..7
2..3

IX.
6)72342
6)11806..2
6)1731..0
6)264..4
6)40..4
6)6..0
1..0

IX.
3)72342
3)23713..2
3)7234..0
3)2371..1
3)723..1
3)237..0
3)72..1
3)23..2
3)7..0
2..1

IX. 72342 = XII. 23702 = VI. 1004402 = III 2102101102

IX.
72342
9
65
9
588
9
5296
9
47666

XII.
23702
12
27
12
331
12
3972
12
47666

VI.
1004402
6
6
6
36
6
220
6
1324
6
7944
6
47666

III
2102101102
3
7
3
21
3
65
3
196
3
588
3
1765

1765
3
5296
3
15888
3
47666

(14)

II.		IV.	IV.
111111	100000	333333	100000
2	2	4	4
—	—	—	—
3	2	15	4
2	2	4	4
—	—	—	—
7	4	63	16
2	2	4	4
—	—	—	—
15	8	255	64
2	2	4	4
—	—	—	—
31	16	1023	256
2	2	4	4
—	—	—	—
63 Greatest.	32 Least.	4095 Greatest.	1024 Least.

VI.	VI.	VIII.	VIII.
555555	100000	777777	100000
6	6	8	8
—	—	—	—
35	6	63	8
6	6	8	8
—	—	—	—
215	36	511	64
6	6	8	8
—	—	—	—
1295	216	4095	512
6	6	8	8
—	—	—	—
7775	1296	32767	4096
6	6	8	8
—	—	—	—
46655 Greatest.	7776 Least.	262143 Greatest.	32768

(Continued on [illegible] page.)

(14 continued.)

XII.	XII.
eeeeee	100000
12	12
143	12
12	12
1727	144
12	12
20735	1728
12	12
248831	20736
12	12
2985983 Greatest.	248832 Least.

(15)

$$1728 = 2^6 \times 3^3.$$

1..2..4..8..16..32..64
1..3..9..27

1..2..4..8..16..32..64..3..6..12..24..48..96..192..9..18..36..72..144..288..576..27..54..108..216..432..864..1728.

Therefore the divisors of 1728 are 1, 2, 3, 4, 6, 8, 9, 12, 16, 18, 24, 27, 32, 36, 48, 54, 64, 72, 96, 108, 144, 192, 216, 288, 432, 576, 864, 1728.

(16)

$$\begin{array}{r|l} 30 & \cancel{2}..\cancel{4}..\cancel{6}..\cancel{8}..\cancel{10}..\cancel{12}..\cancel{14}..\cancel{16}..\cancel{18}..\cancel{20}..\cancel{22}..\cancel{24}..\cancel{26}..\cancel{28}..\cancel{30} \\ 14 & \quad\cancel{2} \quad\quad \cancel{4} \quad\quad\quad \cancel{2}..\cancel{7}..\cancel{8}..3..\cancel{2}..11..\cancel{4}..13..\cancel{14} \\ 12 & \quad\quad\quad\quad \cancel{2} \quad\quad\quad\quad\quad \cancel{4}..\cancel{8} \quad\quad 11..\cancel{2}..13 \\ 143 & \quad\quad\quad\quad\quad\quad\quad\quad\quad\quad\quad\quad\quad \cancel{11} \quad\quad \cancel{13} \end{array}$$

$$30 \times 14 \times 12 \times 143 = 720720 = \text{l. c. m.}$$

(17)

Dissimilar.		Similar.		Similar and Coterminous.
$97·91\dot{3}4\dot{2}$	=	$97·918\dot{4}2\dot{3}$	=	$97·918\dot{4}2342342342\dot{3}$
$18·123\dot{4}56\dot{7}$	=	$19·123\dot{4}56\dot{7}$	=	$18·123\dot{4}5674567456\dot{7}$
		Difference	=	$79·789\dot{9}6667774885\dot{5}$

(18)

$$\begin{array}{rrrr}
20\text{ ft.} & 7' & & \\
19\text{ ft.} & 5 & 7'' & \\
\hline
1 & 0 & 0 & 1''' \\
8 & 6 & 11 & \\
391 & 1 & & \\
\hline
400 & 7 & 11 & 1
\end{array}
= 44\text{ sq. yds.} + \tfrac{4}{9} + \tfrac{7}{108} + \tfrac{11}{1296} + \tfrac{1}{15552} =$$

$$44\tfrac{8053}{15552}\text{ sq. yds.} = 44·517 + \text{ sq. yds.}$$

$$\$2·87\tfrac{1}{2} \times 44·517 = \$127·98 +.$$

(19)

916 acres 3 roods 17 per. 7 yds. = 4437591¼ sq. yds., and 43 acres 1 rood 2 per. 17 yds. = 209407½ sq. yds.

4437591¼ ÷ 209407½ = 4437591·25 ÷ 207407·5 = 21·19117+.

EXERCISE 94—Page 233

(1)

$742·10 × ·05 = $37·10½.

(2)

$1000 × ·11 = $110.

(3)

$734·19 × ·10 = $73·419.

(4)

$1624·50 × ·875 = $1421·4375.

(5)

$994·70 × ·125 = $124·3375.

(6)

$777.50 × ·0875 = $68·03125, or $$68·03\frac{1}{8}$.

(7)

$7135·80 × ·0225 = $160·5555.

(8)

2740 × ·20 = 548.

(9)

$7490 × ·10 = $749
$7490 × ·17 = $1273·30
$7490 × ·27 = $2022·30
$7490 × ·46 = $3445·40

(10)

$740 × ·045 = $33·30
$1680 × ·025 = $42·00
$42·00 — $33·30 = $8·70

(11)

729 × ·11 = 80·19
729 — 80·19 = 648·81 = $648\frac{81}{100}$.

(12)

$763·22 × ·25 = $190·8050
$847·16 × ·16 = 135·5456
$1234·17 × ·0625 = 77·135625

Sum = $403·486225

(13)

$17429·40 × ·43 = $$7494·64\frac{1}{5}$
$17429·40 × ·37 = $6448·87\frac{4}{5}$

$13943·52

$17429·40 — $13943·52 = $3485·88.

(14)

68978 × ·36 = 24832·08.

(15)

29800 × ·17 = 5066
29800 — 5066 = 24734

Exercise 95—Page 235.

(1)

$1000 × ·045 = $45.

(2)

$1678·30 × ·0225 = $37·76175.

(3)

$7531·19 × ·0375=$282·419625.

(4)

$508·60 × ·0125=$6·3575

(5)

$7863·50 × ·0175=$137·61125.

(6)

$878·30 × ·025=$21·9575

(7)

$7193·16 × ·03125=$224·78625.

(8)

$6734·10 × ·17=$1144·797.

(9)

$7·13 × 718 × ·0425 = $217·57195.

(10)

$1·85 × 8243 × ·05625 = $857·7871875.

Exercise 96—Page 236.

(1)

$7893·87 × ·02 = $157·8774.

(2)

$8000 × ·00875 = $70.

(3)

$8643·22 × ·0125 = $108·04025.

(4)

$78963.80 × ·00875 = $690·93325.

(5)

$1987·27 × ·0375 = $74·522625.

EXERCISE 97—Page 237.

(1)

$4000 ÷ 1·0125 = $3950·61728 + = sum to be invested.
$4000 — 3950·61728 = $49·38271 = commission.

(2)

$7500 ÷ 1·045 = $7177·03349 = sum to be expended in laces.
$7500 — $7177·03349 = $322·96651 = commission.

(3)

$8470 ÷ 1·05 = $8066·66⅔ = sum to be invested.
$8066·66⅔ ÷ $6·40 = 1260 5/12 barrels.

(4)

$11000 ÷ 1·00875 = $10904·584882 = sum to be invested.

(5)

$13000 ÷ 1.045 = $12440·1913 + = sum to be invested.
$13000 — $12440·1913 = $559·8086 + = commission.
$12440·1913 + ÷ $3·63 = 3427·0499 yds.

EXERCISE 98—Page 238.

(1)

$9000 ÷ 0·83 = $10843·373.

(2)

$8500 ÷ 1·11 = $7657·6576.

(3)

$17500 ÷ 1·0125 = $17283·951 = amount to be invested.
$17283·951 ÷ 1·07 = $16153·22 = stock.

(4)

$20000 ÷ 1·0175 = $19656·01965 = amount to be invested.
$19656·01965 ÷ 0·97 = $20263·937 = stock remitted.

(5)

$200 × 100 = $20000 = par value of 200 shares.
$1 stock costs $1·055. $1·055 × 20000 = $21100 = cost of stock.
$21100 × ·00875 = $184·625 = brokerage.
$21100 + $184·625 = $21284·625 = whole cost.

Exercise 99—Page 240.

(1)

$7500 × ·0175 = $131·25.

(2)

$8375 × ·0075 = $62·8125.

(3)

$6000 × ·01875 = $112.50

(4)

$5000 × ·0117 = $58·50.

(5)

$6400 × ·0090 = $57·60

(6)

$4500 × ·0035 = $15·75.

(7)

$36000 × ·03 = $1080.

(8)

$27000 × ·0482 × 4 = $5205·60.

(9)

$39000 × ·022 = $858.

(10)

$17800 × ·005 = $89.

(11)

$\$12350 \times \cdot 01\tfrac{3}{7} \times 7 = \$1235.$

EXERCISE 100—Page 241.

(1)

$17000 ÷ 965 = $17616·58.

(2)

$22750 ÷ ·94 = $24202·127.

(3)

$15000÷·9775=$15345·2685.

(4)

$33000 ÷ ·9425=$35013·2625.

EXERCISE 101—Page 243.

(1)

1347 × 5 = 6735 lbs. = gross weight.
6735 × ·06 = 404·1 lbs. tare.

6330·9 lbs. = net at 3½ cents per lb. = 6330·9 × ·035 = $221·58.

(2)

127 × 11 = 1397 lbs. = gross weight.
1397 × ·03 = 41·91 lbs. = tare.

1355·09 lbs. = net at $·012 per lb. = 1355·09 × ·012 = $16·26.

(3)

129 × ·13 = $16·77.

(4)

31 × 207 = 6417 lbs. = gross weight.
207 × 2¼ = 465¾ lbs. = tare.

5951¼ lbs. = net at 5¾ cents per lb. = 5951¼ × ·0575 = $342·1968.

(5)

214 × ·47 = $100·58.

EXERCISE 102—Page 243.

(1)

$17429·80 × ·21 = $3660·2580.

(2)

$2920·16 × ·075 = $219·012.

(3)

$71342·90 × ·25 = $17835·725.

(4)

$913·73 × ·2 = $182·746.

(5)

$14713·19 × ·33 = $4855·3527.

EXERCISE 103—Page 244.

(1)

$23900 ÷ 7142300 = $0·0033462 = rate per dollar.
$·0033462 × 14729·50 = $49·2878 +.

(2)

$100000 ÷ 5793000 = $·017262 = rate per dollar.
$·017262 × 18600 = $321·0732.

(3)

$100000 ÷ 5793000 = $·017262 = rate per dollar.
$·017262 × 7500 = $129·465.

(4)

$100000 ÷ 5793000 = $·017262 = rate per dollar.
$·017262 × 11400 = $196·7868.

EXERCISE 104—Page 252.

(1)

Here $P = \$723 \cdot 19$, $r = \cdot 067$, and $t = 7 \cdot 32$.
Then $I = Prt = 723 \cdot 19 \times \cdot 067 \times 7 \cdot 32 = \$354 \cdot 6813036$.

(2)

Here $P = 857 \cdot 19$, $r = \cdot 065$, and $t = 6\frac{1}{2}$ or $6 \cdot 5$.
Then $A = P(1 + rt) = \$857 \cdot 19 \times 1 \cdot 4225 = \$1219 \cdot 352775$.

(3)

Here $t = 11$, and $r = \cdot 725$.
Then $n = tr + 1 = 11 \times \cdot 725 + 1 = 8 \cdot 975$.

(4)

Here $P = \$654 \cdot 32$, $I = \$234 \cdot 56$, and $r = \cdot 07$.

Then $t = \frac{I}{Pr} = \frac{234 \cdot 56}{654 \cdot 32 \times \cdot 07} = 5 \cdot 12112$ or 5 years 1 m. 13 d.

(5)

Here $A = \$1200$, $P = \$700$, and $t = 5$.

Then $r = \frac{A - P}{Pt} = \frac{1200 - 700}{700 \times 5} = \frac{1}{7} =$ rate per unit $\therefore 14\frac{2}{7} =$ rate per cent.

(6)

Here $n = 4$, and $r = \cdot 23$.

Then $t = \frac{n - 1}{r} = \frac{4 - 1}{\cdot 23} =$ 13 years 15 days.

(7)

Here $P = \$270$, $I = \$87$ and $r = \cdot 07$.

Then $t = \frac{I}{Pr} = \frac{87}{270 \times} =$ 4 years $7\frac{5}{21}$ months.

(8)

Here $P=\$680$, $t=11\frac{1}{2}$, and $r=\cdot11$.
Then $A=P(1+rt)=680\times2\cdot265=\$1540\cdot20$

(9)

Here $A=\$2000$, $t=20$, and $r=\cdot08$.

$$\text{Then } P=\frac{A}{1+rt}=\frac{2000}{2\cdot6}=\$769\cdot23\tfrac{1}{13}.$$

(10)

Here $n=21$, and $t=24$.

$$\text{Then } r=\frac{n-1}{t}=\frac{21-1}{24}=\cdot83\tfrac{1}{3}=\text{rate per unit. } \therefore\ 83\tfrac{1}{3}=\text{rate per cent.}$$

(11)

Here $n=23$, and $r=\cdot16$.

$$\text{Then } t=\frac{n-1}{r}=\frac{23-1}{\cdot16}=137\tfrac{1}{2}\text{ years}$$

(12)

Here $P=\$679\cdot18$, $r=\cdot0775$, and $t=11\cdot73$.
Then $I=Prt=679\cdot18\times\cdot0775\times11\cdot73=\$617\cdot4255$.

(13)

Here $P=\$950$, $A=\$1763\cdot42$, and $t=10$.

$$\text{Then } r=\frac{A-P}{Pt}=\frac{1763\cdot42-950}{950\times10}=\cdot08562=\text{rate per unit}$$

$\therefore\ 8\cdot562=$ rate per cent.

(14)

Here $P = \$666$, $A = \$1347{\cdot}50$, and $r = {\cdot}06$.

Then $t = \dfrac{A - P}{Pr} = \dfrac{1347{\cdot}50 - 666}{666 \times {\cdot}06} = 17{\cdot}054+$ years, or 17 years 19 days.

(15)

Here $P = \$273$, $I = \$100$, and $r = {\cdot}09$

Then $t = \dfrac{I}{Pr} = \dfrac{100}{273 \times {\cdot}09} = 4{\cdot}07$ years $=$ 4 years 25 days.

(16)

Here $P = \$476{\cdot}30$, $A = \$500$, and $t = 2$.

Then $r = \dfrac{A - P}{Pt} = \dfrac{500 - 476{\cdot}30}{476{\cdot}30 \times 2} = {\cdot}0248 =$ rate per unit.

$\therefore 2\frac{12}{25} =$ rate per cent.

(17)

Here $P = \$749{\cdot}49$, $I = \$257$, and $t = 7$.

Then $r = \dfrac{I}{Pt} = \dfrac{257}{749{\cdot}49 \times 7} = {\cdot}04898 =$ rate per unit.

$\therefore 4{\cdot}898 =$ rate per cent.

(18)

Here $A = \$1111{\cdot}11$, $t = 11$, and $r = {\cdot}11$.

Then $P = \dfrac{A}{1 + rt} = \dfrac{1111{\cdot}11}{2{\cdot}21} = \$502{\cdot}7647$.

(19)

$P = £167{\cdot}47$, $r = {\cdot}11$, and $t = 9$.

$I = Prt = 167{\cdot}47 \times .11 \times 9 = £165{\cdot}7953 = £165$ 15s. $10\frac{109}{125}$d.

Exercise 105—Page 253.

(1)

11 ÷ 2 = 5½ cents.

(2)

16 ÷ 2 = 8 cents = $0·08.

(3)

9 years and 8 months = 116 months, and 116 ÷ 2 = 58 cents = $0·58.

(4)

16 years and 3 months = 195 months, and 195 ÷ 2 = 97½ cents = $0·97½.

(5)

11 years and 7 months = 139 months, and 139 ÷ 2 = 69½ cents = $0·695.

(6)

12 years and 5 months = 149 months, and 149 ÷ 2 = 74½ cents = $0·745.

(7)

3 years and 2 months = 38 months, and 38 ÷ 2 = 19 cents = interest of $1 for given rate and time.

$0·19 × 279·40 = $53·086.

(8)

6 years and 7 months = 79 months, and 79 ÷ 2 = 39½ cents = interest of $1 for given rate and time.

$0·395 × 189.70 = $74.9315.

(9)

3 years and 11 months = 47 months, and 47 ÷ 2 = 23½ cents = interest of $1 for given rate and time.

$0·235 × 1463 = $343·805.

(10)

11 years and 1 month = 133 months, and 133 ÷ 2 = 66½ cents = interest of $1 for given rate and time.

$0·665 × 28967·50 = $19263·3875.

EXERCISE 106—Page 254.

(1)

2 ÷ 6 = ⅓ mill = $·0003.

(2)

7 ÷ 6 = 1⅙ mills = $0·001⅙.

(3)

11 ÷ 6 = 1⅚ mills = $0·001⅚.

(4)

27 ÷ 6 = 4½ mills = $0·004½.

(5)

47 ÷ 6 = 7⅚ mills = $0·007⅚.

(6)

8 ÷ 2 = 4 cents = $0·04.

12 ÷ 6 = 2 mills = $0·002 and $0·04 + $0·002 = $0.042

(7)

66 ÷ 6 = 11 mills = $0·011.

(8)

2 years 2 m'ths = 26 months, and 26 ÷ 2 = 13 cents = $0·13.

19 ÷ 6 = 3⅙ mills = $0·003⅙ and $0·13 + $0·003⅙ = $0·133⅙.

(9)

7 years 8 m'ths = 92 months, and $92 \div 2 = 46$ cents = \$0·46.
$9 \div 6 = 1\frac{1}{2}$ mills = \$0·001$\frac{1}{2}$ and \$0.46 + \$0·001$\frac{1}{2}$ = \$0·461$\frac{1}{2}$.

(10)

17 years 11 months = 215 months, and $215 \div 2 = 107\frac{1}{2}$ cents = \$1·075.
$23 \div 6 = 3\frac{5}{6}$ mills = \$0·003$\frac{5}{6}$, and \$1·075 + \$0·003$\frac{5}{6}$ = \$1·078$\frac{5}{6}$.

(11)

12 years 7 months = 151 months, and $151 \div 2 = 75\frac{1}{2}$ cents = \$0·755.
$17 \div 6 = 2\frac{5}{6}$ mills = \$0·002$\frac{5}{6}$, and \$0·755 + \$0·002$\frac{5}{6}$ = \$0·757$\frac{5}{6}$.

EXERCISE 107—Page 255.

(1)

Interest on \$1 for 7 months = \$0·035
Interest on \$1 for 17 days = $2\frac{5}{6}$
Therefore interest on \$1 for 7 months 17 days, = \$0·037$\frac{5}{6}$
Then \$0·037$\frac{5}{6}$ × 917·30 = \$34·704516.

(2)

Interest on \$1 for 3 months = \$0·015
Interest on \$1 for 13 days = $2\frac{1}{6}$
Therefore interest on \$1 for 3 months 13 days = \$0·017$\frac{1}{6}$
Then \$0·017$\frac{1}{6}$ × 842·50 = \$14·462916.

(3)

Interest on \$1 for 2 years 11 months = \$0·175
Interest on \$1 for 10 days = $1\frac{2}{3}$
Therefore interest on \$1 for 2 years 11 m'ths 10 days = \$0 176$\frac{2}{3}$
Then \$0·176$\frac{2}{3}$ × 573·83 = \$101·3766,

(4)

Interest on $1 for 6 years 9 months = $0·405
Interest on $1 for 19 days = 3⅙
Therefore interest on $1 for 6 years 9 m'ths 19 days = $0·408⅙
Then $0·408⅙ × 642·30 = $262·16545.

(5)

Interest on $1 for 5 years 5 months = $0·325
Interest on $1 for 7 days = 1⅙
Therefore interest on $1 for 5 years 5 months 7 days = $0·326⅙
Then $0·326⅙ × 1427·875 = $465·7252.

(6)

Interest on $1 for 4 years 7 months = $0·275
Interest on $1 for 16 days = 2⅔
Therefore interest on $1 for 4 years 7 m'ths 16 days = $0·277⅔
Then $0·277⅔ × 709·63 = 197·040596.

(7)

Interest on $1 for 7 years 7 months = $0·455
Interest on $1 for 22 days = 3⅔
Therefore interest on $1 for 7 years 7 m'ths 22 days = $0·458⅔
Then $0·458⅔ × 2463·20 = $1129·7877 + $2463·20 = $3592·9877.

(8)

Interest on $1 for 9 years 9 months = $0·585
Interest on $1 for 9 days = 1½
Therefore interest on $1 for 9 years 9 months 9 days = $0·586½
Then $0·586½ × 999·99 = $586·494135.

(9)

Interest on $1 for 3 years 4 months = $0.20
Interest on $1 for 27 days = 4½

Therefore interest on $1 for 3 years 4 m'ths 27 days = $0·204½
Then $0·2045 × 68·70 = $14·04915.

(10)

Interest on $1 for 3 years = $0·18
Interest on $1 for 28 days = 4⅔

Therefore interest on $1 for 3 years 28 days = $0·184⅔
Then $0·184⅔ × 742·63 = $137·139.

(11)

Interest on $1 for 7 years 4 months = $0·44
Interest on $1 for 11 days = 1⅚

Therefore interest on $1 for 7 years 4 m'ths 11 days = $0·441⅚
Then $0·441⅚ × 200 = $88·366 + $200 = $288·366.

(12)

Interest on $1 for 9 years 3 months = $0·555
Interest on $1 for 9 days = 1½

Therefore interest on $1 for 9 years 3 months 9 days = $0·556½
Then $0·5565 × 743·63 = $413·830095 + $743·63 = $1157·460095.

Exercise 108—Page 256.

(1)

Interest on $1 at 6 per cent. for given time = $0·526⅔.
Interest on $1234·56 at 6 per cent. for given time = $0.526⅔ × 1234·56 = $650·2016.
Hence interest on $1234·56 at 7 per cent. for given time = $650·2016 + one sixth of $650·2016 = $758·5685.

(2)

Interest on $1 at 6 per cent. for given time = $0·126⅚.

Interest on $9876·54 at 6 per cent. for given time = $0·126⅚ × 9876·54 = $1252·67449.

Hence interest on $9876·54 at 3 per cent. for given time = $1252·67449 ÷ 2 = $626·337245.

(3)

Interest on $1 at 6 per cent. for given time = $0·216⅔.

Interest on $715·30 at 6 per cent. for given time = $0·216⅔ × 715·30 = $154.98166.

Hence interest on $715·30 at 8 per cent. for given time = $154·98166 + one third of $154·98166 = $206·6422.

(4)

Interest on $1 at 6 per cent. for given time = $0·141⅓.

Interest on $555·55 at 6 per cent. for given time = $0·141⅓ × 555·55 = $78.51773.

Hence interest on $555·55 at 12 per cent. for given time = $78·51773 × 2 = $157·03546 ; amount = $157·03546 + $555·55 = $712·58546.

(5)

Interest on $1 at 6 per cent. for given time = $0·016⅔.

Interest on $7766·55 at 6 per cent. for given time = $0·016⅔ × 7766·55 = $129·4425.

Hence interest on $7766.55 at 5 per cent. for given time = $129·4425 — one sixth of $129·4425 = $107·86875.

Amount = $107·86875 + $7766·55 = $7874·41875.

(6)

Interest on $1 at 6 per cent. for given time = $0·521⅓.

Interest on $500 at 6 per cent. for given time = $0·521⅓ × 500 = $260·666⅔.

Hence interest on $500 at 16 per cent. for given time = $260·666⅔ × 2⅔ = $695·111 ; amount = $695,111 + $500 = $1195·111.

(7)

Interest on $1 at 6 per cent. for given time = $0·206⅙.
Interest on $576 at 6 per cent. for given time = $0·206⅙ × 576 = $118·752.
Hence interest on $576 at 5 per cent. for given time = $118·752 —one sixth of $118·752 = $98·96.

(8)

Interest on $1 at 6 per cent. for given time = $0·151⅚.
Interest on $2478·91 at 6 per cent. for given time = $0·151⅚ × 2478·91 = $376·38116.
Hence interest on $2478·91 at 4½ per cent. for given time = $376·38116 — one fourth of $376·38116 = $282·285.

(9)

From May 9th to December 11th = 216 days. Interest on $1 at 6 per cent. for 216 days = $0·036.
Interest on $780 at 6 per cent. for 216 days = $0·036 × 780 = $28·08.

(10)

From August 16th 1851 to June 19th 1852 = 308 days.
Interest on $1 at 6 per cent. for given time = $0·051⅓.
Interest on $1830·63 at 6 per cent. for given time = $0·051⅓ × 1830·63 = $93·97234.
Hence interest on $1830·63 at 7 per cent. for given time = $93·97234 + one sixth of $93·97234 = $109·63439.

(11)

From September 3rd 1858 to January 9th 1859 = 128 days.
Interest on $1 at 6 per cent. for given time = $0·021⅓.
Interest on $6200 at 6 per cent. for given time = $0·021⅓ × 6200 = $132·266.

Amount = $132·266 + $6200 = $6332·266.

EXERCISE 109.—Page 258.

(1)

From June 2nd to July 17th there are 45 days.
" July 17th to October 6th " 81 "
" October 6th to December 11th " 66 "
" December 11th to March 29th " 109 "
" March 29th to October 7th " 192 "

Whole sum $1217·30 for 45 days = $54778·50 for 1 day.
1st endorsement 207·80

Balance $1009·50 for 81 days = $81769·50 for 1 day.
2nd endorsement 209·60

Balance $799·90 for 66 days = $52793·40 for 1 day.
3rd endorsement 320·90

Balance $479·00 for 109 days = $52211·00 for 1 day.
4th endorsement 421·83

Balance $57·17 for 192 days = $10976·64 for 1 day.

Whole interest = that of $252529·04 for 1 day.

Interest on $252529·04 at 6 per cent. for 1 year = $15151·7424.
Hence interest for 1 day = $15151·7424 ÷ 365 = $41·5116.
Then interest due = $41·5116
Balance on Note = $57·17

Principal and interest due = $98·6816

(2)

From 17th June to 5th September there are 80 days.
" 5th September to 7th December " 93 "
" 7th December to 11th June " 186 "
" 11th June to 7th February " 241 "
" 7th February to 19th December " 315 "
" 19th December to 1st May " 133 "

Whole sum $7348·25 for 80 days = $587860·00 for 1 day.
1st endorsement 2463·80

Balance $4884·45 for 93 days = $454253·85 for 1 day.
2nd endorsement 392·20

Balance $4492·25 for 186 days = $835558·50 for 1 day.
3rd endorsement 982·20

Balance $3510·05 for 241 days = $845922.05 for 1 day.
4th endorsement 2842·90

Balance $667·15 for 315 days = $210152·25 for 1 day.
5th endorsement 317·23

Balance $349·92 for 133 days = $46539·36 for 1 day.

Whole interest = that of $2980286·01 for 1 day.

Interest on $2980286·01 at 8 per cent. for 1 year = $238422·8808.
Hence interest for 1 day = $238422·8808 ÷ 365 = $653·2133.

Then interest due = $653 2133
Balance on Note = $349·92

Principal and interest due = $1003.1333

EXERCISE 110.—Page 259.

(1

$1800	Principal.
108	Interest for 1st year.
$1908	Amount for 1 year = principal for 2nd year.
114·48	Interest for 2nd year.
$2022·48	Amount for 2 years = principal for 3rd year.
121·3488	Interest for 3rd year.
$2143·8288	Amount for 3 years = principal for 4th year.
128·629728	Interest for 4th year.
$2272·458528	Amount for 4 years = principal for 5th year.
136·347511	Interest for 5th year.
$2408·806039	Amount for 5 years.
1800	Given Principal.
$608·806 =	Compound interest required.

(2)

$700	Principal.
49	Interest for 1st half year.
$749	Amount for 1 half y. = principal for 2nd half y.
52·43	Interest for 2nd half year.
$801·43	Amount for 1 year = principal for 3rd half y.
56·1001	Interest for 3rd half year.
$857·5301	Amount for 1½ years = principal for 4th half y
60·027107	Interest for 4th half year.
$917·557207	Amount for 2 years = principal for 5th half y
64·229004	Interest for 5th half year.
$981·786211	Amount for 2½ years = principal for 6th half y.
68·725034	Interest for 6th half year.
$1050·511245	Amount for 3 years = principal for 7th half y
73·535787	Interest for 7th half year.
$1124·047032	Amount for 3½ years.
700	Given Principal.
$42[illegible]·047 =	Compound interest required.

(3)

$673·40	Principal.
20·202	Interest for 1st quarter.
$693·602	Amount for 1 quar. = principal for 2nd quarter.
20·80806	Interest for 2nd quarter.
$714.41006	Amount for 1 half y. = principal for 3rd quart.
21·4323018	Interest for 3rd quarter.
$733·8423618	Amount for 3 quarters = principal for 4th quar.
22·0752708	Interest for 4th quarter.
$757·9176326	Amount for 1 year = principal for 5th quarter.
22·7375289	Interest for 5th quarter.
$780 6551615	Amount for 5 quarters = principal for 6th quar.
23·4196548	Interest for 6th quarter.
$804·0748163	Amount for 3 half y. = principal for 7th quarter.
24 1222444	Interest for 7th quarter.
$828·1970607	Amount for 7 quarters = principal for 8th qr.
24·8459118	Interest for 8th quarter.
$853 0429 =	Amount for 2 years required.
673 40	Given Principal.
$179·6429 =	Compound Interest required.

(4)

$860	Principal.
34·4	Interest for 1st half year.
$894·4	Amount for 1 half year = principal for 2nd half y.
35·776	Interest for 2nd half year.
$930·176	Amount for 1 year = principal for 3rd half year.
37·20704	Interest for 3rd half year.
$967·38304	Amount for 3 half years = principal for 4th half y.
38·69532	Interest for 4th half year.
$1006·07836	Amount for 2 years = principal for 5th half year.
40·24313	Interest for 5th half year.
$1046·32149	Amount for 5 half years = principal for 6th half y.
41·85285	Interest for 6th half year.
$1088·17434 =	Amount for 3 years required.
860	Given Principal.
$228·1743 =	Compound Interest required.

EXERCISE 111—Page 261.

(1)

By the table the am't of $1 at 6 per cent. for 11 years = $1·8983.
Then $1·8983 × 875 = $1661.0125 = Amount.
875 Principal.

$786·0125 = Interest.

(2)

By the table the am't of $1 for the given time and rate = $2·77247.
Then $2·77247 × 643·98 = $1785·41523 = Amount.
643·98 Principal.

$1141·43523 = Interest.

(3)

By the table the am't of $1 at 6 per cent. for 45 years = $13·76461.
Then $13·76461 × ·01 = $·137646 = Amount.
·01 Principal.

$·127646 = Interest.

(4)

By the table the am't of $1 for the given time and rate = $2·28793.
Then $2·28793 × 78·2 = $178·916 = Amount.
78 2 Principal.

$100·716 = Interest.

(5)

By the table the am't of $1 for the given rate and time = $2·40662
Then $2.40662 × 777·77 = $1871·7968 = Amount.
777·77 Principal.

$1094·0268 = Interest.

(6)

£44 5s. 9d. = £44·2875.

By the table the am't of £1 at 6 per cent. for 11 years = £1·8983.
Then £1·8983 × 44·2875 = £84·07096 = £84 1 5 = Amount.
44 5 9 Principal.
£39 15 8 = Interest.

(7)

£32 4s. 9¾d. = £32·240625.

By the table the amount of £1 for the given time and rate = £1·26532. Then £1·26532 × 32·240625 =
£40·7947076 = £40 15 10¾ nearly = Amount.
32 4 9¾ Principal.
£8 11 1 = Interest.

Exercise 112—Page 262.

(1)

Amount of $1 for 7 years at 4 per cent. = $1·31593.
$7439·87 ÷ 1·31593 = $5653·697.

(2)

Amount of $1 at 5 per cent. for 20 years = $2·6533.
$9193·90 ÷ 2·6533 = $3465·081.

(3)

£595 10s. 2⅖d. = £595·51
Amount of £1 at 6 per cent. for 3 years = £1·19102.
£595·51 ÷ 1·19102 = £500.

(4)

Amount of $1 at 6 per cent. for 7 years = $1·50363.
$7111·11 ÷ 1·50363 = $4729·295.

(5)

$$£268\ 0s.\ 4\tfrac{4}{5}d. = £268{\cdot}02.$$

Amount of £1 at 5 per cent for 6 years = £1·3401.

$$£268{\cdot}02 \div 1{\cdot}3401 = £200.$$

EXERCISE 113—Page 263.

(1)

Here $A = \$962$, $r = {\cdot}04$, and $t = 1$. Whence $1 + rt = 1{\cdot}04$.

$$\text{Then}\quad P = \frac{A}{1 + rt} = \frac{962}{1{\cdot}04} = \$925.$$

(2)

Here $A = \$2202$, $r = {\cdot}06$, and $t = 5{\cdot}75$. Whence $1 + rt = 1{\cdot}345$.

$$\text{Then}\quad P = \frac{A}{1 + rt} = \frac{2202}{1{\cdot}345} = \$1637{\cdot}174.$$

(3)

Here $A = \$1003{\cdot}50$, $r = {\cdot}06$, and $t = \frac{2}{3}$ year. Whence $1 + rt = 1{\cdot}04$.

$$\text{Then}\quad P = \frac{A}{1 + rt} = \frac{1003{\cdot}50}{1{\cdot}04} = \$964{\cdot}9038.$$

(4)

Here $A = \$716$, $r = {\cdot}08$, and $t = \frac{7}{12}$ year. Whence $1 + rt = 1{\cdot}04\frac{2}{3}$.

$$\text{Then}\quad P = \frac{A}{1 + rt} = \frac{716}{1{\cdot}04\frac{2}{3}} = \$684{\cdot}0764.$$

(5)

Here $A = \$1342\cdot50$, $r = \cdot065$, and $t = \frac{25}{73}$ year. Whence $1 + rt = 1\cdot022\frac{19}{73}$.

$$\text{Then}\quad P = \frac{A}{1+rt} = \frac{1342\cdot50}{1\cdot022\frac{19}{73}} = \$1313\cdot266.$$

(6)

Here $A = \$2400$, $r = \cdot05$, and $t = \frac{236}{365}$ year. Whence $1 + rt = 1\cdot03\frac{17}{73}$.

$$\text{Then}\quad P = \frac{A}{1+rt} = \frac{2400}{1\cdot03\frac{17}{73}} = \$2324\cdot84.$$

(7)

Here $A = \$2202$, $r = \cdot05$, and $t = \cdot75$ year. Whence $1 + rt = 1\cdot0375$.

$\$2202 \div 1\ .0375 = \$2122\cdot40963+ =$ Present worth.

$\$2202 - \$2122\cdot40963+ = \$79\cdot59036 =$ Discount.

(8)

Here $A = \$4360$, $r = \cdot06$, and $t = 1\frac{5}{12}$. Whence $1 + rt = 1\cdot085$

$$\text{Then}\quad P = \frac{A}{1+rt} = \frac{4360}{1\cdot085} = \$4018\cdot43317.$$

(9)

Here $A = \$1647$, $r = \cdot06$, and $t = \frac{11}{12}$ year. Whence $1 + rt = 1\cdot055$.

$$\text{Then}\quad P = \frac{A}{1+rt} = \frac{1647}{1\cdot055} = \$1561\cdot13744.$$

(10)

Here $A = \$2000$, $r = \cdot06$, and $t = 3\frac{7}{12}$. Whence $1 + rt = 1\cdot215$

$$\text{Then}\quad P = \frac{A}{1+rt} = \frac{2000}{1\cdot215} = \$1646\cdot09053.$$

(11)

Here $A = \$2070{\cdot}90$, $r = {\cdot}05$, and $t = 1\frac{7}{12}$. Whence $1 + rt = 1{\cdot}07\frac{11}{12}$.

$$\text{Then } P = \frac{A}{1 + rt} = \frac{2070{\cdot}90}{1{\cdot}07\frac{11}{12}} = \$1918{\cdot}9806.$$

$\$2070 - \$1918{\cdot}9806 = \$151{\cdot}019 =$ Discount required.

(12)

Here $A = \$970{\cdot}63$, $r = {\cdot}08$, and $t = \frac{11}{12}$ year. Whence $1 + rt = 1{\cdot}07\frac{1}{3}$.

$$\text{Then } P = \frac{A}{1 + rt} = \frac{970{\cdot}63}{1{\cdot}07\frac{1}{3}} = \$904{\cdot}313.$$

(13)

Here in first case $A = \$1512$, $r = {\cdot}07$, and $t = {\cdot}5$ year. Whence $1 + rt = 1{\cdot}035$.

$$\text{Then } P = \frac{A}{1 + rt} = \frac{1512}{1{\cdot}035} = \$1460{\cdot}8695.$$

Also $A = 1512$, $r = {\cdot}07$, and $t = 1$. Whence $1 + rt = 1{\cdot}07$.

$$\text{Then } P = \frac{A}{1 + rt} = \frac{1512}{1{\cdot}07} = \$1413{\cdot}0841.$$

$\$1460{\cdot}8695 + \$1413{\cdot}0841 = \$2873{\cdot}9536 =$ Present worth of whole amount.

$\$3024 - \$2873{\cdot}9536 = \$150{\cdot}0464 =$ Discount required.

(14)

Here in first case $A = \$440$, $r = {\cdot}08$, and $t = 1{\cdot}25$. Whence $1 + rt = 1{\cdot}1$.

$$\text{Then } P = \frac{A}{1 + rt} = \frac{440}{1{\cdot}1} = \$400.$$

In second case $A = \$896$, $r = {\cdot}08$, and $t = 1{\cdot}5$. Whence $1 + rt = 1{\cdot}12$.

$$\text{Then } P = \frac{A}{1 + rt} = \frac{896}{1{\cdot}12} = \$800.$$

$\$400 + \$800 = \$1200.$

EXERCISE 114—Page 265.

(1)

Here the time the note has to run is 2 years 3 months 3 days. Interest of $1 at 7 per cent. for 2 yrs., 3 m., 3 days = $0·1580$\frac{5}{6}$. Interest of $986 at 7 per cent. for 2 years, 3 months, 3 days = $0·1580$\frac{5}{6}$ × 986 = $155·8701.

(2)

Here the time the note has to run is 103 days = 3 months 13 days. Interest of $1 at 8 per cent. for 3 months 13 days = $0·022$\frac{8}{9}$. Interest of $640 at 8 per cent. for 3 months, 13 days = $0·022$\frac{8}{9}$ × 640 = $14·6488.

(3)

Here the time the note has to run is 94 days = 3 months 4 days. Interest of $1 at 6 per cent. for 3 months 4 days = $0·015$\frac{2}{3}$. Interest of $563·80 at 6 per cent. for 3 months 4 days = $0·015$\frac{2}{3}$ × 563·80 = $8·8323 and $563·80 — $8·8328 = $554·967.

EXERCISE 115–Page 266.

(1)

Interest on $1 for 93 days at 7 p. c. = $0·0180$\frac{5}{6}$, and this taken from $1 gives a remainder of $0·9819$\frac{1}{6}$ = present worth of $1.
Then $3755 ÷ 0·9819$\frac{1}{6}$ = $3824·15.

(2)

Interest on $1 for 6 months 3 days at 5 per cent. = $0·0254$\frac{1}{6}$ and this taken from $1 gives a remainder $0·9745$\frac{5}{6}$ = present worth of $1.
Then $1147·80 ÷ 0·9745$\frac{5}{6}$ = $1177·734.

(3)

Interest on \$1 for 48 days at $3\frac{1}{2}$ per cent. $= \$0\cdot004\frac{2}{3}$, and this taken from \$1 gives a remainder $\$0\cdot995\frac{1}{3}$ = present worth of \$1.

Then $\$713\cdot90 \div 0\cdot995\frac{1}{3} = \$717\cdot2471$.

EXERCISE 116—Page 268.

(1)

$\$200 \times 3 = 600$
$150 \times 4 = 600$
$250 \times 6 = 1500$

$600 \quad 600)2700(4\frac{1}{2}$ months.
2400
$\left.\frac{300}{600}\right\} = \frac{1}{2}$

(2)

$\frac{1}{4} \times 0 = 0$
$\frac{1}{4} \times 3 = \frac{3}{4}$
$\frac{1}{4} \times 6 = 1\frac{1}{2}$
$\frac{1}{4} \times 9 = 2\frac{1}{4}$

$1 \quad 1)4\frac{1}{2}$
$4\frac{1}{2}$ months.

(3)

$\$50 \times 2 = 100$
$40 \times 5 = 200$
$30 \times 7 = 210$

$120 \quad 120)510(4\frac{1}{4}$ months.
480
$\left.\frac{30}{120}\right\} = \frac{1}{4}$

(4)

$\$1000 \times 0 = 0$
$1500 \times 1 = 1500$
$600 \times 3 = 1800$
$700 \times 5 = 3500$
$1400 \times 7 = 9800$

$5200 \quad 5200)16600(3\frac{5}{26}$ months.
15600
$\left.\frac{1000}{5200}\right\} = \frac{5}{26}$

(5)

Six months from 15th January = 15th July, and from 1st July to 15th July there are 14 days.

Six months from 10th February = 10th August, and from 1st July to 10th August there are 40 days.

Six months from 6th March = 6th September, and from 1st July to 6th September there are 67 days.

Six months from 8th June = 8th December, and from 1st July to 8th December there are 160 days.

$$\begin{array}{rcrcr}
\$3750 & \times & 14 & = & 52500 \\
3000 & \times & 40 & = & 120000 \\
2400 & \times & 67 & = & 160800 \\
2250 & \times & 160 & = & 360000 \\
\hline
11400 & & & 11400) & 693300(60\tfrac{4}{5} \text{ days.} \\
 & & & & 684000 \\
\hline
 & & & & \left.\dfrac{9300}{11400}\right\} = \tfrac{4}{5}
\end{array}$$

Therefore the note must be made payable on the 61st day from the 1st of July, which is the 31st of August.

EXERCISE 117.—Page 269.

(1)

Whole stock : A's stock :: whole profit : A's profit.

That is, \$4300·\$3000::\$1117: $\dfrac{1117 \times 3000}{4300}$ = \$779·392 + = A's sh

\$1117 − \$779·302 + = \$337·697 = B's share.

(2)

Whole stock = \$6470 + \$3780 + \$9860 = \$20110.

Whole stock : A's stock :: whole profit : A's profit.

That is, $\$20110:\$6470::\$7890:\frac{7890\times 6470}{20110}=\$2538{\cdot}453+=\text{A's sh.}$

Again, whole stock : B's stock :: whole profit : B's profit.

That is, $\$20110:\$3780::\$7890:\frac{7890\times 3780}{20110}=\$1483{\cdot}053+=\text{B's sh.}$

Lastly, whole stock : C's stock :: whole profit : C's profit.

That is, $\$20110:\$9860::\$7890:\frac{7890\times 9860}{20110}=\$3868{\cdot}493+=\text{C's sh.}$

(3)

Whole stock : B's stock :: whole gain : B's gain.

That is, $\$320 : \$120 :: \$80 : \frac{80\times 120}{320}=\$30=\text{B's gain.}$

Again, whole stock : C's stock :: whole gain : C's gain.

That is, $\$320 : \$200 :: \$80 : \frac{80\times 200}{320}=\$50=\text{C's share.}$

(4)

Whole stock : B's stock :: whole gain : B's gain.

That is, $\$2800 : \$1200 :: \$728 : \frac{728\times 1200}{2800}=\$312=\text{B's gain.}$

Again, whole stock : C's stock :: whole gain : C's gain.

That is, $\$2800 : \$1600 :: \$728 : \frac{728\times 1600}{2800}=\$416=\text{C's gain.}$

(5)

Whole stock : B's stock : : whole amount to be divided : B's share

$$\text{That is, } \$3 : \$2 :: \$100 : \frac{100\times2}{3} = \$66{\cdot}66\tfrac{2}{3} = \text{B's share.}$$

Again, whole st'k : C's st'k :: whole amo't to be divided : C's sh'e

$$\text{That is, } \$3 : \$1 :: \$100 : \frac{100\times1}{3} = \$33{\cdot}33\tfrac{1}{3} = \text{C's share.}$$

(6)

$$£1400 : £500 :: £1100 : \frac{1100 \times 500}{1400} = £392\tfrac{6}{7} = \text{B's share.}$$

$$£1100 - £392\tfrac{6}{7} = £707\tfrac{1}{7} = \text{C's share.}$$

(7)

$$\begin{array}{l} \text{casks. casks.} \\ 900 : 200 :: 180 : \dfrac{180 \times 200}{900} = 40 \text{ casks} = \text{B's loss.} \\ 900 : 300 :: 180 : \dfrac{180\times300}{900} = 60 \text{ casks} = \text{C's loss.} \end{array}$$

$$180 - (40 + 60) = 80 \text{ casks} = \text{D's loss.}$$

(8

$$\$1800 : \$800 :: \$100 : \frac{100\times800}{1800} = \$44{\cdot}44\tfrac{4}{9} = \text{B's share.}$$

$$\$1800 : \$600 :: \$100 : \frac{100\times600}{1800} = \$33{\cdot}33\tfrac{1}{3} = \text{C's share.}$$

$$\$44{\cdot}44\tfrac{4}{9} + \$33{\cdot}33\tfrac{1}{3} = \$77{\cdot}77\tfrac{7}{9}, \text{ and } \$100 - \$77{\cdot}77\tfrac{7}{9} =$$
$$\$22{\cdot}22\tfrac{2}{9} = \text{D's share.}$$

(9)

$$6:1::120:\frac{120\times1}{6}=20$$

$$6:2::120:\frac{120\times2}{6}=40$$

$$6:3::120:\frac{120\times3}{6}=60$$

(10)

Whole loss = \$900 — \$540 = \$360.

$$8:1::\$360:\frac{360}{8}=\$45=\text{B's loss.}$$

$$8:2::\$360:\frac{360\times2}{8}=\$90=\text{C's loss.}$$

\$45+90=\$135, and \$360—135=\$225=D's loss.

(11)

$$\$12:\$6::\$1320:\frac{1320\times6}{12}=\$660=\text{B's gain.}$$

$$\$12:\$4::\$1320:\frac{1320\times4}{12}=\$440=\text{C's gain.}$$

$$\$12:\$2::\$1320:\frac{1320\times2}{12}=\$220=\text{D's gain.}$$

(12)

£35 + £29 = £64, and £110—£64 = £46 = D's profit.

D's profit : B's profit :: D's stock : B's stock.

$$\text{That is, }£46:£35::£1090:\frac{1090\times35}{46}=£829\text{ 6s. 11[illegible]d}=\text{B's st.}$$

Again, D's profit : C's profit :: D's stock : C's stock.

$$\text{That is, }£46:£29::£1090:\frac{1090\times29}{46}=£687\text{ 3s. [illegible]d.}=\text{C's st.}$$

EXERCISE 118.—Page 271.

(1)

$$\left.\begin{array}{l}\$357 \times 5 = \$1785 \text{ for one month} \\ 371 \times 7 = 2597 \text{ for one month} \\ 154 \times 11 = 1694 \text{ for one month}\end{array}\right\} = \$6076 \text{ for one month.}$$

$$\$6076 : \$1785 :: \$347{\cdot}20 : \frac{347{\cdot}20 \times 1785}{6076} = \$102$$

$$\$6076 : \$2597 :: \$347{\cdot}20 : \frac{347{\cdot}20 \times 2597}{6076} = \$148{\cdot}40.$$

$$\$6076 : \$1694 :: \$347{\cdot}20 : \frac{347{\cdot}20 \times 1694}{6076} = \$96{\cdot}80$$

(2)

$$\left.\begin{array}{l}40 \times 6 = 240 \text{ for one month} \\ 30 \times 5 = 150 \text{ for one month} \\ 50 \times 1 = 50 \text{ for one month}\end{array}\right\} = 440 \text{ for one month.}$$

$$440 : 240 :: \$160 : \frac{160 \times 240}{440} = \$87{\cdot}27\tfrac{3}{11}\text{; B's share.}$$

$$440 : 150 :: \$160 : \frac{160 \times 150}{440} = \$54{\cdot}54\tfrac{6}{11}\text{; C's share.}$$

$$440 : 50 :: \$160 : \frac{160 \times 50}{440} = \$18{\cdot}18\tfrac{2}{11}\text{; D's share.}$$

(3)

$$\left.\begin{array}{l}£150 \times 6 = £900 \text{ for one month} \\ 200 \times 3 = 600 \text{ for one month} \\ 125 \times 16 = 2000 \text{ for one month}\end{array}\right\} = £3500 \text{ for one month.}$$

$$£3500 : £900 :: £291\ 13s.\ 4d. : \frac{£291\ 13s.\ 4d. \times 900}{3500} = £75.$$

$$£3500 : £600 :: £291\ 13s.\ 4d. : \frac{£291\ 13s.\ 4d. \times 600}{3500} = £50.$$

$$£3500 : £2000 :: £291\ 13s.\ 4d. : \frac{£291\ 13s.\ 4d. \times 2000}{3500} = £166\ 13s.\ 4d.$$

(4)

$$\left.\begin{array}{l}\$4000 \times 12 = \$48000 \text{ for one month}\\ 3000 \times 15 = 45000 \text{ for one month}\\ 5000 \times 8 = 40000 \text{ for one month}\end{array}\right\} = \$133000 \text{ for one month}$$

$$\$133000 : \$48000 :: \$665 : \frac{665 \times 48000}{133000} = \$240 ; \text{ B's share.}$$

$$\$133000 : \$45000 :: \$665 : \frac{665 \times 45000}{133000} = \$225 ; \text{ C's share.}$$

$$\$133000 : \$40000 :: \$665 : \frac{665 \times 40000}{133000} = \$200 ; \text{ D's share.}$$

(5)

$$\left.\begin{array}{l}56 \times 12 = 672 \text{ for one day}\\ 64 \times 15 = 960 \text{ for one day}\\ 80 \times 18 = 1440 \text{ for one day}\end{array}\right\} = 3072 \text{ for one day.}$$

$$3072 : 672 :: \$320 : \frac{320 \times 672}{3072} = \$70 = \text{rent to be paid by 1st troop.}$$

$$3072 : 960 :: \$320 : \frac{320 \times 960}{3072} = \$100 = \text{" " " 2nd "}$$

$$3072 : 1440 :: \$320 : \frac{320 \times 1440}{3072} = \$150 = \text{" " " 3rd "}$$

(7)

Sum of profits $= 240 + 800 + 400 = \$1440$.

Whole profit : A's profit :: Whole stock for 1 m. : A's st. for 1 m.

That is, $1440 : 240 :: 34560 : \dfrac{34560 \times 240}{1440} = 5760 =$ A's stock for one month. Hence, since A's stock was in for 6 months, it will be $\$5760 \div 6 = \$960$.

(Continued on next page.)

(7 Continued.)

Whole profit : B's profit :: Whole stock for 1 m. : B's st. for 1 m.

$$1440 : 800 :: 34560 : \frac{34560 \times 800}{1440} = 19200 = \text{B's stock for one}$$

month. And, since B's stock was in for 12 months, $19200 \div 12 = \$1600$ will be his stock.

Whole profit : C's profit :: whole stock for 1 m. : C's st. for 1 m.

$$1440 : 400 :: 34560 : \frac{34560 \times 400}{1440} = \$9600 = \text{C's stock for one}$$

month, and hence his stock will be $\$9600 \div 15 = \$640$.

(8)

A's profit was $240 for 6 months = $40 for 1 month.
B's profit was $800 for 12 months = $66⅔ for 1 month.
C's profit was $400 for 15 months = $26⅔ for 1 month.

Sum of profits for 1 month = $133⅓

Whole profit for 1 m. : A's profit for 1 m. :: whole stock : A's st.

$$133\tfrac{1}{3} : 40 :: 3200 : \frac{3200 \times 40}{133\tfrac{1}{3}} = \$960 = \text{A's stock.}$$

$$133\tfrac{1}{3} : 66\tfrac{2}{3} :: 3200 : \frac{3200 \times 66\tfrac{2}{3}}{133\tfrac{1}{3}} = \$1600 = \text{B's stock.}$$

$$133\tfrac{1}{3} : 26\tfrac{2}{3} :: 3200 : \frac{3200 + 26\tfrac{2}{3}}{133\tfrac{1}{3}} = \$640 = \text{C's stock.}$$

EXERCISE 119—Page 275.

(1)

$0·12½ = selling price.
$0·09 = buying price.

$0·03½ = gain per lb.
$0·03½ × 317 = $11·095.

(2)

$1·20 = selling price
$0·87½ = buying price.

$0·32½ = gain per bushel
$0·32½ × 2138 = $694·85.

(3)

$0·15×317×13=$618·15=cost of 13 barrels at $0·15 per lb.
$735—618·15 =$116·85 gain.

(4

$3·15×22×17=$1178·10 = price of 17 kegs at $3·15 per gal.
$0·37½× 1178·1=$441·7875 =ad valorem duty.
$1178·10+$441·7875+$26·33=$1646·2175=whole cost.
$1646·2175—$1625=$21·2175 loss.

Exercise 120—Page 276.

(1)

Here for every $1 I expend I wish to receive $1·30, and hence the selling price will be $3·25 × 1·30=$4·22½.

(2)

Here for every $1 I expend I wish to receive $1·05, and hence the selling price will be $1·05 × 13420 = $14091.

(3)

Here for every $1 I expend I desire to receive $1·15, and hence the selling price will be $1·15×·11 = $0·1265 =12$\frac{13}{20}$ cents.

(4)

Here for every $1 I expend I wish to receive $1·23, and hence the selling price will be $1·23 × 15·25 = $18·75¾.

(5)

Here for every $1 I expend I am willing to receive $0·89, and hence the selling price will be $0·89×7890=$7022·10.

EXERCISE 121—Page 277.

(1)

Here the whole gain is \$0·87½ — \$0·60 = \$0·27½.

That is, \$0·60 gains \$0·27½, and therefore 1 cent gains $\frac{27\frac{1}{2}}{60} = \frac{55}{120} = \frac{11}{24}$ of a cent.

And hence, the gain per cent $= \frac{11}{24} \times 100 = \frac{1100}{24} = 45\frac{5}{6}$ per cent.

(2)

Here the loss on each lb. is 2 cents.

That is, every 13 cents invested gives a loss of 2 cents.

Therefore every cent invested loses $\frac{1}{13}$ of $2 = \frac{2}{13}$ cents.

And hence, the loss per cent $= \frac{2}{13} \times 100 = \frac{200}{13} = 15\frac{5}{13}$ per c.

(3)

Here the gain on each barrel is \$1·60.

That is, every \$6·20 invested gives a gain of \$1·60.

Therefore every \$1 invested gains $\frac{1}{620}$ of $160 = \frac{8}{31}$ of a \$.

And hence, the gain per cent $= \frac{8}{31} \times \frac{100}{1} = 25·8 = 25\frac{4}{5}$ p. c.

(4)

Here the gain on each yard is 35 cents.

That is, every \$2·75 invested gives a gain of 35 cents.

Therefore every \$1 invested gains $\frac{1}{275}$ of $35 = \frac{35}{275} = \frac{7}{55}$ of a dollar.

And hence the gain per cent $= \frac{7}{55} \times 100 = \frac{700}{55} = 12\frac{8}{11}$ p. c.

(5)

Here the gain on every bushel is 9 cents.

That is, every 47 cents invested gives a gain of 9 cents.

Therefore every cent invested gains $\frac{1}{47}$ of $9 = \frac{9}{47}$ cents.

And hence the gain per cent $= \frac{9}{47} \times 100 = \frac{900}{47} = 19\frac{7}{47}$ p. c.

(6)

Here the loss on each lb. is $1\frac{1}{2}$ cents.

That is every 12 cents invested gives a loss of $1\frac{1}{2}$ cents.

Therefore every cent invested loses $\frac{1}{12}$ of $1\frac{1}{2} = \frac{1}{8}$ of a cent.

And hence, the loss per cent $= \frac{1}{8} \times 100 = \frac{100}{8} = 12\frac{1}{2}$ p. c.

(7)

Here the whole gain is \$127 — \$93 = \$34.

That is, \$93 gain \$34, and therefore \$1 gains $\frac{34}{93}$ of a dollar.

Hence, gain per cent $= \$\frac{34}{93} \times 100 = \frac{3400}{93} = 36\frac{52}{93}$ per cent.

(8)

Here the loss is \$6742·50 — \$6000 = \$742·50.

That is, \$6742·50 loses \$742·50, and therefore \$1 loses $\frac{1}{674250}$ of 742·50 $= \frac{99}{899}$ of a dollar.

Hence loss per cent $= \$\frac{99}{899} \times 100 = \frac{9900}{899} = 11\frac{11}{899}$ per cent.

(9)

Here \$5700 + \$275 + \$1987.32 = \$7962·32 = whole sum expended.

Whole gain = \$8750 — \$7962·32 = \$787·68.

That is, \$7962·32 gains \$787·68, and therefore \$1 gains $\frac{1}{796232}$ of 787.68 $= \frac{9846}{99529}$ of a \$.

Hence gain per cent $= \$\frac{9846}{99529} \times 100 = \frac{984600}{99529} = 9·89$ or nearly 10 per cent.

(10)

\$4·25 × 723 = \$3072·75 = price of 723 yds. @ \$4·25.

\$3072.75 × ·07 = \$215·0925 = amount for Insurance.

\$3072·75 × ·22 = \$676·005 = amount for ad valorem duty.

Then whole cost = \$3072·75 + \$215·0925 + \$23·70 + \$2·70 + \$3·16 + \$676·005 = \$3993·4075.

Whole gain = \$5270 — \$3993·4075 = \$1276·5925.

That is, \$3993·4075 gains \$1276·5925 ∴ \$1 gains $\frac{1}{39934075}$ of \$1276·5925 $= \frac{510637}{1597363}$ of a \$.

Hence gain per cent $= \$\frac{510637}{1597363} \times 100 = 31·96749$ or nearly 32 per cent.

EXERCISE 122—Page 278.

(1)

Loss on $1 is 4 cents, or for every $1 paid I receive $0·96. Hence cost $= \$24·60 \div 0·96 = \$25·625$.

(2)

Loss on $1 is 10 cents, or for every $1 paid he receives $0.90. Hence cost $= \$2360 \div ·90 = \$2622·22$.

(3)

Gain on $1 is 11 cents, or for every $1 paid he receives $1·11. Hence cost $= \$7400 \div 1·11 = \$6666·666$.

(4)

Gain on $1 is 17 cents, or for every $1 paid he receives $1·17.

$$\$117 : \$100 :: \$3789·40 : \frac{3789·40 \times 100}{117} = \$3238·803.$$

(5)

Loss on $1 is 13 cents, or for every $1 paid I receive $0·87.

$$\$87 : \$100 :: \$2740 : \frac{2740 \times 100}{87} = \$3149·425.$$

EXERCISE 123—Page 279.

(1)

$2 gains 50 cents.

$$\text{Hence } \$0·50 : \$0·10 :: \$2·00 : \frac{2·00 \times 10}{50} = 40 \text{ cents.}$$

(2)

$$\$2\cdot00 : \$2\cdot80 :: \$2\cdot50 : \frac{2\cdot50\times2.80}{2\cdot00}=\$3\cdot50.$$

(3)

8 cents gain 5 cents in 9 months.

Hence 9 mo's : 6 mo's :: 5 cents : $\frac{5\times6}{9}=3\frac{1}{3}=$ gain for 6 mo's.

8 cts. : 12 cts. :: $3\frac{1}{3}$: $\frac{3\frac{1}{3}\times12}{8}=5$ cts. gain on 12 cts. for 6 mo's.

Therefore $12+5=17$ cents $=$ his selling price.

(4)

$\$1\cdot60 : \$1\cdot85 :: \$0\cdot55 : \frac{1\cdot85\times\cdot55}{1\cdot60}=\$0\cdot6359375=$ what L ought to get in order to sell at the same profit as K.

But L only gets 60 cents, therefore K has the advantage.

70 yds. of cloth at $\$1\cdot85=\$1\cdot85\times70=\$129\cdot50.$

$$\$129\cdot50\div\$\cdot60=215\tfrac{5}{6}.$$

(5)

5 tons of butter at $102	$= \$102\times5$	$= \$510$
10½ tons of tallow at $135	$= \$135\times10\frac{1}{2}$	$= \$1417\cdot50$
Total value		$= \$1927\cdot50$
Deduct ready money,		$\$600\cdot30$
		$\$1327\cdot20$

$\$1327\cdot20\div\$4\cdot20=316$ barrels.

EXERCISE 124—Page 281.

(1)

7 oz. × 22 = 154 carats.
$12\frac{1}{2}$ " × 21 = $262\frac{1}{2}$ "
17 " × 9 = 153 "

$36\frac{1}{2}$ $36\frac{1}{2})569\frac{1}{2}$ "
2 2 "

73) 1139($15\frac{44}{73}$ carats.
73
409
365
44

(2)

2 gallons @ 14s. = 28s.
1 " @ 12s. = 12s.
2 " @ 9s. = 18s.
4 " @ 8s. = 32s.

9 9)90s.
10s.

(3)

15 bushels @ $1·20 = $18·00
30 " @ $1·50 = $45·00
60 " @ $1·10 = $66·00
83 " @ $1·75 = $145·25

188 188)$274·25($1·458
188
86·2
75·2
11·05
9·40
1·650
1·504
·146

(4)

12 lbs. @ 50 cents = 600 cents.
16 " @ 72 " = 1152 "
22 " @ 65 " = 1430 "
18 " @ 85 " = 1530 "
100 " @ 42 " = 4200 "
168

168)8912 cents (53 $\frac{1}{21}$ cents.
840
512
504

$$\left.\begin{array}{c} 8 \\ \hline 168 \end{array}\right\} = \frac{1}{21}.$$

Exercise 125—Page 283.

(1)

Prices. Differences. Prices.

$$125 = \left\{\begin{array}{l} 160 - \overbrace{35 \text{———} 15}^{} + 110 \\ 140 - 15 \text{———} 25 + 100 \end{array}\right\} = 125$$

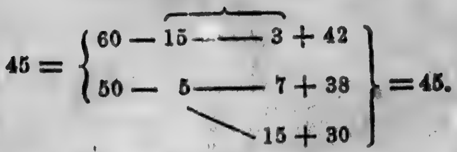
Prices. Differences. Prices.

$$125 = \left\{\begin{array}{l} 160 - \overbrace{35 \quad\;\; 15}^{} + 110 \\ \qquad\qquad\; \times \\ 140 - 15 \quad\;\; 25 + 100 \end{array}\right\} = 125.$$

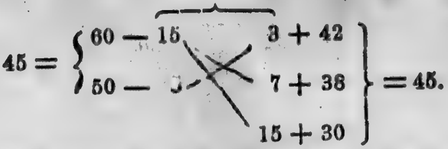
Ans. 35 bush. @ $1·10, 15 @ $1·60, 15 @ $1, and 25 @ $1·40.
35 bush. @ $1·00, 15 @ $1·40, 15 @ $1·10, and 25 @ $1·60.

(2)

Prices. Differences. Prices.



Prices. Differences. Prices.

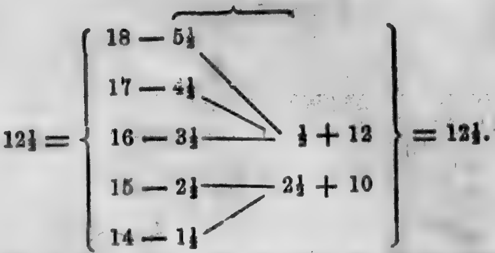


Ans. 15 quarts @ 42 cents, 3 @ 60 cents, 5 @ 38 cents, 5 @ 30 cents, and 7 + 15 = 22 @ 50 cents.

15 quarts @ 38 cents, 3 @ 50 cents, 5 @ 42 cents, 15 @ 30 cents, and 7 + 15 = 22 @ 60 cents.

(3)

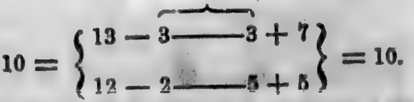
Prices. Differences. Prices.



Ans. ½ lb. @ 18 cents, ½ @ 17 cents, ½ @ 16 cents, 2½ @ 15 cents, 2½ @ 14 cents, 5½ + 4½ + 3½ = 13½ @ 12 cents, and 2½ + 1½ = 4 @ 10 cents.

(4)

Prices. Differences. Prices.



Ans. 3 lbs. @ 7d., 3 @ 13d., 2 @ 5d., and 5 @ 12d.

EXERCISE 126—Page 284.

(1)

By Case I we find that 17 quarts @ 31 cents, 6 @ 16 cents, 6 @ 19 cents, and 6 @ 23 cents will make a mixture worth 25 cents per quart.

Therefore 17 qts. : 87 qts. :: 6 qts. : $\frac{6 \times 87}{17} = 30\frac{12}{17}$ quarts @ 16 cents, and as there are 6 lbs. at each of the other prices, the same statement may be used, and the answer is therefore $30\frac{12}{17}$ quarts @ each price.

(2)

To produce a mixture worth 75 cents per bushel, we require 45 bushels @ 80 cents, 5 @ 37 cents, and 5 @ 68 cents.

Therefore 45 bush. : 70 bush. :: 5 bush. : $\frac{5 \times 70}{45} = 7\frac{7}{9}$ bush. oats @ 37 cents.

45 bush. : 70 bush. :: 5 bush. : $\frac{5 \times 70}{45} = 7\frac{7}{9}$ bush. barley @ 68 cents.

(3)

To produce a mixture worth 1s. per lb., we require $1\frac{1}{2}$ lbs. @ 16d., $1\frac{1}{2}$ @ 14d., and 6 @ $10\frac{1}{2}$d.

Then $1\frac{1}{2}$ lbs. : 50 lbs. :: $1\frac{1}{2}$ lbs. : 50 lbs. brass @ 14d.

$1\frac{1}{2}$ lbs. : 50 lbs. :: 6 lbs. : 200 lbs. pewter @ $10\frac{1}{2}$d.

(4)

By Case I we find that 1 oz. of 20 carats fine, 1 of 21 carats fine and 3 of 23 carats fine, will make a mixture 22 carats fine.

Then 1 oz. : 30 oz. :: 1 oz. : 30 oz. of 21 carats fine.

1 oz. : 30 oz. :: 3 oz. : 90 oz. of 23 carats fine.

EXERCISE 127—Page 285.

(1)

To produce a mixture worth $1·40 per lb., we require 20 lbs. @ $1·00, 40 @ $1·20, 40 @ $1·60, and 20 @ $1·80. But all of these added together, will make 120 lbs.

$$\text{Therefore } \overset{\text{lbs.}}{120} : \overset{\text{lbs.}}{20} :: \overset{\text{lbs.}}{168} : \overset{\text{lbs.}}{\frac{168 \times 20}{120}} = 28 \text{ lbs., the required quantity @ \$1·00.}$$

$$120 : 40 :: 168 : \frac{168 \times 40}{120} = 56 \text{ lbs., the required quantity @ \$1·20.}$$

$$120 : 40 :: 168 : \frac{168 \times 40}{120} = 56 \text{ lbs., the required quantity @ \$1·60.}$$

$$120 : 20 :: 168 : \frac{168 \times 20}{120} = 28 \text{ lbs., the required quantity @ \$ 1·80.}$$

(2)

To produce a mixture worth 4s. 4d. per lb., we require 10 lbs. @ 5s. and 8 @ 3s. 6d. But these added together make 18 lbs.

$$\text{Therefore } \overset{\text{lbs.}}{18} : \overset{\text{lbs.}}{10} :: \overset{\text{lbs.}}{27} : \overset{\text{lbs.}}{\frac{27 \times 10}{18}} = 15 \text{ lbs., the required quantity of tea @ 5s.}$$

$$18 : 8 :: 27 : \frac{27 \times 8}{18} = 12 \text{ lbs., the required quantity of tea @ 3s. 6d.}$$

(3)

To produce a mixture worth \$2·70 per gallon, we require 20 gallons @ \$2·40, 10 @ \$2 60, 10 @ \$2·80, and 30 @ \$2·90. But all of these added together will make 70 gallons. Therefore

gals. gals gals. gals.

$70 : 20 :: 63 : \frac{63\times 20}{70} =$ 18 gallons, the required quantity of brandy @ \$2·40.

$70 : 10 :: 63 : \frac{63\times 10}{70} =$ 9 gallons, the required quantity of brandy @ \$2·60.

$70 : 10 :: 63 : \frac{63\times 10}{70} =$ 9 gallons, the required quantity of brandy @ \$2·80.

$70 : 30 :: 63 : \frac{63\times 30}{70} =$ 27 gallons, the required quantity of brandy @ \$2.90.

EXERCISE 128—Page 289.

(1)

$1974·80 \times \frac{3}{8} =$ £740·55 = £740 11s.

(2)

$765·43 \times \frac{2}{5} =$ £306·172 = £306 3s. $5\frac{7}{25}$d.

(3)

$8172·19 \times \frac{1}{4} =$ £2043·0475 = £2C43 0s. $11\frac{2}{5}$d.

EXERCISE 129—Page 289.

(1)

£743 18s. 11d. = £743·94583 and $743·94583 \div \frac{3}{10} =$ £2479·8194.

(2)

£119 9s. 8¼d. = £119·484375 and 119·484375 ÷ $\frac{3}{8}$ = $318·625.

(3)

£473 17s. 1¾d = £473·8572916, and 473·8572916 ÷ $\frac{7}{30}$ = $2030·816964.

Exercise 130—Page 290.

(1)

1006·90 ÷ 4·867 = £206·88309 = £206 17s. 7¾d.

(2)

916·87 ÷ 4·867 = £188·38504 = £188 7s. 8¼d.

(3)

2114·81 ÷ 4·867 = £434·52023 = £434 10s. 4¾d.

Exercise 131—Page 290.

(1)

£2043 11s. 3d. = £2043·5625 and 2043·5625 × 4·867 = $9946·01868.

(2)

£777 7s. 7d. = £777·37916 and 777·37916 × 4·867 = $3783·50437.

(3)

£557 19s. 5½d. = £557·972916 and 557·972916 × 4·867 = $2715·65418.

EXERCISE 132—Page 294.

(1)

$16785·25 × 5·04 = 84597 francs 66 centimes.

(2)

Commercial value of the marc banco = 35 cents.
Add 1 per cent 35
3535
Then $0·3535 × 4000 = $1414.

(3)

$35678 × 1·0225 = $36480·755.

(4)

The par value of 1 ruble = 75 cents.
Deduct 2 per cent 15
·735
Then $0·735 × 2560 = $1881·60.

(5)

Old commercial par of £1 sterling $= \$4{\cdot}44\dot{4} = \$4{\cdot}44444$
Add 8 per cent ·35555
$4·79999
Then $\$4{\cdot}79999 \times 800 = \$3839{\cdot}99\dot{9} = \$3840{\cdot}00.$

EXERCISE 133—Page 298.

(1)

$$£1 = 420\text{d.}$$
$$19\tfrac{1}{3}\text{d.} = 1 \text{ franc.}$$
$$300 \text{ francs} = 60 \text{ ducats.}$$
$$1 \text{ ducat} = 360 \text{ maravedis.}$$
$$x = £1000.$$

$$x = \frac{\overset{84}{\cancel{420}} \times 1 \times \cancel{60} \times 360 \times 1000}{19\tfrac{1}{3} \times \underset{5}{\cancel{300}}} = 1564138 \text{ maravedis by cir. ex.}$$

$$42\tfrac{1}{2}\text{d.} : £1000 :: 272 \text{ maravedis} : \frac{272 \times 1000 \times 20 \times 12}{42\tfrac{1}{2}} =$$

$$\frac{\overset{16}{\cancel{272}} \times 1000 \times 8 \times 12}{\cancel{17}} = 1536000 \text{ maravedis by direct exchange.}$$

Difference = 1564138—1536000 = 28138 maravedis.

```
34)28138
  8)827 reals 20 maravedis
   103 piastres 3 reals 20 maravedis.
```

(2)

Old commercial par of £1 sterling = \$4·444
To which add 10 per cent. of itself = ·4444
Gives price of £1 sterling = \$4·8884

\$4888·40÷\$4·8884 = £1000 = amount of bill he receives if he remits direct to London.

$$\left.\begin{array}{r} \$1 = 515 \text{ centimes.} \\ 2580 \text{ cen.} = £1 \text{ sterling.} \\ x = \$4888{\cdot}40 \end{array}\right\} x = \frac{515 \times 4888{\cdot}40}{2580} = £975{\cdot}78526.$$

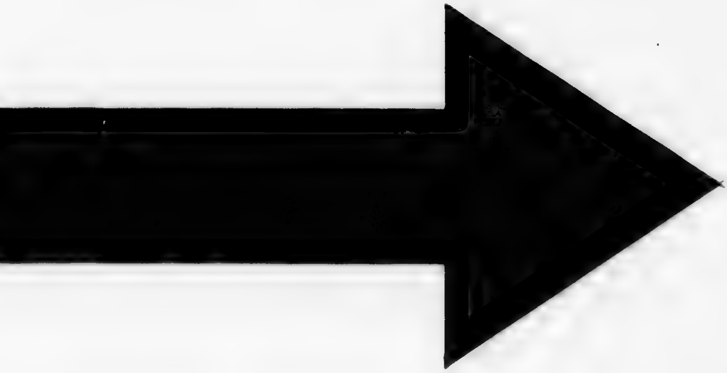
= £975 15s. 8¼d.+ = amount of bill he receives if he remits through Paris.

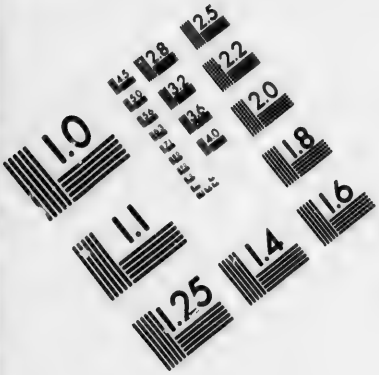
$$35 \text{ cents} = 1 \text{ marc.}$$
$$3\tfrac{3}{4} \text{ marcs} = £1 \text{ sterling.}$$
$$x = \$4888{\cdot}40.$$

$$x = \frac{4888{\cdot}40}{{\cdot}35 \times 13\tfrac{3}{4}} = \frac{391072}{385} = £1015{\cdot}77142 = £1015 \text{ 15s. 5d.}+ =$$

amount of bill he receives by remitting through Hamburg.

[redacted]





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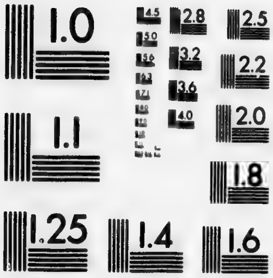
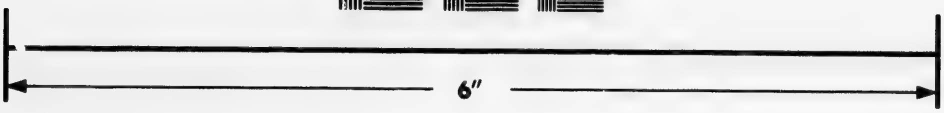
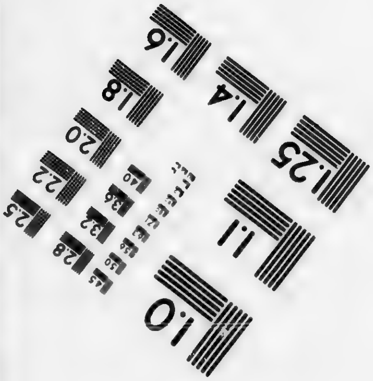
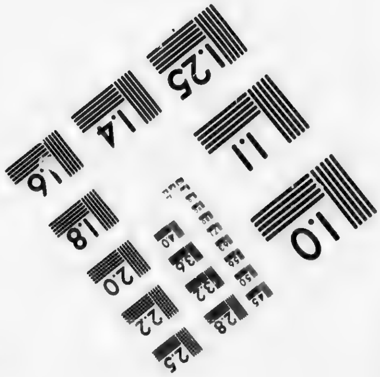
TEST TARGET (MT-3)









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1.0
2.8
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3.2
3.6
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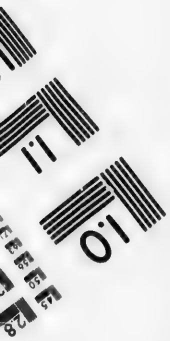
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2.8

(3)

18 cents. = 1 franc.
25 francs. = 240d.
180d. = 3 milrees,
5 milrees. = 18 marcs ban.
1200 marcs ban = x

$$x = \frac{\cancel{18} \times 25 \times \overset{60}{\cancel{180}} \times \overset{5}{\cancel{1200}} \times 5}{\cancel{240} \times \cancel{3} \times \cancel{18}}$$

= $375 = circuitous exchange or sum he pays for 1200 marks.

1200 × ·35 = $420 = direct exchange or sum paid for 1200 marks. $420 − $375 = $45 = gain by circuitous exchange.

EXERCISE 134—Page 298.

(1)

$$(3)^5 = 3 \times 3 \times 3 \times 3 \times 3 = 243.$$

(2)

$$(20)^{10} = 20 \times 20 \times 20 \times 20 \times 20 \times 20 \times 20 \times 20 \times 20 \times 20 = 10240000000000.$$

(3)

$$(1{\cdot}05)^6 = 1{\cdot}05 \times 1{\cdot}05 \times 1{\cdot}05 \times 1{\cdot}05 \times 1{\cdot}05 \times 1{\cdot}05 = 1{\cdot}340095640625.$$

(4)

$$(\tfrac{3}{4})^7 = \tfrac{3}{4} \times \tfrac{3}{4} \times \tfrac{3}{4} \times \tfrac{3}{4} \times \tfrac{3}{4} \times \tfrac{3}{4} \times \tfrac{3}{4} = \tfrac{2187}{16384}.$$

(5)

$$(\tfrac{5}{9})^5 = \tfrac{5}{9} \times \tfrac{5}{9} \times \tfrac{5}{9} \times \tfrac{5}{9} \times \tfrac{5}{9} = \tfrac{3125}{59049}.$$

(6)

$$11\tfrac{2}{5} = \tfrac{57}{5}.\quad (\tfrac{57}{5})^3 = \tfrac{57}{5} \times \tfrac{57}{5} \times \tfrac{57}{5} = \tfrac{185193}{125} = 1481\tfrac{68}{125}.$$

EXERCISE 135—Page 299.

(1)

$$4^2 \times 4^4 \times 4^5 \times 4^7 = 4^{2+4+5+7} = 4^{18}.$$

(2)

$$13^{11} \div 13^2 = 13^{11-2} = 13^9.$$

(3)

$$(3^3)^5 = 3^{3\times5} = 3^{15}.$$

(4)

$$\left\{(7^4 \times 7^3) \div (7^2 \times 7^2)\right\}^6 = \left\{(7^{4+3}) \div (7^{2+2})\right\}^6 = \left\{7^7 \div 7^4\right\}^6 = (7^{7-4})^6 = (7^3)^6 = 7^{3\times6} = 7^{18}.$$

(5)

$$\left\{(5^3 \times 5^4 \times 5^{11} \times 5^9) \div (5^3 \times 5^2 \times 5^7 \times 5^5)\right\}^3 = \left\{(5^{3+4+11+9}) \div (5^{3+2+7+5})\right\}^3 = \left\{5^{27} \div 5^{17}\right\}^3 = (5^{27-17})^3 = (5^{10})^3 = 5^{10\times3} = 5^{30}.$$

EXERCISE 137—Page 304.

(1)

$$\begin{array}{rl} & \dot{1}9\dot{5}3\dot{6}4(442 \\ & 16 \\ \hline 84) & 353 \\ & 336 \\ \hline 882) & 1764 \\ & 1764 \end{array}$$

(2)

$$\begin{array}{rl} & \cdot0\dot{6}7\dot{6}(\cdot26 \\ & 4 \\ \hline 46) & 276 \\ & 276 \end{array}$$

(3)

$$\begin{array}{rl} & \dot{9}8\dot{4}0\dot{6}4(992 \\ & 81 \\ \hline 189) & 1740 \\ & 1701 \\ \hline 1982) & 3964 \\ & 3964 \end{array}$$

(4)

```
 .  .  .  .  .  .
 5·0000000000(2·23606
 4
 -
42)1·00
    ·84
    ---
443)·1600
     ·1329
     -----
4466)27100
      26796
      -----
447206)3040000
        2683236
        -------
         356764
```

(5)

```
  .  .  .  .  .  .
 ·500000000000(·707106
  49
  --
1407)10000
      9849
      -----
14141)15100
      14141
      -----
1414206)9590000
        8485236
        -------
        1104764
```

(6)

```
 .   .  .  .
 60.487129(7·777
 49
 --
147)1148
    1029
    -----
1547)11971
     10829
     ------
15547)114229
      108829
      ------
        5400
```

(7)

```
 .  .  .  .  .  .  .  .  .
 79792266297612001(282475249
 4
 -
48)397
   384
   ----
562)1392
    1124
    -----
5644)26826
     22576
     ------
56487)425062
      395409
      -------
564945)2965397
       2824725
       --------
5649502)14067261
        11299004
        ---------
56495044)276825720
         225980176
         ----------
564950489)5084554401
          5084554401
```

(8)

```
   .  .  .  .  .
 ·0000012321(·00111
       1
       -
     21)23
        21
        --
    221)221
        221
```

EXERCISE 138—Page 304.

(1)

$$\dot{1} = \tfrac{1}{9} \text{ and } \sqrt{\tfrac{1}{9}} = \tfrac{1}{3}.$$

(3)

$$5\tfrac{1}{7} = 5{\cdot}142857142857 \text{ and } \sqrt{5{\cdot}142857142857} = 2{\cdot}267786.$$

(4)

$$\tfrac{217}{538} = {\cdot}4033457249 \text{ and } \sqrt{{\cdot}4033457249} = {\cdot}63509.$$

(5)

$$13\tfrac{1}{5} = 13{\cdot}2 \text{ and } \sqrt{13{\cdot}2} = 3{\cdot}63318$$

EXERCISE 139—Page 305.

(1)

```
      .  .  .  .
      11333311(2626
       4
       -
    46)433
       411
       ---
   552)2233
        1434
        ----
  5546)46611
        46611
```

(2)

```
       .  .  .  .
       33233344(4344
       24
       --
   123)523
        413
        ---
  1304)11033
        10024
        -----
 13124)100544
        100544
```

(3)

4234·101230(42·412
31
133)1134
1004
1414)130·10
122·21
14231)2·3412
1·4231
142322)·413130
·340144
22431

(4)

888888·8880(888·88
71
178)1788
1601
1878)18788
16801
18878)1887·88
1688·01
188878)188·8780
168·8801
18·8878

(5)

248664e t69(54373
21
t4)386
354
t83)3264
2809
t867)657e t
62et 1
t8723)281969
281969

Exercise 140—Page 307.

(1)

$100^2 = 10000$
$60^2 = 3600$

Difference = 6400 and $\sqrt{6400} = 80$.

(2)

$50^2 = 2500$
$80^2 = 6400$

Sum $= 8900$ and $\sqrt{8900} = 94{\cdot}34$ nearly

(3)

$24^2 = 576 \div 2 = 288$ and $\sqrt{288} = 16{\cdot}97$.

(4)

$36^2 = 1296$
$20^2 = 400$

Difference $= 896$ and $\sqrt{896} = 29{\cdot}933$.

(5)

$40^2 = 1600$
$14^2 = 196$

Difference $= 1404$ and $\sqrt{1404} = 37{\cdot}469$.
$40^2 = 1600$
$26^2 = 676$

Difference $= 924$ and $\sqrt{924} = 30{\cdot}397$.
$37{\cdot}469 + 30{\cdot}397 = 67{\cdot}866$ and $67{\cdot}866 \div 3 = 22{\cdot}622$.

(6)

1760 sq. yds. $= 15840$ sq. ft. and $\sqrt{15840} = 125{\cdot}857$.

(7)

$\sqrt{141376} = 376$.

(8)

$3^2 = 9$
$3^2 = 9$

Sum $= 18$ and $\sqrt{18} = 4{\cdot}24264$.

(9)

$$\begin{aligned} 16^2 &= 256 \\ 12^2 &= 144 \end{aligned}$$

Sum $= 400$ and $\sqrt{400} = 20$

(10)

$3^2 + 3^2 + 3^2 = 27$ and $\sqrt{27} = 5 \cdot 196$.

(11)

$(\frac{1}{10})^2 = \frac{1}{100}$ and $(1)^2 = 1$.

Then $\frac{1}{100} : 1 :: 450 : \dfrac{450}{\frac{1}{100}} = 45000$.

(12)

1 sq. acre = 160 sq. perches. $160 \div 3 \cdot 1416 = 50 \cdot 929462$ and $\sqrt{50 \cdot 929462} = 7 \cdot 136$.

EXERCISE 141—Page 311.

(1)

$$\begin{array}{rrl}
 & & \dot{6}27\dot{1}27\dot{7}28\dot{3}17(3973 \\
 & & 27 \\
3^2 \times 300 = & 2700 & 35712 \\
3 \times 9 \times 30 = & 810 & \\
9^2 = & 81 & \\
\hline
 & 3591 & 32319 \\
\hline
39^2 \times 300 = & 456300 & 3393728 \\
39 \times 7 \times 30 = & 8190 & \\
7^2 & 49 & \\
\hline
 & 464539 & 3251773 \\
\hline
397^2 \times 300 = & 47282700 & 141955317 \\
397 \times 3 \times 30 = & 35730 & \\
3^2 = & 9 & \\
\hline
 & 47318439 & 141955317
\end{array}$$

(2)

		1953125(125
		1
$1^2\times300=$	300	953
$1\times2\times30=$	60	
$2^2=$	4	
	364	728
$12^2\times300=$	43200	225125
$12\times5\times30=$	1800	
$5^2=$	25	
	45025	225125

(3)

		1076890625(1025
		1
$1^2\times300=$	300	76
$10^2\times300=$	30000	76890
$10\times2\times30=$	600	
$2^2=$	4	
	30604	61208
$102^2\times300=$	3121200	15682625
$102\times5\times30=$	15300	
$5^2=$	25	
	3136525	15682625

(4)

	·697864103(·887
	512
$8^2\times300=19200$	185864
$8\times8\times30=1920$	
$8^2=64$	
21184	169472
$88^2\times300=2323200$	16392103
$88\times7\times30=18480$	
$7^2=49$	
2341729	16392103

(5)

	102503·232(46·8
	64
$4^2\times300=4800$	38503
$4\times6\times30=720$	
$6^2=36$	
5556	33336
$46^2\times300=634800$	5167·232
$46\times8\times30=11040$	
$8^2=64$	
645904	5167·232

(6)

		179597·069288(56·42
		125
$5^2\times300=$	7500	54597
$5\times6\times30=$	900	
$6^2=$	36	
	8436	50616
$56^2\times300=$	940800	3981·069
$56\times4\times30=$	6720	
$4^2=$	16	
	947536	3790·144
$564^2\times300=$	95428800	190·925288
$564\times2\times30=$	33840	
$2^2=$	4	
	95462644	190·925288

(7)

		483·736625(7·85
		343
$7^2\times300=$	14700	140·736
$7\times8\times30=$	1680	
$8^2=$	64	
	16444	131·552
$78^2\times300=$	1825200	9·184625
$78\times5\times30=$	11700	
$5^2=$	25	
	1836925	9·184625

(8)

$$
\begin{array}{rl}
 & \cdot\dot{6}36\dot{0}56(\cdot 86 \\
 & 512 \\
 & \overline{} \\
8^2\times 300 = 19200 & 124056 \\
8\times 6\times 30 = 1440 & \\
6^2 = 36 & \\
\overline{} & \\
20676 & 124056
\end{array}
$$

EXERCISE 142.—Page 312.

(1)

$\frac{2}{19} = \cdot 105263157894 +$ and $\sqrt[3]{\cdot 105263157894} = \cdot 4721.$

(2)

$\frac{3}{17} = \cdot 176470588235 +$ and $\sqrt[3]{\cdot 176470588235} = \cdot 5609.$

(3)

$\frac{1}{3}$ of $2\frac{1}{2} = \frac{5}{6} = \cdot 833333333 +$ and $\sqrt[3]{\cdot 833333333} = \cdot 941.$

(4)

$28\frac{3}{4} = 28\cdot 75$ and $\sqrt[3]{28\cdot 75} = 3\cdot 063$

(5)

$32\frac{8}{11} = 32\cdot\dot{7}\dot{2}$ and $\sqrt[3]{32\cdot\dot{7}\dot{2}} = 3\cdot 198.$

EXERCISE 143.—Page 312.

(1)

One million = 33233344 senary.

		33233344(244.
		12
$2^2=4\times300=$	2000	21233
$2\times30=100\times4=$	400	
$4^2=$	24	
	2424	14544
$24^2=1104\times300=$	332000	2245344
$24\times30=1200\times4=$	5200	
$4^2=$	24	
	341224	2245344

(2)

		6131271·000000(165·32.
		1
$1^2\times300=$	300	5131
$1\times30\times6=$	220	
$6^2=$	44	
	564	4270
$16^2=304\times300=$	111400	641271
$16\times30=520\times5=$	3220	
$5^2=$	31	
	114651	600113
$165^2=32571\times300=$	12015300	41154·000
$165\times30=5370\times3=$	20350	
$3^2=$	11	
	12035661	36131·423
$1653^2=3272071\times300=$	1205625300	3022·355000
$1653\times30=54010\times2=$	130020	
$2^2=$	4	
	1205755324	2413·732650
		406·422130

(3)

		$10221012\cdot102000000$
		$\underline{1}\;\big\vert\,\overline{112\cdot012}=$ root.
$1\times1000=$	1000	2221
$1\times1\times100=$	100	
$1^2=$	$\underline{1}$	
	$\underline{1101}$	$\underline{1101}$
$11^2=121\times1000=$	121000	1120012
$11\times100=1100\times2=$	2200	
$2^2=$	$\underline{11}$	
	$\underline{200211}$	$\underline{1101122}$
$112^2=21021\times1000=$	$\underline{21021000}$	$11120\cdot102$
$1120^2=2102100\times1000=$	2102100000	$11120\cdot102000$
$1120\times100=112000\times1=$	112000	
$1^2=$	$\underline{1}$	
	$\underline{2102212001}$	$\underline{2102\cdot212001}$
$11201^2=211010101\times1000=$	211010101000	$2010\cdot112222000$
$11201\times100=1120100\times2=$	10010200	
$2^2=$	$\underline{11}$	
	211020111211	$\underline{1122\cdot111000122}$
		$111\cdot001221101$

(4)

		teteet·000000(e7·t2.
		92e
$e^2 = t1 \times 300 =$	26300	18eeet
$e \times 30 = 290 \times 7 =$	1730	
$7^2 =$	41	
	27t71	167217
$e7^2 = e221 \times 300 =$	2966300	249t3·000
$e7 \times 30 = 2t90 \times t =$	24e60	
$t^2 =$	84	
	298e324	24154·7e4
$e7t^2 = e39544 \times 300 =$	29e441000	84t·408000
$e7t \times 30 = 2te60 \times 2 =$	59e00	
$2^2 =$	4	
	29e49te04	57t·979t08
		28e·64t1e4

(5)

		421030·441200000(44·004
		224
$4^2 = 31 \times 300 =$	14300	142030
$4 \times 30 = 220 \times 4 =$	1430	
$4^2 =$	31	
	21311	141244
$44^2 = 4301 \times 300 =$	2340300	231·441
$440^2 = 430100 \times 300 =$	234030000	231·441200
$4400^2 = 43010000 \times 300 =$	23403000000	231·441200000
$4400 \times 30 = 242000 \times 4 =$	2123000	
$4^2 =$	31	
	23410123031	210·141102224
		21·300042221

EXERCISE 144.—Page 314.

(2)

$3^3 : 6^3 :: 4\text{lb.} : \text{Ans.} = 32\text{ lbs.}$

(3)

$1^3 : \left(\frac{7}{2}\right)^3 :: \$120 : \text{Ans.} = \$5145.$

(4)

$(70)^3 ; \left(\frac{623}{5}\right)^3 :: 180\text{ lbs.} : \text{Ans.}$
$; \ 343000 : \frac{241804367}{125} :: 180 : \text{Ans.} =$
$180 \times \frac{241804367}{125} \times \frac{1}{343000} = 1015 \cdot 1\text{ lbs.}$

(5)

$973^3 = 921167317.$
$45^3 = 91125$
$62^3 = 238328$
$30^3 = 27000$
$80^3 = 512000$
$20^3 = 8000$
$921167317 - (91125 + 238328 + 27000 + 512000 + 8000) =$
920290864 and $\sqrt[3]{920290864} = 972 \cdot 69.$

(6)

8 feet 3 inches = 99 inches, 3 feet = 36 inches, and 2 feet 7 inches = 31 inches.

$99 \times 36 \times 31 = 110484$ and $\sqrt[3]{110484} = 47 \cdot 9843.$

(7)

After the first has wound off her portion, there will remain ¾ of the thread.

Then the whole ball : part remaining :: cube of diameter of whole ball : cube of diameter of part remaining.

That is, $1 : \frac{3}{4} :: 3^3 : x^3$, and hence $x = 3 \times \sqrt[3]{\frac{3}{4}} = 3 \times \sqrt[3]{\cdot 75} = \cdot 90856 \times 3 = 2\cdot 72568$ = diameter of the ball after the first has wound off her portion.

Similarly after the second has wound off her portion, there will remain $\frac{1}{2}$ of the ball, and after the third has taken her portion, $\frac{1}{4}$ of the ball.

Hence $1 : \frac{1}{2} :: 3^3 : x^3$, whence $x = 3 \times \sqrt[3]{\frac{1}{2}} = 3 \times \sqrt[3]{.5} = 3 \times \cdot 79370 = 2\cdot 38110$ = diameter after the second has taken her portion.

$1 : \frac{1}{4} :: 3^3 : x^3$, whence $x = 3 \times \sqrt[3]{\frac{1}{4}} = 3 \times \sqrt[3]{.25} = 3 \times \cdot 62996 = 1\cdot 88988$ = diameter after the third has taken her portion.

Hence 1st takes off	$3 - 2.72568 =$	$\cdot 27432$	inches.
2nd " "	$2.72568 - 2\cdot 38110 =$	$\cdot 34458$	"
3rd " "	$2.38110 - 1\cdot 88988 =$	$\cdot 49122$	"
4th " "	remaining	1.88988	"

Exercise 145—Page 315

(1)

$\sqrt{19987173376} = 141376$, and $\sqrt{141376} = 376$.

(2)

$\sqrt[3]{308915776} = 676$, and $\sqrt{676} = 26$.

(3)

$\sqrt[3]{40353607} = 343$, and $\sqrt[3]{343} = 7$.

(4)

$\sqrt[3]{387420489} = 729$. $\sqrt[3]{729} = 9$, and $\sqrt{9} = 3$.

(5)

$$\sqrt[3]{134217728} = 512,\ \sqrt[3]{512} = 8, \text{ and } \sqrt[3]{8} = 2.$$

EXERCISE 148—Page 321.

(1)

The mantissa of the logarithm of 8193 (the first four digits) = ·913443, and the next following mantissa is ·913496.

Then from ·913496
Subtract.. ·913443

Difference, 53; and 53×217 (remaining digits of given number) = 11501, from which we cut off three digits, since we multiplied by a number of three digits, and since the highest digit cut off is not less than 5, we add unity to the part retained, which gives us 12.

Then mantissa of logarithm of first four digits	.913443
Add,	12
Mantissa of logarithm of given number,	.913455

To which attach the characteristic 6 and required logarithm = 6·913455.

The mantissa of the logarithm of 7392 (the first four digits) = ·868762, and the next following mantissa is ·868821.

Then from ·868821
Subtract.. ·868762

Difference, 59; and 59 × 45 (remaining digits of given number) = 2655, from which we cut off two digits, since we multiplied by a number of two digits, and since the highest digit cut off is not less than 5, we add unity to the part retained which gives us 27.

Then mantissa of logarithm of first four digits,	.868762
Add,	27
Mantissa of logarithm of given number,	·868789

(Continued on next page.)

(1 continued.)

To which attach the characteristic 1 and required logarithm = 1·868789.

The mantissa of the logarithm of 8437 (the first four digits) = ·926188, and the next following mantissa is ·926240.

Then from ·926240
Subtract.. ·926188

Difference, 52; and 52 × 42 (remaining digits of given number) = 2184, from which we cut off two digits, since we multiplied by a number of two digits, and since the highest digit cut off is not less than 5, we add unity to the part retained, which becomes 22.

Then mantissa of logarithm of first four digits ·926188
Add, 22

Mantissa of logarithm of given number, ·926210

To which attach the characteristic $\bar{1}$ and required logarithm = $\bar{1}.926210$.

(2)

The mantissa of the logarithm of 2345 = ·370143, and the next following mantissa is ·370328.

Then from ·370328
Subtract.. ·370143

Difference, 185; and 185 × 64 = 11840, from which we cut off two digits, since we multiplied by a number of two digits, which gives us 118.

Then mantissa of logarithm of 2345 = ·370143
Add, 118

Mantissa of logarithm of given number = ·370261

To which attach the characteristic $\bar{4}$ and required logarithm = $\bar{4}$·370261.

(Continued on next page.)

(2 continued.)

The mantissa of the logarithm of 1007 = ·003029, and the next following mantissa is ·003461.

Then from ·003461

Subtract.. ·003029

Difference, 432; and 432 × 013 = 5616, from which we cut off three digits, since we multiplied by a number of three digits, and since the highest digit cut off is not less then 5, we add unity to the part retained, which gives us 6.

Then mantissa of logarithm of 1007 = ·003029

Add, 6

Mantissa of logarithm of given number ·003035

To which attach the characteristic $\bar{3}$, and required logarithm = $\bar{3}$·003035.

(3)

Mantissa of logarithm of 5237	·719083
Difference from column D = 83; and 83 × 6 = 498 from which we cut off 1 digit and add..........	50
And also attach the characteristic 1, and required logarithm = ...	1.719133
Mantissa of logarithm of 1294........................	·111934
Difference from column D = 335; and 335 × 76 = 25460 from which we cut off two digits and add,	255
And also attach the characteristic 2 and required logarithm =................................	2·112189

(4)

Mantissa of logarithm of ·0004713 = ·673297
P. P. corresponding to ·00000009 = 83
P. P " to ·000000008 = 74
Sum, = ·6733874

Therefore required mantissa = ·673387 and required logarithm = $\bar{4}$·673387.

Mantissa of logarithm of 9136000 = ·960756
P. P. corresponding to 700 = 33
P. P. " to 10 = 5
P. P. " to 2 = 9
Sum, = ·96078959

Therefore required mantissa = ·960790 and required logarithm = 6·960790.

(5)

Mantissa of logarithm of 4·23400 = ·626751
P. P. corresponding to 20 = 20
P. P. " to 9 = 92
Sum, = ·6267802

Therefore required logarithm is 0·626780.

Mantissa of logarithm of 763·1 = ·882581
P. P. corresponding to ·02 = 11
P. P. " to ·009 = 51
P. P. " to ·0008 = 46
P. P. " to ·00009 = 40
Sum, = ·882597600

Therefore required logarithm is 2·882598.

EXERCISE 149.—Page 323.

(1)

Given logarithm, ·137139
Next lower in table, ·137037 = log. of 1371.

Difference 102, Tabular difference = 316.

Then 1020000 ÷ 316 gives 3227 for digits in 5th, 6th, 7th, and 8th places.

Hence the digits of the natural number are 13713227; and since the characteristic is 4, i.e., one less than the number of digits to the left of the decimal point the required number is 13713·227.

Given logarithm, ·718134
Next lower in table, ·718086 = log. of 5225.

Difference, 48, Tabular difference = 83.

Then 48000 ÷ 83 gives 578 for digits in 5th, 6th, and 7th places.

Hence the digits of the natural number are 5225578, and since the characteristic is 0, i.e., one less than the number of digits to the left of the decimal point, the required number is 5·225578.

Given logarithm, ·635421
Next lower in table, ·635383 = log. of 4319.

Difference, 38, Tabular difference = 101.

Then 38000 ÷ gives 376 for digits in 5th, 6th, and 7th places.

Hence the digits of the natural number are 4319376, and since the characteristic is $\bar{4}$, i.e., one more than the number of ciphers between the decimal point and the first figure to the right, the required number is ·0004319376.

(2)

Given log. ·921686 = log. of 8350.

And since the characteristic is 2, i.e, one less than the number of digits to the left of the decimal point, the required number is 835.

Given logarithm, ·922165
Next lower in table, ·922154 = log. of 8359.

Difference = 11, Tabular difference = 52.

Then 11000 ÷ 52 gives 211 for digits in 5th, 6th, and 7th places.

Hence the digits of the natural number are 8359211; and since the characteristic is $\bar{1}$, i.e., one more than the number of ciphers between the decimal point and first figure to the right, the required number is ·8359211.

(3)

Given logarithm, ·407968
Next lower in table, ·407901 = log. of 2558.

Difference, = 67
Highest P. P. not greater than 67 = 51 corresponds to 3 for 5th place.
160
Highest P. P. not greater than 160 = 153 corresponds to 9 for 6th place.
70
Highest P. P. not greater than 70 = 68 corresponds to 4 for 7th place.
2

Therefore digits of required number are 2558394; and since the characteristic is 5, there must be six digits to the left of the decimal point.

Hence required number is 255839·4.

(Continued on next page.)

(3 continued.)

Given logarithm, ·408386
Next lower in table, ·408240 = log. of 2560.

Difference, = 146
Highest P.P. not greater than 146 = 136 corresponds to 8 in 5th place.
100
Highest P.P. not greater than 100 = 85 corresponds to 5 in 6th place.
150
Highest P.P. not greater than 150 = 136 corresponds to 8 in 7th place.
140
Highest P.P. not greater than 140 = 136 corresponds to 8 in 8th place.

Therefore digits of required number are 25608588; and since the characteristic is 7, there must be eight digits to the left of the decimal point.

Hence required number is 25608588.

Given logarithm, ·416369
Next lower in table, ·416308 = log. of 2608.

Difference, = 61
Highest P.P. not greater than 61 = 49 corresponds to 3 in 5th place.
12

Therefore digits of required number are 26083; and since the characteristic is $\bar{3}$, there must be two ciphers between the decimal point and first figure.

Hence required number is ·0026083.

(4)

Given logarithm, ·877777
Next lower in table, ·877774 = log. of 7547.

Difference, = 3
There is no P.P. not greater than 3 0 corresponds to 0 in 5th place.
30
Highest P.P. not greater than 30 = 29 corresponds to 5 in 6th place.
10
Highest P.P. not greater than 10 = 6 corresponds to 1 in 7th place.
40
Highest P.P. not greater than 40 = 35 corresponds to 6 in 8th place.
50
Highest P.P. not greater than 50 = 46 corresponds to 8 in 9th place.
4

Therefore digits of required number are 754705168; and since the characteristic is 4, there must be five digits to the left of the decimal point.

Hence required number is 75470.5168.

Given logarithm, ·555555
Next lower in table, ·555457 = log. of 3593.

Difference, = 98
Highest P.P. not greater than 98 = 98 corresponds to 8 in 5th place.

Therefore digits of required number are 35938; and since the characteristic is 0, there must be one digit to the left of the decimal point.

Hence required number is 3·5938.

0

EXERCISE 150.—Page 324.

(1)

10 — 5.631642 = 4·368358.
10 — 0·714000 = 9·286000.

(2)

10 — $\overline{3}$·123456 = 12·876544.
10 — $\overline{7}$·213149 = 16·786851.

(3)

10 — 6·124357 = 3·875643 and 10 — $\overline{2}$·000837 = 11·999163.

EXERCISE 151.—Page 325.

(1)

Logarithm of 61 = 1·785330
" 22 = 1·342423
" 65 = 1·812913

Sum = 4·940666 = logarithm of 87230.

(2)

Logarithm of 52 = 1·716003
" 734 = 2·865696
" 6 = 0·778151

Sum = 5·359850
5·359835 = logarithm of 229000

15 = 8

Ans. 229008

(3)

Logarithm of 35.86 = 1·554610

" 2·1046 = 0·323169

" ·8372 = $\bar{1}$·922829

" ·00294 = $\bar{3}$·468347

Sum = $\bar{1}$·268955

$\bar{1}$·268812 = logarithm of ·185706

143 = 61

Ans. ·185761

(4)

Log. of ·00008764 = $\bar{5}$·942702

" ·86359 = $\bar{1}$·936308

Sum = $\bar{5}$·879010

$\bar{5}$·878981 = logarithm of ·000075680

29 = 5

Ans. ·000075685

EXERCISE 152.—Page 326.

(1)

Logarithm of ·6734 = $\bar{1}$·828273

" ·0009278 = $\bar{4}$·967454

Difference = 2·860819

2·860817 = logarithm of 725·8000

2 = 33

Ans. 725·8033

(2)

Logarithm of 437·89 = 2·641365

" 62·735 = 1·797510

Difference = ·843855 = logarithm of 6·98

(3)

Logarithm of 98·217 = 1·969495

" ·0007132 = $\bar{4}$·853211

Difference = 5·116284

5·116276 = logarithm of 130700·0

8 = 2·4

Ans. 130702·4

(4)

Logarithm of 23 = 1·361728

" 189 = 2·276462

" 2·748 = 0·439017

Sum = 4·077207

Logarithm of 9835267 = 6·992786

4·077207

Difference = 2·915579

2·915558 = logarithm of 823·300

21 = 39

Ans. 823·339

EXERCISE 153.—Page 326.

(1)

Logarithm of 5 = 0·698970.

Then 0·698970 × 5 = 3·494850 = logarithm of 3125.

(2)

Logarithm of 1,073 = ·030600.
Then ·030600 × 6 = ·183600 = logarithm of 1·5261.

(3)

Logarithm of ·0279 = $\bar{2}$·445604.
Then $\bar{2}$·445604 × 4 = $\bar{7}$·782416 = logarithm of ·00000060592.

(4)

Logarithm of 1·111 = ·045714.
Then ·045714 × 11 = ·502854 = logarithm of 3·1831.

EXERCISE 154.—Page 327.

(1)

Logarithm of 913426000 = 8·960673.
8·960673 ÷ 7 = 1·2800961 = logarithm of 19·0588.

(2)

Logarithm of 1·61342 = ·207747.
·207747 ÷ 11 = ·01888609 = logarithm of 1·0444.

(3)

Logarithm of ·000007139 = $\bar{6}$·853637 = $\overline{10}$ + 4·853637.
($\overline{10}$ + 4·853637) ÷ 5 = $\bar{2}$·970727 = logarithm of ·0934817.

(4)

Logarithm of ·002147 = $\bar{3}$·331832 = $\bar{7}$ + 4·331832.
($\bar{7}$ + 4.331832) ÷ 7 = $\bar{1}$·6188331 = logarithm of ·41575.

EXERCISE 155.—Page 328.

(1)

14000 = 7 × 2 × 1000 ∴ log. 14000 = (log. 7) + (log. 2) + (log. 1000).

Log. 7 = 0·845098
Log. 2 = 0·301030
Log. 1000 = 3

Sum, = 4·146128 = log. 14000

4·9 = 7^2 ÷ 10 ∴ log. 4·9 = (log. 7) × 2 — (log. 10).

Log. 7 = 0·845098 × 2 = 1·690196
Log. 10 = 1

Difference = ·690196 = log. 4·9

·00196 = 49 × 4 ÷ 100000 = 7^2 × 2^2 ÷ 100000
∴ log ·00196 = (log. 7) × 2 + (log. 2) × 2 — (log. 100000).

Log. 7 = 0·845098 × 2 = 1.690196
Log. 2 = 0.301030 × 2 = 0.602060

Sum = 2·292256

Log. of 100000 = 5 and 2·292256 — 5 = $\bar{3}$·292256 = log of .00196.

Since 5 = 10 ÷ 2, the logarithm of 5 = log. 10 — log. 2 = 1 — 0·301030 = 0· 698970.

1750 = 5^2 × 7 × 10 ∴ log. 1750 = (log. 5) × 2 + (log. 7) + (log. 10)

Log. 5 = 0·698970 × 2 = 1·397940
Log. 7 = ·845098
Log. 10 = 1

Sum, = 3·243038 = log. of 1750

$1428\cdot\dot{5}7142\dot{8}$ = $\frac{1}{7}$ × 10000 ∴ log. $1428\cdot\dot{5}7142\dot{8}$ = (log. $\frac{1}{7}$) + log. 10000.

(1 continued.)

Log. $\frac{1}{7}$ = (log. 1) — (log. 7) = 0 — 0·845098 = $\bar{1}$·154902
Log. 10000 = 4

$\therefore$ log. of 1428·571428 = sum = 3·154902

·00000112 = $2^4 \times 7 \div 100000000$ $\therefore$ log. ·00000112 =
(log. 2) × 4 + (log. 7) — (log. 100000000).
Log. 2 = 0·301030 × 4 = 1·204120
Log. 7 = 0·845098

Sum = 2·049218 = and log. 100000000 = 8
2·049218 — 8 = $\bar{6}$·049218 = log. ·00000112

3·0625 = $\frac{49}{16}$ $\therefore$ log. 3·0625 = (log. 49) — (log. 16) =
(log. 7) × 2 — (log. 2) × 4.

Log. 7 = 0·845098 × 2 = 1·690196
Log. 2 = 0·301030 × 4 = 1·204120

Difference = 0·486076 = log. of 3·0625.

3 $\frac{0625}{1000}$

(2)

$49\frac{1}{2} = \frac{99}{2} = 3^2 \times 11 \times \frac{1}{2}$ $\therefore$ log. $49\frac{1}{2}$ = (log. 3) × 2 + (log 11)
+ (log. $\frac{1}{2}$).
Log. 3 = 0·477121 × 2 = 0·954242
Log. 11 = 1·041393
Log. $\frac{1}{2}$ = $\bar{1}$·698970

Sum = 1·694605 = log. of $49\frac{1}{2}$.

363 = $11^2 \times 3$ $\therefore$ log. 363 = (log. 11) × 2 + (log. 3).
Log. 11 = 1·041393 × 2 = 2·082786
Log. 3 = 0·477121

Sum = 2·559907 = log. of 363.

Log. ·5 or $\frac{1}{2}$ = 1·698970, and by altering the characteristic we get 0·698970 for log. of 5.

(Continued on next page.)

(2 continued.)

$4{\cdot}\dot{0}\dot{9} = 4\frac{1}{11} = \frac{45}{11} = 3^2 \times 5 \div 11$ $\therefore$ log. $4{\cdot}\dot{0}\dot{9} =$ (log. 3) × 2 + (log. 5) — (log. 11).

Log. 3 = ·477121 × 2 = 0·954242
Log. 5 = ·698970

1·653212

Log. 11 = 1·041393 and 1·653212 — 1·041393 = 0·611819 = log. of $4{\cdot}\dot{0}\dot{9}$.

$2{\cdot}\dot{4} = 2\frac{4}{9} = \frac{22}{9} = 11 \times 2 \div 9$ $\therefore$ log. $2{\cdot}\dot{4} =$ (log. 11) + (log. 2) — (log. 3) × 2.

Log. 2 = (log. 10) — (log. 5) = 1 — 0·698970 = 0·301030.

Log. 11 = 1·041393
Log. 2 = 0·301030

1·342423

Log. 3 = 0·477121 × 2 = 0·954242 and 1·342423 — 0·954242 = 0·388181 = log. of $2{\cdot}\dot{4}$.

$392{\cdot}\dot{7}\dot{2} = 392\frac{8}{11} = \frac{4320}{11} = 2^4 \times 3^3 \times 10 \div 11$ $\therefore$ log. $392{\cdot}\dot{7}\dot{2}$ = (log. 2) × 4 + (log. 3) × 3 + (log. 10) — (log. 11).

Log. 2 = 0·301030 × 4 = 1·204120
Log. 3 = 0·477121 × 3 = 1·431363
Log. 10 = 1

Sum = 3·635483

Log. 11 = 1·041393 and 3·635483 — 1·041393 = 2·594090 = log. of $392.\dot{7}\dot{2}$.

$293333\frac{1}{3} = \frac{880000}{3} = 2^3 \times 11 \times 10000 \div 3$ $\therefore$ log. $293333\frac{1}{3}$ = (log. 2) × 3 + (log. 11) + (log. 10000) — (log. 3).

Log. 2 = 0·301030 × 3 = 0·903090
Log. 11 = 1·041393
Log. 10000 = 4

Sum = 5·944483

(Continued on next page.)

(2 continued.)

Log. 3 $= 0{\cdot}477121$ and $5{\cdot}944483 - 0{\cdot}477121 = 5{\cdot}467362 =$ log. of $293333\frac{1}{3}$.

$19{\cdot}965 = 11^3 \times 5 \times 3 \div 1000$ ∴ log. $19{\cdot}965 =$ (log. 11) $\times$ 3 + (log. 5) + (log. 3) − (log. 1000).

$$\begin{aligned} \text{Log. } 11 &= 1{\cdot}041393 \times 3 = 3{\cdot}124179 \\ \text{Log. } 5 &= 0{\cdot}698970 \\ \text{Log. } 3 &= 0.477121 \\ \text{Sum} &= 4{\cdot}300270 \end{aligned}$$

Log. 1000 $= 3$ and $4{\cdot}300270 - 3 = 1{\cdot}300270 =$ log. of $19{\cdot}965$.

EXERCISE 156—Page 336.

(1)

Here we have given the first term 4, the number of terms 17 and the sum of the series 884, to find l, the last term.

$$\text{Then } l = \frac{2s}{n} - a = \frac{884 \times 2}{17} - 4 = 104 - 4 = 100.$$

(2)

Here we have given the first term 21, the last term 497 and the number of terms 41, to find the common difference.

$$\text{Then } d = \frac{l-a}{n-1} = \frac{497-21}{41-1} = \tfrac{476}{40} = \tfrac{119}{10} = 11\tfrac{9}{10}.$$

(3)

Here we have given a, l, and d, to find n, and since $a = 12$, $l = 96$, and $d = 6$, we have

$$n = \frac{l-a}{d} + 1 = \frac{96-12}{6} + 1 = \tfrac{84}{6} + 1 = 14 + 1 = 15.$$

(4)

Here we have given l, d, and s, to find n, and since $l = 14$, $d = 1$, and $s = 105$, we have

$$n = \frac{2l+d}{2d} + \sqrt{\left(\frac{2l+d}{2d}\right)^2 - \frac{2s}{d}} = \frac{2\times14+1}{2\times1} + \sqrt{\left(\frac{2\times14+1}{2\times1}\right)^2 - \frac{2\times105}{1}} = 14\tfrac{1}{2} + \sqrt{(\tfrac{29}{2})^2 - 210} = 14\tfrac{1}{2} + \sqrt{\tfrac{841}{4} - 210} = 14\tfrac{1}{2} + \sqrt{\tfrac{1}{4}} = 14\tfrac{1}{2} + \tfrac{1}{2} = 15.$$

(5)

Here we have given a, d, and s, to find l, and since $a = \frac{2}{3}$, $d = \frac{2}{3}$, and $s = 1180$, we have

$$l = -\tfrac{1}{2}d + \sqrt{2ds + (a - \tfrac{1}{2}d)^2} = -\tfrac{1}{2} \text{ of } \tfrac{2}{3} + \sqrt{2 \times \tfrac{2}{3} \times 1180 + (\tfrac{2}{3} - \tfrac{1}{2} \times \tfrac{2}{3})^2} = -\tfrac{1}{3} + \sqrt{\tfrac{4720}{3} + (\tfrac{1}{3})^2} = -\tfrac{1}{3} + \sqrt{\tfrac{4720}{3} + \tfrac{1}{9}} = -\tfrac{1}{3} + \sqrt{\tfrac{14161}{9}} = -\tfrac{1}{3} + \tfrac{119}{3} = \tfrac{118}{3} = 39\tfrac{1}{3}.$$

(6)

Here we have given a, l, and s, to find d, and since $a = 8$, $l = 170$, and $s = 4895$, we have

$$d = \frac{(l+a)\ (l-a)}{2s - l - a} = \frac{(170+8)\ (170-8)}{2\times4895 - 170 - 8} = \frac{178 \times 162}{9790 - 178} = \tfrac{28836}{9612} = 3.$$

(7)

Here we have given a, l, and d, to find n, and since $a = 5$, $l = 27\frac{1}{2}$, and $d = 2\frac{1}{4}$, we have

$$n = \frac{l-a}{d} + 1 = \frac{27\frac{1}{2} - 5}{2\frac{1}{4}} + 1 = \frac{22\frac{1}{2}}{2\frac{1}{4}} + 1 = \frac{\frac{45}{2}}{\frac{9}{4}} + 1 = 10 + 1 = 11$$

(8)

Here we have given a, l, and n, to find s, and since $a = 2$, $l = 478$, and $n = 86$, we have

$$s = (a + l)\tfrac{n}{2} = (2 + 478)\tfrac{86}{2} = 480 \times 43 = 20640.$$

(9)

Here we have given a, l, and d, to find s, and since $a = 2$, $l = 998$, and $d = 6$, we have

$$s = \frac{(l+a)\ (l-a)}{2d} + \frac{l+a}{2} = \frac{(998+2)\ (998-2)}{2 \times 6} + \frac{998+2}{2} =$$

$$\frac{1000 \times 996}{12} + \tfrac{1000}{2} = 83000 + 500 = 83500.$$

(10)

Here we have given a, n, and d, to find l, and since $a = 5$, $n = 11$, and $d = 2\tfrac{1}{4}$, we have

$$l = a + (n - 1)\,d = 5 + (11 - 1)2\tfrac{1}{4} = 5 + (10 \times 2\tfrac{1}{4}) = 5 + \tfrac{45}{2} = \tfrac{55}{2} = 27\tfrac{1}{2}.$$

(11)

Here we have given l, d, and n, to find s, and since $l = 199$, $d = 11$, and $n = 19$, we have

$$s = \left\{2l - (n - 1)d\right\}\tfrac{n}{2} = \left\{2 \times 199 - (19 - 1)11\right\}\tfrac{19}{2} =$$

$$\left\{398 - (18 \times 11)\right\}\tfrac{19}{2} = 200 \times \tfrac{19}{2} = 1900.$$

(12)

Here we have given s, a, and l, to find n, and since $s = 39840$, $a = 2$, and $l = 478$, we have

$$n = \frac{2s}{l + a} = \frac{2 \times 39840}{478+2} = \tfrac{79680}{480} = 166.$$

(13)

Here we have given s, l, and a, to find d, and since $s = 83500$ $l = 998$, and $a = 2$, we have

$$d = \frac{(l+a)\,(l-a)}{2s-l-a} = \frac{(998+2)\ (998-2)}{(2\times 83500)-998-2} = \frac{1000\times 996}{167000-1000} = \frac{996000}{166000} = 6$$

(14)

Here we have given s, a, and d, to find n, and since $s = 360$, $a = 2$, and $d = 2$, we have

$$n = \frac{d-2a}{2d} + \sqrt{\frac{2s}{d} + \left(\frac{2a-d}{2d}\right)^2} = \frac{2-(2\times 2)}{2\times 2} + \sqrt{\frac{2\times 260}{2} + \left(\frac{(2\times 2)-2}{2\times 2}\right)^2} = -\tfrac{1}{2} + \sqrt{260 + (\tfrac{1}{2})^2} =$$

$$-\tfrac{1}{2} + \sqrt{260\tfrac{1}{4}} = -\tfrac{1}{2} + 16{\cdot}13226 = 15{\cdot}63226 \text{ days} = 15 \text{ days},$$

15 hours, 10 minutes, 27·264 seconds.

(15)

Here we have given s, a, and d, to find l, and since $s = 83500$, $a = 2$, and $d = 6$, we have

$$l = -\tfrac{1}{2}d + \sqrt{2ds + (a-\tfrac{1}{2}d)^2} = -\tfrac{1}{2}\times 6 +$$

$$\sqrt{2\times 6\times 83500 + (2-\tfrac{1}{2}\times 6)^2} = -3 + \sqrt{1002000 + (2-3)^2}$$

$$= -3 + \sqrt{1002001} = -3 + 1001 = 998.$$

(16)

Here we have given s, n, and l, to find a, and since $s = \$1125$, $n = 18$, and $l = 120$, we have

$$a = \frac{2s}{n} - l = \frac{2\times 1125}{18} - 120 = 125 - 120 = 5.$$

(17)

Here we have given a, l, and n, to find d, and since $a = 5$, $l = 27\frac{1}{2}$, and $n = 11$ we have

$$d = \frac{l - a}{n - 1} = \frac{27\frac{1}{2} - 5}{11 - 1} = \frac{22\frac{1}{2}}{10} = 2\frac{1}{4}.$$

(18)

Here we have a, d, and n given, to find s, and since to deposit one stone he must walk 5 yards, and the distance travelled for each succeeding stone is 5 yards, therefore $a = 5$, $d = 5$, and $n = 220$.

Then $s = \left\{2a + (n - 1)d\right\}\frac{n}{2} = \left\{2 \times 5 + (220 - 1)5\right\}\frac{220}{2}$

$$= \left\{10 + (219 \times 5)\right\}110 =$$

$$1105 \times 110 = 121550 \text{ yards} = 69\tfrac{1}{16} \text{ miles.}$$

(19)

Here we have s, n, and l given, to find a, and since $s = 39840$, $n = 166$, and $l = 478$, we have

$$a = \frac{2s}{n} - l = \frac{2 \times 39840}{166} - 478 = 480 - 478 = 2.$$

(20)

Here we have n, a, and d given, to find s, and since $n = 12$, $a = 4$, and $d = 2$, we have

$$s = \left\{2a + (n - 1)d\right\}\frac{n}{2} = \left\{2 \times 4 + (12 - 1)2\right\}\frac{12}{2} = \left\{8 + (11 \times 2)\right\}6 =$$

$$30 \times 6 = 180.$$

(21)

Here we have given a, l, and n, to find s, and $a = 1$, $l = 24$, and $n = 24$.

Then $s = (a + l)\frac{n}{2} = (1 + 24)\frac{24}{2} = 25 \times 12 = 300.$

EXERCISE 157—Page 342.

(1)

Here $n = 11$, $a = £1024$, and $r = 1\frac{1}{2}$.

Then $l = ar^{n-1} = 1024 \times (\frac{3}{2})^{10} = 1024 \times \frac{59049}{1024} = £59049$

$$s = \frac{rl - a}{r - 1} = \frac{\frac{3}{2} \times 59049 - 1024}{\frac{3}{2} - 1} = \frac{\frac{177147}{2} - 1024}{\frac{1}{2}} = \frac{\frac{175099}{2}}{\frac{1}{2}} =$$

£175099 = whole fortune.

(2)

Here $a = 7$, $l = 1240029$ and $s = 1860040$.

$$\text{Then } r = \frac{s - a}{s - l} = \frac{1860040 - 7}{1860040 - 1240029} = \frac{1860033}{620011} = 3.$$

(3)

Here $n = 12$, $a = £1$, and $l = £2048$.

$$\text{Then } r = \left(\frac{l}{a}\right)^{\frac{1}{n-1}} = \left(\frac{2048}{1}\right)^{\frac{1}{12-1}} = \sqrt[11]{2048} = 2.$$

$$s = \frac{rl - a}{r - 1} = \frac{(2 \times 2048) - 1}{2 - 1} = 4096 - 1 = £4095.$$

(4)

Here $r = \frac{3}{2}$, $n = 8$, and $l = 106\frac{403}{512}$.

$$\text{Then } s = \frac{l(r^n - 1)}{(r - 1)r^{n-1}} = \frac{106\frac{403}{512} \times [(\frac{3}{2})^8 - 1]}{(\frac{3}{2} - 1)\ (\frac{3}{2})^7} = \frac{\frac{54675}{512} \times \frac{6305}{256}}{\frac{1}{2} \times \frac{2187}{128}} =$$

$$\frac{25 \times 6305}{512} = 307\frac{441}{512}.$$

(5)

Here $a = 1$, $n = 7$, and $r = 3$.

$$\text{Then } s = \frac{a(r^n-1)}{r-1} = \frac{1\times(3^7-1)}{3-1} = \tfrac{2186}{2} = 1093.$$

(6)

Here $a = 1$, $l = 10077696$, and $n = 10$.

$$\text{Then } s = \frac{l^{\frac{n}{n-1}} - a^{\frac{n}{n-1}}}{l^{\frac{1}{n-1}} - a^{\frac{1}{n-1}}} = \frac{(10077696)^{\frac{10}{10-1}} - 1^{\frac{10}{10-1}}}{(10077696)^{\frac{1}{10-1}} - 1^{\frac{1}{10-1}}} =$$

$$\frac{\sqrt[9]{(10077696)^{10}} - 1}{\sqrt[9]{10077696} - 1} = \frac{\sqrt[3]{(216)^{10}} - 1}{\sqrt[3]{216} - 1} = \frac{6^{10}-1}{6-1} = \frac{60466176-1}{5}$$

$$= \tfrac{60466175}{5} = 12093235.$$

(7

Here $a = 6$, $l = 3072$, and $s = 6138$.

$$\text{Then } r = \frac{s-a}{s-l} = \frac{6138-6}{6138-3072} = \tfrac{6132}{3066} = 2.$$

(8

Here $r = 2$, $n = 11$, and $s = 20470$.

$$\text{Then } l = \frac{(r-1)sr^{n-1}}{r^n-1} = \frac{(2-1)\times20470\times2^{10}}{2^{11}-1} = \frac{20470\times1024}{2047}$$

$$= 10240.$$

(9)

Here $a = 1s.$, $n = 12$, and $r = 2$.

$$\text{Then } s = \frac{a(r^n-1)}{r-1} = \frac{1\times(2^{12}-1)}{2-1} = \frac{4095}{1} = 4095s.$$

$$= £204\ 15s.$$

(10)

Here $a = 1$ farthing, $r = 2$ and $n = 32$.

$$\text{Then } s = \frac{a(r^n-1)}{r-1} = \frac{1\times(2^{32}-1)}{2-1} = 4294967295 \text{ far.} =$$

£4473924 5s. 3¾d.

(11)

Here $a = 4$, $l = 78732$, and $n = 10$.

$$\text{Then } r = \left(\frac{l}{a}\right)^{\frac{1}{n-1}} = \left(\frac{78732}{4}\right)^{\frac{1}{10-1}} = \sqrt[9]{19683} = 3.$$

(12)

Here $a = 5$, $r = 2$, and $n = 7$.

Then $l = ar^{n-1} = 5 \times 2^{7-1} = 5 \times 2^6 = 5 \times 64 = 320$.

(13)

Here $a = 5$, $l = 327680$, and $r = 4$.

$$\text{Then } s = \frac{rl-a}{r-1} = \frac{(327680\times4)-5}{4-1} = \frac{1310715}{3} = 436905.$$

(14)

Here $a = 1$, $r = 2$, and $n = 64$.

$$\text{Then } s = \frac{a(r^n-1)}{r-1} = \frac{1\times(2^{64}-1)}{2-1} = 18446744073709551615 \text{ gr.}$$

$$18446744073709551615 \div (7680 \times 64) = 37529996894754 \text{ bush.}$$

$$\$1{\cdot}50 \times 37529996894754 = \$56294995342131$$

(15)

Here $r = 3$, $n = 10$, and $s = 295240$.

$$\text{Then } l = \frac{(r-1)sr^{n-1}}{r^n-1} = \frac{(3-1)\times295240\times3^9}{3^{10}-1} = \frac{2\times295240\times19683}{59048}$$

$$= 196830.$$

(16)

Here $a = 1$, $l = 2048$, and $n = 12$.

$$\text{Then } s = \frac{l^{\frac{n}{n-1}} - a^{\frac{n}{n-1}}}{l^{\frac{1}{n-1}} - a^{\frac{1}{n-1}}} = \frac{2048^{\frac{12}{12-1}} - 1^{\frac{12}{12-1}}}{2048^{\frac{1}{12-1}} - 1^{\frac{1}{12-1}}} =$$

$$\frac{\sqrt[11]{(2048)^{12}}-1}{\sqrt[11]{2048}-1} = \frac{2^{12}-1}{2-1} = 2^{12}-1 = 4095.$$

17)

Here $a = 5$, $r = 4$, and $n = 9$.

Then $l = ar^{n-1} = 5\times4^{9-1} = 5\times4^8 = 5\times65536 = 327680$.

EXERCISE 158.—Page 344.

(1)

Here $a = \frac{3}{7}$, and $r = \frac{2}{5}$.

$$\text{Then } s = \frac{a}{1-r} = \frac{\frac{3}{7}}{1-\frac{2}{5}} = \frac{\frac{3}{7}}{\frac{3}{5}} = \frac{5}{7}.$$

(2)

Here $a = 4$, and $r = \frac{1}{2}$,

$$\text{Then } s = \frac{a}{-r} = \frac{4}{1-\frac{1}{2}} = \frac{4}{\frac{1}{2}} = 8.$$

(3)

Here $a = \frac{79}{100}$, and $r = \frac{1}{100}$.

$$\text{Then } s = \frac{a}{1-r} = \frac{\frac{79}{100}}{1-\frac{1}{100}} = \frac{\frac{79}{100}}{\frac{99}{100}} = \frac{79}{99}.$$

(4)

Here $a = \frac{1234}{10000}$, and $r = \frac{1}{10000}$.

$$\text{Then } s = \frac{a}{1-r} = \frac{\frac{1234}{10000}}{1-\frac{1}{10000}} = \frac{\frac{1234}{10000}}{\frac{9999}{10000}} = \frac{1234}{9999}.$$

EXERCISE 159.—Page 345.

(1)

Since there are 9 means and 2 extremes the number of terms is 11.

$$\text{Then } d = \frac{l-a}{n-1} = \frac{92-2}{11-1} = \tfrac{90}{10} = 9.$$

1st term = 2; 2nd = 2 + 9 = 11; 3rd = 11 + 9 = 20; 4th = 20 + 9 = 29; 5th = 29 + 9 = 38; 6th = 38 + 9 = 47; and so on.

And series is 2, 11, 20, 29, 38, 47, 56, 65, 74, 83, 92.

(2)

Since there are 4 means and 2 extremes the number of terms is 6.

$$\text{Then } d = \frac{l-a}{n-1} = \frac{50-7}{6-1} = \tfrac{43}{5} = 8\tfrac{3}{5}.$$

1st term = 7; 2nd = $7 + 8\frac{3}{5} = 15\frac{3}{5}$; 3rd = $15\frac{3}{5} + 8\frac{3}{5} = 24\frac{1}{5}$; 4th = $24\frac{1}{5} + 8\frac{3}{5} = 32\frac{4}{5}$; 5th = $32\frac{4}{5} + 8\frac{3}{5} = 41\frac{2}{5}$; and 6th = $41\frac{2}{5} + 8\frac{3}{5} = 50$.

And series is 7, $15\frac{3}{5}$, $24\frac{1}{5}$, $32\frac{4}{5}$, $41\frac{2}{5}$, 50.

(3)

Since there are 8 means and 2 extremes the number of terms is 10.

$$\text{Then } r = \left(\frac{l}{a}\right)^{\frac{1}{n-1}} = \left(\tfrac{8}{4096}\right)^{\frac{1}{10-1}} = \left(\tfrac{1}{512}\right)^{\frac{1}{9}} = \tfrac{1}{2}.$$

1st term = 4096; 2nd = 4096 × $\frac{1}{2}$ = 2048; 3rd = 2048 × $\frac{1}{2}$ = 1024; 4th = 1024 × $\frac{1}{2}$ = 512; 5th = 512 × $\frac{1}{2}$ = 256, and so on.

And the means are 2048, 1024, 512, 256, 128, 64, 32, and 16.

(4)

Since there are 7 means and 2 extremes the number of terms is 9.

$$\text{Then } r = \left(\frac{l}{a}\right)^{\frac{1}{n-1}} = \left(\tfrac{23514624}{14}\right)^{\frac{1}{9-1}} = (1679616)^{\frac{1}{8}} = 6.$$

1st term = 14; 2nd = 14 × 6 = 84; 3rd = 84 × 6 = 504; 4th = 504 × 6 = 3024; 5th = 3024 × 6 = 18144, and so on.

And the means are 84, 504, 3024, 18144, 108864, 653184, and 3919104.

EXERCISE 160.—Page 347

(1)

Assume 4 to be the number of men.
Then 2 × 4 = 8 = number of women.
And 8 × 3 = 24 = number of children.

6d. × 4 = 24d. = amount received by the men.
4d. × 8 = 32d. = " " " women.
2d. × 24 = 48d. = " " " children.

Sum, = 104d., but it should, by question, = 78d.

$$\text{Then } 104 : 78 :: 4 : \frac{78 \times 4}{104} = 3 = \text{number of men.}$$

3×2=6=number of women, and 6×3=18=number of children.

(2)

Assume £8 to be the price of the harness.
Then £8 × 2 = 16 = price of horse.
And £8 + £16 = £24×2 = 48 = " chaise.

Sum, = £ 72, but it should by question = £60.

$$\text{Then } £72 : £60 :: £8 : \frac{8 \times 60}{72} = £6\ 13\ 4 = \text{price of harness.}$$

£6 13 4 × 2 = 13 6 8 = " horse.
£6 13 4 + £13 6 8 = £20 × 2 = 40 0 0 = " chaise.

(3)

Assume 20 as C's age.

Then $20 \times 3 = 60 =$ B's age.

And $60 \times 2 = 120 =$ A's age.

Sum $= 200$, but by question it should $= 140$.

Then $200 : 140 :: 20 : \dfrac{20 \times 140}{200} = 14 =$ C's age.

$14 \times 3 = 42 =$ B's age, and $42 \times 2 = 84 =$ A's age.

(4)

Assume 100.

One fourth of $100 = 25$ and remainder $= 100 - 25 = 75$.

One fifth of $75 = 15$ and remainder $= 75 - 15 = 60$, but it should by the question $= 72$.

Then $60 : 72 :: 100 : \dfrac{100 \times 72}{60} = 120$.

(5)

A can do the work in 7 days ∴ he will do $\frac{1}{7}$ of it in 1 day.
B " " 5 " ∴ " $\frac{1}{5}$ " "
C " " 6 " ∴ " $\frac{1}{6}$ " "

Then all working together will do $\frac{1}{7} + \frac{1}{5} + \frac{1}{6} = \frac{107}{210}$ in 1 day.

Therefore to do the whole work it will take them $\dfrac{1}{\frac{107}{210}} = \frac{210}{107} =$ $1\frac{103}{107}$ days.

(6)*

A and B working together can do it in 10 days ∴ they will do $\frac{1}{10}$ of it in 1 day.

A can do it in 15 days ∴ he will do $\frac{1}{15}$ of it in 1 day.

Therefore $\frac{1}{10} - \frac{1}{15} = \frac{1}{30} =$ amount done by B in 1 day.

Then if he does $\frac{1}{30}$ in 1 day, it will take him 30 days to do the whole.

---

* The mode of working these questions by position is so simple that they cannot trouble any one; it has therefore been thought advisable to work them by simple analysis.

(7)*

The first pipe empties the whole of it in 1 hour.
The second pipe empties $\frac{1}{2}$ of it in 1 hour.
The third pipe empties $\frac{1}{3}$ of it in 1 hour.

Then all these pipes running together will empty $1 + \frac{1}{2} + \frac{1}{3} = \frac{11}{6}$ in 1 hour.

Therefore to empty the cistern it will take $1 \div \frac{11}{6} = \frac{6}{11}$ hours.

(8)

Assume 84
One third of 84 = 28
One sixth of 84 = 14
One seventh of 84 = 12

Sum = 54, but by question it should = 27.

$$\text{Then } 54 : 27 :: 84 : \frac{84 \times 27}{54} = 42.$$

(9)

All 5 mills working together will grind $7 + 5 + 4 + 3 + 1 = 20$ bushels in 1 hour.

Therefore to grind 500 bushels it will take them $500 \div 20 = 25$ hours.

(10)*

One pipe fills $\frac{1}{12}$ of the cistern in 1 hour, and the other empties $\frac{1}{18}$ of it in 1 hour.

Then $\frac{1}{12} - \frac{1}{18} = \frac{1}{36}$ = part of the cistern filled in 1 hour when both are left open.

And if $\frac{1}{36}$ of it is filled in 1 hour, the whole will be filled in

$$\frac{1}{\frac{1}{36}} = 36 \text{ hours.}$$

---

* See note on page 227.

Exercise 161.—Page 352.

(1)

Assume 60 for father's age, then 15 = son's.

$$\begin{array}{r} 5 \\ \hline 5)55 \\ \hline 11 \\ 10 \\ \hline -\ 1 \end{array} \qquad\qquad \begin{array}{r} 5 \\ \hline 10 \end{array}$$

Assume 100 for father's age, then 25 = son's.

$$\begin{array}{r} 5 \\ \hline 5)95 \\ \hline 19 \\ 20 \\ \hline +\ 1 \end{array} \qquad\qquad \begin{array}{r} 5 \\ \hline 20 \end{array}$$

Errors.		Assumed numbers.		
− 1	×	100	=	100
+ 1	×	60	=	60

Sum of errors = 2 Sum of products = 160

Therefore result required = 160 ÷ 2 = 80 = father's age, and ¼ of 80 = 20 = son's age.

(2)

$$\begin{array}{r} \text{Assume } 80 \\ 34 \\ \hline 46 \\ 3 \\ \hline 138 \\ 80 \\ \hline 58 \\ \tfrac{1}{4} \text{ of } 80 = 20 \\ \hline +\ 38 \end{array} \qquad\qquad \begin{array}{r} \text{Assume } 44 \\ 34 \\ \hline 10 \\ 3 \\ \hline 30 \\ 44 \\ \hline -\ 14 \\ \tfrac{1}{4} \text{ of } 44 = 11 \\ \hline -25 \end{array}$$

(Continued on next page.)

(2 continued.)

Errors.		Assumed numbers.		
− 25	×	80	=	2000
+ 38	×	44	=	1672
Sum of errors = 63				3672

Therefore result required = 3672 ÷ 63 = $58\frac{2}{7}$.

(4)

Assume 18 and 7
One half of 18 = 9 2 × 7 = 14
14
− 5

Assume 22 and 3
One half of 22 = 11 2 × 3 = 6
6
+ 5

Errors.		Assumed numbers.		
− 5	×	22	=	110
+ 5	×	18	=	90
Sum of errors = 10		Sum of products		= 200

Then 200 ÷ 10 = 20 = one number, and 25 − 20 = 5 = other number.

(5)

A. B.
Suppose 8 6
$22\frac{1}{2}$ 9
12
180 15
132 18
21
8)48 24
27
+ 6
6 132
36
72
3)36
12

A. B.
Suppose 6 6
$22\frac{1}{2}$ 9
12
135 15
81 18
21
6)54
81
+ 9
8
72

9 − 6 = 3 = difference of errors.

(7)

Assume 30.

$\frac{1}{2}$ of 30 = 15; $\frac{1}{4}$ of 30 = $7\frac{1}{2}$;
$\frac{1}{5}$ of 30 = 6; and $\frac{1}{6}$ of 30 = 5;
$15 \times 7\frac{1}{2} \times 6 \times 5 = 3375$;
$3375 - 6998\frac{2}{5} = -3623{\cdot}4$ = error.

Assume 60.

$\frac{1}{2}$ of 60 = 30; $\frac{1}{4}$ of 60 = 15;
$\frac{1}{5}$ of 60 = 12; and $\frac{1}{6}$ of 60 = 10.
$30 \times 15 \times 12 \times 10 = 54000$.
$54000 - 6998\frac{2}{5} = +47001{\cdot}6$ = error.

$$\begin{array}{rl} & 30^4 = 810000, \text{ and } 60^4 = 12960000 \\ -3623{\cdot}4 \times 12960000 & = 46959264000 \\ +47001{\cdot}6 \times 810000 & = 38071296000 \\ \hline \text{Sum} = 50625 & \text{Sum} = 85030560000 \end{array}$$

$85030560000 \div 50625 = 1679616$

4th root = square root of square root.

$\sqrt{1679616} = 1296$, and $\sqrt{1296} = 36$ = required number.

NOTE.—For reason why we multiply by the 4th powers of the assumed numbers and then take the 4th root of the quotient, see Arith. page 353, Example 11.

It may, however, perhaps be clearer from the following illustration:

Let x = the number required.

$$\text{Then } \frac{x}{2} \times \frac{x}{4} \times \frac{x}{5} \times \frac{x}{6} = \frac{x^4}{240} = 6998\tfrac{2}{5}$$

$$\therefore x^4 = 1679616$$

$$\therefore x = \sqrt[4]{1679616} = 36.$$

(8)

Suppose A had 9s. at first.
Then $9 + 1 = 10$; $10 \div 2 = 5$; $5 + 1 = 6$ = what B had at first.
$6 + 1 = 7$, but should $= 9 - 1 = 8$.
Error $= 7 - 8 = -1$.

Suppose A had 11s. at first.
Then $11 + 1 = 12$; $12 \div 2 = 6$; $6 + 1 = 7$ = what B had at first.
$7 + 1 = 8$, but should $= 11 - 1 = 10$.
Error $= 8 - 10 = -2$.

(Continued on next page.)

(8 continued.)

Errors.

$$\begin{array}{r}
-2 \times 9 = 18 \\
-1 \times 11 = 11 \\
\hline
\text{Diff.} = 1 \quad \text{diff.} = 7
\end{array}$$

7 ÷ 1 = 7 = shillings A had at first.

7 + 1 = 8; 8 ÷ 2 = 4; 4 + 1 = 5 = shillings B had at first.

(9)

Assume 24 and 6.

$$\tfrac{24}{2} + \tfrac{24}{3} + \tfrac{24}{6} = 24.$$

$$\tfrac{6}{2} + \tfrac{3}{4} \text{ of } 6 + \tfrac{6}{4} = 9.$$

24 − 9 = + 15 = error.

Assume 20 and 10.

$$\tfrac{20}{2} + \tfrac{20}{3} + \tfrac{20}{6} = 20.$$

$$\tfrac{10}{2} + \tfrac{3}{4} \text{ of } 10 + \tfrac{10}{4} = 15.$$

20 − 15 = + 5 = error.

Errors.

$$\begin{array}{r}
+15 \times 20 = 300 \\
+5 \times 24 = 120 \\
\hline
\text{Diff.} = 10 \quad \text{diff.} = 180
\end{array}$$

180 ÷ 10 = 18 = one number.

30 − 18 = 12 = other number.

(10)

Suppose 1st horse to be worth £20.

20 + 50 = 70; 70 ÷ 2 = £35 = value of 2nd horse.
35 + 50 = 85, but it should equal 60, i. e. (20 × 3).
Then 60 − 85 = − 25 = error.

Suppose 1st horse to be worth £60.

£60 + £50 = £110; £110 ÷ 2 = £55 = worth of 2nd horse.
55 + 50 = 105, but it should equal 180, i. e. (60 × 3).
180 − 105 = + 75 = error.

Errors.

$$\begin{array}{r}
+75 \times 20 = 1500. \\
-25 \times 60 = 1500 \\
\hline
\text{Sum} = 100 \quad \text{Sum} = 3000
\end{array}$$

3000 ÷ 100 = £30 = value of 1st horse.

£30 + £50 = £80; £80 ÷ 2 = £40 = value of 2nd horse.

(11)

Suppose there were 11 beggars.
11 × 4 = 44; 44 + 6 = 50 = number of pence he had.
11 × 6 = 66; 66 − 12 = 54 = " " "
54 − 50 = + 4 = error.

Suppose there were 12 beggars.
12 × 4 = 48; 48 + 6 = 54 = pence he had.
12 × 6 = 72; 72 ÷ 12 = 60 = pence he had.
60 − 54 = + 6 = error.

Errors.
+ 6 × 11 = 66
+ 4 × 12 = 48

Diff. = 2 diff. = 18, and 18 ÷ 2 = 9 = number of beggars.

EXERCISE 162.—Page 357.

(1)

Here P = \$713·29, r = ·045, and t = 14.
Then $A = P(1+r)^t$, or log. A = log. P + log. $(1+r) \times t$
= 2·853267 + (·019116 × 14) = 3·120891 = log. of *Ans.*
Hence amount = \$1320·96.

(2)

Here n = 7, r = ·015.

$$\text{Then } t = \frac{\log.\ n.}{\log.\ (1+r)} = \frac{\cdot845098}{\cdot006466} = 130\cdot698 \text{ payments, and}$$

130·698 ÷ 4 = 32·674 years = 32 years 8 months 2 days.

(3)

Here A = \$1111·11, P = 111·11, and r = ·08.

$$\text{Then } t = \frac{\log.\ A - \log.\ P}{\log.\ (1+r)} = \frac{3\cdot045757 - 2\cdot045753}{\cdot033424} = \frac{1\cdot000004}{\cdot033424}$$

= 29·918 years = 29 years 11 months.

(4)

Here $A = \$3333\cdot33$, $P = \$222\cdot22$, and $t = 120$.

Then $r = \sqrt[t]{\dfrac{A}{P}} - 1$; or $\log.\,(1 + r) = \dfrac{\log.\,A - \log.\,P}{t} = \dfrac{3\cdot522878 - 2\cdot346783}{120} = \dfrac{1\cdot176095}{120} = \cdot0098007$. Hence $1 + r = 1\cdot0228$, $r = \cdot0228$, and rate per cent. $= 2\frac{7}{25}$.

(5)

Here $n = 2$ and $r = \cdot07$.

Then $t = \dfrac{\log.\,n.}{\log.\,(1 + r)} = \dfrac{0\cdot301030}{0\cdot029384} = 10\cdot2446$ years $=$ 10 yrs. 2 months 28 days.

(6)

Here $A = \$100$, $r = \cdot0225$, and $t = 28$.

Then $P = \dfrac{A}{(1 + r)^t}$, or $\log P = \log.\,A - \log.\,(1 + r) \times t$.

$\text{Log.}\,P = 2 - (0\cdot009664 \times 28) = 2 - 0\cdot270592 = 1\cdot729408$.

Hence $P = \$53\cdot63$.

(7)

Here $P = \$2468\cdot13$, $r = \cdot0375$, and $t = 26$.

Then $A = P\,(1 + r)^t$, or $\log.\,A = \log.\,P + \log.\,(1 + r) \times t$.

$\text{Log.}\,A = 3\cdot392368 + (0\cdot015988 \times 26) = 3\cdot392368 + 0\cdot415688$
$= 3\cdot808056$.

Hence $A = \$6427\cdot705$.

(8)

Here $A = \$7137\cdot40$, $r = \cdot0425$, and $t = 22$.

Then $P = \dfrac{A}{(1 + r)^t}$, or $\log.\,P = \log.\,A - \log.\,(1 + r) \times t$.

$\text{Log.}\,P = 3\cdot853540 - (0\cdot018076 \times 22) = 3\cdot853540 - 0\cdot397672$
$= 3\cdot455868$.

Hence $P = \$2856\cdot723$.

(16)

Here $n = 19$, and $r = \cdot 0525$.

$$\text{Then } t = \frac{\log. n}{\log. (1 + r)} = \frac{1\cdot 278754}{0\cdot 022222} = 57\cdot 5445 \text{ payments} =$$

28·7722 years = 28 years 9 months 8 days.

EXERCISE 163.—Page 360.

(1)

Here $r = \cdot 03$, $a = 500$, $A = 8365$.

$$\text{Formula IV. } t = \frac{\sqrt{\left\{\frac{8rA}{a} + (2 - r)^2\right\}} - (2 - r)}{2r}$$

$$= \frac{\sqrt{\left\{\frac{8 \times \cdot 03 \times 8365}{500} + (2 - .03)^2\right\}} - (2 - \cdot 03)}{2 \times \cdot 03}$$

$$= \frac{\sqrt{\left\{\frac{2007\cdot 6}{500} + 3\cdot 8809\right\}} - 1\cdot 97.}{\cdot 06}$$

$$= \frac{\sqrt{(4\cdot 0152 + 3\cdot 8809)} - 1\cdot 97}{\cdot 06} = \frac{\sqrt{7\cdot 8961} - 1\cdot 97}{\cdot 06}$$

$$= \frac{2\cdot 81 - 1\cdot 97}{\cdot 06} = \frac{\cdot 84}{\cdot 06} = \frac{84}{6} = 14 \text{ payments} = 7 \text{ years.}$$

(2)

Here $a = 112\cdot 50$, $r = \cdot 015$, $t = 44$.

$$\text{Formula I. } A = at\left(1 + \frac{(t - 1)r}{2}\right)$$

$$= 112\cdot 50 \times 44\left(1 + \frac{(44 - 1) \times \cdot 015}{2}\right) = 4950 \times 1\cdot 3225$$

$$= \$6546\cdot 375.$$

(3)

Here $a = 300$, $A = 1680$, and $t = 5$.

Formula III. $r = \dfrac{2\,(A - at)}{at\,(t-1)} = \dfrac{2\,\{1680 - (300 \times 5)\}}{300 \times 5\,(5-1)}$

$$= \frac{2\,(1680 - 1500)}{300 \times 5 \times 4} = \frac{2 \times 180}{6000} = \frac{360}{6000} = \cdot 06$$

∴ Rate per cent $= \cdot 06 \times 100 = 6$.

(4)

Here $A = 2080$, $r = \cdot 04$, and $t = 16$.

Formula II. $a = \dfrac{2\,A}{t\,\{2 + (t-1)\,r\}} = \dfrac{2 \times 2080}{16\,\{2 + (16-1)\,\cdot 04\}}$

$$= \frac{4160}{16 \times \{2 + (15 \times \cdot 04)\}} = \frac{4160}{16 \times 2\cdot 6} = \frac{4160}{41\cdot 6} = \frac{41600}{416}$$

= \$100 = 1 payment or rent for half a year, hence yearly rent = \$100 × 2 = \$200.

EXERCISE 164.—Page 366.

(1)

Here $r = \cdot 04$, and $v = \$3000$.
Then $a = vr = 3000 \times \cdot 04 = \$120$.

(2)

Here $a = 563$, and $v = 11260$

Then $r = \dfrac{a}{v} = \dfrac{563}{11260} = \dfrac{1}{20} = \cdot 05$, and hence rate per cent. $= 5$.

(3)

Here $a = 75$, $r = \cdot 05$, and $s = 14$.

$$\text{Then } v = \frac{a}{r\,(1+r)^s} = \frac{75}{\cdot 05 \times (1\cdot 05)^{14}}$$

$$\begin{aligned} \log. v &= \log. 75 - \{(\log. 1\cdot 05 \times 14) + \log. \cdot 05\} \\ &= 1\cdot 875061 - (0\cdot 021189 \times 14 + \log. \cdot 05) \\ &= 1\cdot 875061 - (0\cdot 296646 + \bar{2}\cdot 698970.) \\ &= 2\cdot 879445. \end{aligned}$$

$\therefore v =$ nat. number corresponding to the logarithm $2\cdot 879445$, which is \$757·608.

(4)

Here $a = \$90$, $r = \cdot 04$, $t = 12$, $s = 7$, and $\therefore s + t = 19$.

$$\text{Formula VIII. } v = \frac{a}{r}\left\{\frac{1}{(1+r)^t} - \frac{1}{(1+r)^{s+t}}\right\}$$

$$= \frac{90}{\cdot 04}\left\{\frac{1}{(1\cdot 04)^{12}} - \frac{1}{(1\cdot 04)^{19}}\right\} = \frac{9000}{4}\left\{\frac{1}{1\cdot 60101} - \frac{1}{2\cdot 10682}\right\}$$

$$= 2250 \times (\cdot 624605 - \cdot 474649) = 2250 \times \cdot 149956$$

$$= \$337\cdot 401.$$

(5)

Here $a = 1500$, and $r = \cdot 05$.

$$\text{Formula IX. } v = \frac{a}{r} = \frac{1500}{\cdot 05} = \frac{150000}{5} = \$30000$$

$= 20 \times 1500$ or 20 years' purchase.

(6)

Here $a = 22$, $v = 308\cdot 64166$, and $r = \cdot 04$.

$$\text{Then Formula VII. } t = \frac{\log. a - \log. (a - vr)}{\log. (1+r)}$$

$$= \frac{\log. 22 - \log. (22 - 308\cdot 6416 \times \cdot 04)}{\log. (1\cdot 04)}$$

$$= \frac{1\cdot 342423 - \log. (9\cdot 65425)}{0\cdot 017033} = \frac{1\cdot 342423 - 0\cdot 984707}{0\cdot 017033}$$

$$= \frac{0\cdot 357716}{0\cdot 017033} = \frac{357716}{17033} = 21 +$$

(7)

Here $a = 154$, $t = 19$, and $r = \cdot 05$.

$$\text{Formula V.}\quad v = \frac{a}{r}\left\{1 - \frac{1}{(1+r)^t}\right\}$$

$$= \frac{154}{\cdot 05} \times \left\{1 - \frac{1}{(1\cdot 05)^{19}}\right\} = \frac{15400}{5} \times \left\{1 - \frac{1}{2\cdot 5269}\right\}$$

$$= 3080 \times (1 - \cdot 39574) = 3080 \times \cdot 60426 = \$1861\cdot 12 +$$

(8)

Here $A = 600$, $t = 40$, and $r = \cdot 0375$.

$$\text{Formula II.}\quad a = \frac{Ar}{(1+r)^t - 1} = \frac{600 \times \cdot 0375}{(1\cdot 0375)^{40} - 1}$$

$$= \frac{22\cdot 5}{4\cdot 36034 - 1} = \frac{22\cdot 5}{3\cdot 36034} = \frac{2250000}{336034}$$

$$= £6\cdot 6957 = £6\ 13\text{s.}\ 10\tfrac{3}{4}\text{d} +.$$

(9)

Here $a = 8$, $A = 187\cdot 315625$, and $r = \cdot 03$.

$$\text{Formula IV.}\quad t = \frac{\log.(Ar + a) - \log.\ a}{\log.(1 + r)}$$

$$= \frac{\log.(187\cdot 315625 \times \cdot 03 + 8) - \log 8}{\log.\ 1\cdot 03}$$

$$= \frac{\log(5\cdot 61946875 + 8) - \log 8}{\log 1\cdot 03}$$

$$= \frac{\log.\ 13\cdot 61946875 - \log.\ 8}{\log 1\cdot 03} = \frac{1\cdot 134160 - 0\cdot 903090}{0\cdot 012837}$$

$$= \frac{0\cdot 231070}{0\cdot 012837} = \frac{231070}{12837} = 18.$$

(10)

Here $a = 74$, $r = \cdot 04$, and $t = 30$

$$\text{Formula I.}\quad A = \frac{a\{(1+r)^t - 1\}}{r} = \frac{74 \times \{(1\cdot 04)^{30} - 1\}}{\cdot 04}$$

$$= \frac{74}{\cdot 04} \times (3\cdot 24332 - 1) = \frac{7400}{4} \times 2\cdot 24332 = \$4150\cdot 142$$

By Table, page 362. Amount of \$1 for 30 years, at 4 per cent. = \$56·08494

Then \$56·08494 × 74 = \$4150·28.

EXERCISE 165—Page 367.

EXAMINATION PROBLEMS.

FIRST SERIES.

(2)

\$7580 × ·19 = \$1440·20, and \$7580 — \$1440·20 = \$6139·80.
D is to have one third as much as A, B, and C together, therefore he will have one-fourth of the whole. $\frac{1}{4}$ of \$6139·80 = \$1534·95 = D's share.

\$6139·80 — \$1534·95 = \$4604·85 = amount to be divided among A, B, and C.

B is to have \$90·90 more than C.
A is to have \$111·11 + \$90·90 = 202·01 " " "

\$292·91

\$4604·85 — \$292·91 = \$4311·94 = three times C's share.
\$4311·94 ÷ 3 = \$1437·31$\frac{1}{3}$ = C's share.
\$1437·31$\frac{1}{3}$ + \$90·90 = \$1528·21$\frac{1}{3}$ = B's share.
\$1528·21$\frac{1}{3}$ + \$111·11 = \$1639·32$\frac{1}{3}$ = A's share.

(3)

A and B working together can do the work in 96 hours, therefore in one hour they will do $\frac{1}{96}$ of it.

A by himself can do the work in 192 hours; therefore in 1 hour he can do $\frac{1}{192}$ of it. $\frac{1}{96} - \frac{1}{192} = \frac{1}{192}$ = part B can do in one hour. Therefore he will require as many hours to finish it as $\frac{1}{192}$ is contained times in the whole, i. e. $1 \div \frac{1}{192}$ = 192 hours. Then 192 ÷ 14 = $13\frac{5}{7}$ days.

(4)

£179 14s. $8\frac{3}{4}$d. = \$718·94$\frac{7}{12}$ = \$718·9458$\dot{3}$.
\$718·9458$\dot{3}$ ÷ ·00000048 = \$71894583333·3 ÷ 48 = \$1497803819·4444.

(5)

$$\begin{array}{r|l} 77 & \cancel{44}..18..30..\cancel{77}..\cancel{56}..27 \\ 30 & \cancel{4}..\cancel{18}..\cancel{30} \quad\quad \cancel{8}..\cancel{27} \\ 36 & \cancel{2}..\;\cancel{3} \quad\quad\quad\quad \cancel{4}..\;\cancel{9} \end{array}$$

77 × 30 × 36=83160 = l. c. m.

(6)

Here $n = 20$, and $r = \cdot0525$.

Then $t = \dfrac{n-1}{r} = \dfrac{20-1}{\cdot0525} = \dfrac{19}{\cdot0525} = 361 \cdot 9048$ years = 361 years 10 months 25 days.

(7)

7342163 octenary = 7t0e57 duodenary, and 61351 nonary = 1e454 duodenary.

7t0e57 ÷ 1e454 = 40·38 duodenary.

(8)

$783\frac12 = 3\frac12 + 10 \times 8 + 10 \times 10 \times 7.$

lbs.	oz.	dwt.	grs.				lbs.	oz.	dwt.	grs.
43	3	17	11	×	$3\frac12$	=	151	7	11	$2\frac12$
			10							
433	2	14	14	×	8	=	3465	9	16	16
			10							
4332	3	5	20	×	7	=	30325	11	0	20
							33943	4	8	$14\frac12$

(9)

Here $a = 1$, and $r = \frac12$.

Then $S = \dfrac{a}{1-r} = \dfrac{1}{1-\frac12} = \dfrac{1}{\frac12} = 2.$

(10)

$$\tfrac12 \text{ of } \tfrac23 \text{ of } 192 \div \frac{\frac{2\frac12}{3}}{\frac{\frac{4\frac12}{2}}{\frac34}} = 64 \div \frac{\frac52}{3}{\Big/}\frac{\frac92}{\frac{2}{\frac34}} = 64 \div \frac{\frac56}{\frac{\frac92}{\frac83}} = 64 \div \frac{\frac56}{\frac{27}{16}}$$

$$= 64 \times \frac{81}{40} = 129\tfrac35.$$

(11)

Logarithm of 129140163 = 8·111061.
8·111061 ÷ 17 =·477121 =logarithm of 3.

(12)

$$\begin{array}{r} \text{Suppose } 48 \\ 18 \\ \hline 66 \\ 84 \\ \hline -\,18 \end{array} \qquad\qquad \begin{array}{r} \text{Suppose } 36 \\ 18 \\ \hline 54 \\ 63 \\ \hline -\,9 \end{array}$$

$$\begin{array}{ccccc} \text{Errors.} & \multicolumn{3}{c}{\text{Assumed numbers.}} & \\ -18 & \times & 36 & = & 648 \\ -\;9 & \times & 48 & = & 432 \\ \hline \end{array}$$

Difference of errors = 9 $\qquad$ 9)216 = sum of products.

$$\underline{\quad\quad}$$
$$24$$

SECOND SERIES.

(13)

B is to have \$69·18 more than C.
A is to have \$69·18 + \$93·40 = \$162·58 " " "

$$\underline{\quad\quad\quad}$$
\$231·76

\$897·43 — \$231·76 = \$665·67 = Amount to be divided equally amongst A, B, and C.

\$665·67 ÷ 3 = \$221·89 = C's share.
\$221·89 + \$69·18 = \$291·07 = B's "
\$291·07 + \$93·40 = \$384·47 = A's "

(14)

$$\left.\begin{array}{rlrl} 7 & \text{lbs. wheat} & = 9 & \text{lbs. rye} \\ 5 & \text{" rye} & = 8 & \text{" oats} \\ 13 & \text{" oats} & = 21 & \text{" buckwheat} \\ 27 & \text{" buckwheat} & = 20 & \text{" barley} \\ 24 & \text{" barley} & = 26 & \text{" peas} \\ 11 & \text{" peas} & = 35 & \text{" potatoes} \\ x & \text{" potatoes} & = 16 & \text{" wheat} \end{array}\right\} \qquad \begin{array}{rrcrr} & \cancel{7} & = & \cancel{9} & \\ & \cancel{5} & = & \cancel{8} & \\ & \cancel{13} & = & \cancel{21} & \cancel{7} \\ 3 & \cancel{27} & = & \cancel{20} & 4 \\ 3 & \cancel{24} & = & \cancel{26} & 2 \\ & 11 & = & 35 & \\ & x & = & 16 & \end{array}$$

Ans. $\dfrac{4 \times 2 \times 35 \times 16}{3 \times 11} = \dfrac{4480}{33} = 135\frac{25}{33}$.

(15)

$\frac{2}{3}$ of $4\frac{1}{2}$ of $7\frac{4}{5}$ of $\dfrac{9}{19\frac{1}{2}}$ of $\frac{5}{9}$ of 3 oz. 4 drs. 2 scr. 5 grs. $=\frac{2}{3}$ of $\frac{9}{2}$ of $\frac{39}{5}$ of $\frac{18}{39}$ of $\frac{5}{9}$ of 1725 grs. = 10350 grs.

$\frac{6}{11}$ of $\cdot\dot{6}\dot{3}$ of $2\frac{37}{42}$ of $\frac{3}{13}$ of $6\frac{1}{2}$ times 7 lbs. 3 oz. $=\frac{6}{11}$ of $\frac{7}{11}$ of $\frac{121}{42}$ of $\frac{3}{13}$ of $\frac{13}{2}$ of 41760 grs. = 62640 grs.

10350 ÷ 62640 = ·165229.

(16)

Dissimilar.		Similar.		Similar and Coterminous.
$623\cdot42\dot{7}9\dot{3}$	=	$623\cdot42793\dot{7}9\dot{3}$	=	$623\cdot42793793\dot{7}9\dot{3}$
$93\cdot42671\dot{9}\dot{2}$	=	$93\cdot42671\dot{9}\dot{2}$	=	$93\cdot426719292\dot{9}\dot{2}$

Difference $=530\cdot00121\dot{8}6450\dot{0}$

(17)

\$1·00 — \$0·046 = \$0·954, and \$7493 ÷ 0·954 = \$7854·29.

18)

$$\left.\begin{array}{rl} 36: & 20 \text{ weeks} \\ 6: & 5 \text{ days} \\ 9: & 11 \text{ hours} \\ 11: & 24 \text{ cellars} \\ 20: & 22 \text{ feet long} \\ 16: & 22 \text{ feet wide} \\ 5: & 4 \text{ feet deep} \end{array}\right\} :: 18 \text{ men} : \frac{\overset{6}{\cancel{18}}\times\cancel{20}\times\cancel{5}\times\cancel{11}\times\overset{2}{\cancel{24}}\times\overset{11}{\cancel{22}}\times22\times\cancel{4}}{\underset{3}{\cancel{36}}\times\cancel{6}\times9\times\cancel{11}\times\cancel{20}\times\underset{\underset{2}{\cancel{4}}}{\cancel{16}}\times\cancel{5}}$$

$$=\frac{11\times22}{9}=26\tfrac{8}{9}.$$

(19)

$\frac{1}{2}$ cf $\frac{3}{5}$ of $\frac{4}{7}=\frac{6}{35}$; and if $\frac{6}{35}$ of a certain number $=\frac{72}{35}$, $\frac{1}{35}=\frac{12}{35}$ and $\frac{35}{35}=\frac{12}{35}\times35=12$.

$(\{[(\{[(12\times12\tfrac{1}{2})+31]\times3\}-33)\times300]\div17\}\times9)$ $=81000$

(20)

$$\begin{array}{r|l} 1176 & \cancel{480}\,..\,\cancel{768}\,..\,\cancel{348}\,..\,\cancel{1176} \\ 32 & \cancel{20}\,..\,\cancel{32}\,..\,29 \\ 145 & \cancel{5}\qquad\quad\ \cancel{29} \end{array}$$

1176 × 32 × 145 = 5456640.

(21)

```
                          838)171347(204
                               1676
                               ----
                                3747
                                3352
                                ----
                                 395)838(2
                                     790
                                     ---
                                      48)395(8
                                         384
                                         ---
                                          11)48(4
                                             44
                                             --
                                              4)11(2
                                                 8
                                                 -
                                                3)4(1
                                                  3
                                                  -
                                                  1)3
                                                    -
                                                    3.

17598)46090(2
      35196
      -----
      10894)17598(1
            10894
            -----
             6704)10894(1
                   6704
                   ----
                   4190)6704(1
                        4190
                        ----
                        2514)4190(1
                             2514
                             ----
                             1676)2514(1
                                  1676
                                  ----
                                   838)1676(2
                                       1676
```

As no number greater than unity will divide all of them without a remainder, they have no G. C. M.

(22)

$12000 × 4 = $48000

$12000 + $8000 = $20000 × 2 = $40000

$88000 = product of A's stock and time.

$25000 × 3 = $75000

$25000 — $10000 = $15000 × 3 = $45000

$120000 = product of B's stock and time.

$35000 × 2 = $70000

Continued on next page.)

(22 continued.)

$\frac{2}{7}$ of \$35000 = \$10000. \$35000 − \$10000 = \$25000 × 4 = 100000

$\underline{}$

\$170000

= product of C's stock and time.

\$88000 + \$120000 + \$170000 = \$378000 = sum of the products of stocks and times.

Then \$378000 : \$88000 :: \$15000 : $\frac{15000 \times 88000}{378000}$ = \$3492·06 = A's share.

\$378000 : \$170000 :: \$15000 : $\frac{15000 \times 170000}{378000}$ = \$6746·03 = C's share.

\$15000 − (\$3492·06 + \$6746·03) = \$4761·91 = B's share.

(23)

A's gain in 5 months = \$125 ∴ his gain for 9 months
= $1\frac{4}{5}$ × \$125.................................. = \$225

B's gain in 6 months = \$125 ∴ his gain for 9 months
= $1\frac{1}{2}$ × \$125.................................. = \$$187\frac{1}{2}$

C's gain in 9 months.................................. = \$125

Sum = \$$537\frac{1}{2}$

Then \$$537\frac{1}{2}$: \$225 :: \$400 : $\frac{400 \times 225}{537\frac{1}{2}}$ = \$$167\frac{19}{43}$ = A's stock.

\$$537\frac{1}{2}$: \$$187\frac{1}{2}$ :: \$400 : $\frac{400 \times 187\frac{1}{2}}{537\frac{1}{2}}$ = \$$139\frac{23}{43}$ = B's stock.

\$$537\frac{1}{2}$: \$125 :: \$400 : $\frac{400 \times 125}{537\frac{1}{2}}$ = \$$93\frac{1}{43}$ = C's stock.

(24)

$\frac{1}{8} + \frac{1}{6} + \frac{1}{10} + \frac{1}{12} = \frac{57}{120} = \frac{19}{40}$ = part of the cistern filled in one hour when the four pipes are left open.

$\frac{1}{6} + \frac{1}{5} + \frac{1}{4} + \frac{1}{3} = \frac{114}{120} = \frac{38}{40}$ = part of the cistern emptied in one hour when the four are left open.

$\frac{38}{40} - \frac{19}{40} = \frac{19}{40}$ = part of the cistern which remains filled after the eight pipes have been left open for one hour. And if $\frac{19}{40}$ of the cistern are emptied in one hour, it will take $1 \div \frac{19}{40} = 2\frac{2}{19}$ hours to empty the whole of it.

THIRD SERIES.

(26)

As often as the first receives 4 the second receives 3, therefore as often as the first receives 6 the second receives $4\frac{1}{2}$. Then $6 + 4\frac{1}{2} + 7 = 17\frac{1}{2}$.

$$17\tfrac{1}{2} : 6 :: 2310 : \frac{2310 \times 6}{17\frac{1}{2}} = 792 \text{ loaves} = \text{number the first receives.}$$

$$17\tfrac{1}{2} : 4\tfrac{1}{2} :: 2310 : \frac{2310 \times 4\frac{1}{2}}{17\frac{1}{2}} = 594 \quad " = " \quad \text{second} \quad "$$

$$17\tfrac{1}{2} : 7 :: 2310 : \frac{2310 \times 7}{17\frac{1}{2}} = 924 \quad " = " \quad \text{third} \quad "$$

(27)

To produce a mixture worth 8 cents a pound, we require 4 lbs. @ 12 cents, 4 @ 4 cents, 1 @ 5 cents, and 3 @ 9 cents, or 3 lbs @ 12 cents, 1 @ 4 cents, 4 @ 5 cents, and 4 @ 9 cents,

	lbs. lbs. lbs.		lbs. lbs. lbs.
Then	4 : 72 :: 4 : 72 lbs. @ 4 cts.	or	3 : 72 :: 1 : 24 lbs. @ 4 cts.
	4 : 72 :: 1 : 18 lbs. @ 5 cts.		3 : 72 :: 4 : 96 lbs. @ 5 cts.
	4 : 72 :: 3 : 54 lbs. @ 9 cts.		3 : 72 :: 4 : 96 lbs. @ 9 cts.

(28)

Here $A = \$4444{\cdot}44$, $r = {\cdot}0444$, and $t = 4{\cdot}3\frac{4}{9}$

$$\text{Then } P = \frac{A}{1 + rt} = \frac{\$4444{\cdot}44}{1 + ({\cdot}0444 \times 4{\cdot}3\frac{4}{9})} = \frac{\$4444{\cdot}44}{1{\cdot}19289\frac{1}{3}} = \$3725{\cdot}764.$$

(29)

$\$1{\cdot}00 - \$0{\cdot}0225 = \$0{\cdot}9775$. $\$23470 \div 0{\cdot}9775 = \$24010{\cdot}23$.

(30)

Here $A = \$7493{\cdot}47$, $r = {\cdot}07$, and $t = 8$.

$$\text{Then } P = \frac{A}{1 + rt} = \frac{7493{\cdot}47}{1 + ({\cdot}07 \times 8)} = \frac{7493{\cdot}47}{1{\cdot}56} = \$4803{\cdot}5064.$$

(31)

\$17460 ÷ 1·03125 = \$16930.909 = sum to be invested.
16930·909 ÷ 2·95 = 5739·29 yds. cloth.
16930·909 × ·02½ = \$423·27272 = ad valorem duty.
\$17460 + \$1347·90 + \$479·40 + \$169·83 + \$423·27272 = \$19880·40272 = whole cost.
\$25000 — \$19880·40272 = \$5119·59728 = whole gain.

Then \$19880·40272 : \$100 :: \$5119·59728 : $\frac{5119{\cdot}59728 \times 100}{19880{\cdot}40272}$ = 25·75 = 25¾ per cent.

(32)

V.	III.	VIII.	XII.
134234 =	21122021 =	12701 =	3281
5	3	8	12
8	7	10	38
5	3	8	12
44	22	87	464
5	3	8	12
222	68	696	5569 den.
5	3	8	
1113	206	5569 den.	
5	3		
5569 den.	618		
	3		
	1856		
	3		
	5569 den.		

(33)

[illegible] of 4½ of $\frac{9\frac{3}{4}}{\frac{13}{20}}$ of $\frac{1}{15}$ of $\frac{7}{8}$ of £43 18s. 11½d. £43 18s. 11½d. = \$175·79⅙.

(Continued on next page.)

(33 continued.)

$$\frac{3}{7}\text{ of }\frac{9}{2}\text{ of }\frac{\overset{\cancel{2}}{3\frac{3}{4}}}{\frac{1}{2}\frac{3}{0}}\text{ of }\frac{1}{15}\text{ of }\frac{7}{9}\text{ of }\$175{\cdot}79\tfrac{1}{6}, = \frac{3}{\cancel{7}}\text{ of }\frac{\cancel{9}}{2}\text{ of }\frac{\cancel{15}}{1}\text{ of }\frac{1}{\cancel{15}}\text{ of }\frac{\cancel{7}}{\cancel{9}}$$

of $175·79⅙ = $\frac{3}{2}$ of $175·79⅙ = $263·6875,

$$3\tfrac{8}{9}\text{ of }\frac{1}{17\frac{1}{2}}\text{ of }{\cdot}56\text{ of }1{\cdot}75\text{ of }6\tfrac{1}{2}\text{ times }\$97{\cdot}18 =$$

$$\tfrac{35}{9}\text{ of }\frac{1}{\frac{35}{2}}\text{ of }\tfrac{56}{100}\text{ of }\tfrac{175}{100}\text{ of }6\tfrac{1}{2}\text{ times }\$97{\cdot}18;\ 6\tfrac{1}{2}\text{ times }\$97{\cdot}18$$

= $631·67.

$$\frac{\cancel{35}}{9}\text{ of }\frac{\cancel{2}}{\cancel{35}}\text{ of }\frac{\overset{\overset{7}{\cancel{14}}}{\cancel{56}}}{\underset{\underset{25}{\cancel{50}}}{\cancel{100}}}\text{ of }\frac{\overset{7}{\cancel{175}}}{\underset{\cancel{25}}{\cancel{100}}}\text{ of }\$631{\cdot}67 = \frac{49}{9\times 25}\text{ of }\$631{\cdot}67$$

= $\frac{49}{225}$ of $631·67

$\frac{49}{225}$ of $631·67 = $137·5636.

Then $263.6875 — $137·5636 = $126·1239 = difference.

(34)

$\frac{1}{13} = 1 \div 13 \therefore \log.\ \frac{1}{13} = \log.\ 1 - \log.\ 13 = 0 - 1{\cdot}113943$
$= \bar{2}{\cdot}886057.$

$19{\cdot}5 = 3 \times 13 \times 5 \div 10 \therefore \log.\ 19{\cdot}5 = \log.\ 3 + \log.\ 13 + \log.\ 5 - \log.\ 10.$

log. 3 = 0·477121
log. 13 = 1·113943
log. 5 = log. 10 — log. 2 = 1 — 0·301030 ∴ log. 5 = 0.698970

Sum = 2·290034
From which take log. 10 = 1

Rem. = 1·290034
= log. 19·5.

$1125 = 5^3 \times 3^2 \therefore \log.\ 1125 = (\log.\ 5) \times 3 + (\log.\ 3) \times 2.$

log. 5 = 0·698970 × 3 = 2·096910
log. 3 = 0·477121 × 2 = 0·954242

Sum = 3·051152 = log. of 1125.

(Continued on next page.)

(34 continued.)

$28\cdot1\dot{6} = 28\frac{1}{6} = \frac{169}{6} = 13^2 \div 6$ ∴ log. $28\cdot1\dot{6}$ = (log. 13) × 2 − (log. 2 + log. 3.)

log. 13 = 1·113943 × 2 = 2·227886
(log. 2 + log. 3) = (0·301030 + 0·477121) = 0·778151
Diff. = 1·449735
= log. $28\cdot1\dot{6}$.

65000 = 13 × 5 × 1000 ∴ log. 65000 = log. 13 + log. 5 + log. 1000.

log. 13 = 1·113943
log. 5 = 0·698970
log. 1000 = 3
Sum = 4·812913 = log. of 65000.

log. ·0005 = log. 5 with characteristic changed to − 4
= $\bar{4}$·698970.

$152\cdot1 = 3^2 \times 13^2 \div 10$ ∴ log. 152·1 = (log. 3) × 2 + (log. 13) × 2 − log. 10.

log. 3 = 0·477121 × 2 = 0·954242
log. 13 = 1·113943 × 2 = 2·227886
Sum = 3·182128
From which take log. 10 = 1
Diff. = 2·182128 = log. 152·1

$8\cdot112 = 2^4 \times 13^2 \times 3 \div 1000$ ∴ log. 8·112 = (log. 2) × 4 + (log. 13) × 2 + log. 3 − log. 1000.

log. 2 = 0·301030 × 4 = 1·204120
log. 13 = 1·113943 × 2 = 2·227886
log. 3 = 0·477121
Sum = 3·909127
From which take log. 1000 = 3
Diff. = 0·909127 = log. 8·112.

(85)

$t^2 \times 300 = 21000$
$t \times 8 \times 30 = 1800$
$8^2 = 54$
22854

$t8^2 \times 300 = 2454000$
$t8 \times t \times 30 = 22800$
$t^2 = 84$
2476884

$t8t^2 \times 300 = 249961000$
$t8t \times 2 \times 30 = 54500$
$2^2 = 4$
$2499e5504$

XII.

871tet·72 (t8·t2
6e4
179tet
159768
20352720
1et372e4
517428000
4977ttt08
3e8391e4

(86)

$\frac{1}{6}+\frac{1}{12}+\frac{1}{7}+5$ years $=\frac{11}{28}$ of life time $+5$ years $=$ age at birth of son.

$\frac{28}{28}-(\frac{11}{28}+5)=\frac{17}{28}$ of his life time $-$ 5 years $=$ time he lived after birth of son.

$\frac{17}{28}$ of father's life time $-$ 5 years $-$ 4 years $=$ age of son $=\frac{1}{2}$ father's age.

$\frac{17}{28}$ of father's life time $-$ 9 years $=\frac{1}{2}$ father's age.

∴ 9 years is the difference between $\frac{17}{28}$ and $\frac{14}{28}$ of father's age.

∴ 9 years is equal to $\frac{3}{28}$ of father's age.

If 9 years is $\frac{3}{28}$ of his age, $\frac{1}{28}$ will be the $\frac{1}{3}$ of 9 which is 3 years.

If $\frac{1}{28}$ is 3 years, $\frac{28}{28}$ or the whole age will be $3 \times 28 = 84$ years.

Or by Position.

Assume 42 for father's age at death, the son's age $=21$.

$\frac{1}{6}+\frac{1}{12}+\frac{1}{7}+5=\frac{11}{28}+5$; $\frac{11}{28}$ of $42=16\frac{1}{2}$ and $16\frac{1}{2}+5=21\frac{1}{2}=$ age of father when son was born.

∴ he lived after birth of his son $42-21\frac{1}{2}=20\frac{1}{2}$ years.

(Continued on next page.)

(36 continued.)

By the question he lived 21 + 4 = 25 years.

The error 25 − 20½ = − 4½.

Assume 98 for father's age, then son's age = ½ of 98 = 49.
$\frac{1}{6} + \frac{1}{12} + \frac{1}{7} + 5 = \frac{11}{28} + 5$; $\frac{11}{28}$ of 98 = 38½, and 38½ + 5 = 43½
= age of father at birth of son.

∴ he lived after birth of his son 98 — 43½ = 54½ years.

But by the question he lived 49 + 4 years = 53 years.

Then 53 − 54½ = + 1½ = error.

Errors.

$$\begin{array}{lr} -4\tfrac{1}{2} \times 98 = & 441 \\ +1\tfrac{1}{2} \times 42 = & 63 \\ \hline \text{Sum} = 6 & 504 \end{array}$$

504 ÷ 6 = 84 = father's age.

(37)

m.	fur.	per.	yds.	ft.	in.		fur.	per.	yds.
63	3	7	3	2	7	÷	7	23	3½
8							40		
507							303		
40							5½		
20287							1518¾		
5½							151½		
101438							1670¼		
10143½							3		
111581½							5010¾		
3							12		
334746½							60129		
12									
4016965									

(Continued on next page.)

(37 continued.)

60129)4016965(66·80578 times
360774

409225
360774

484510
481032

347800
300645

471550
420903

506470
481032

(38)

6·3 ÷ ·000000274

274)6300000000(22992700·$\dot{7}$299270$\dot{0}$
548

820
548

2720
2466

2540
2466

740
548

1920
1918

2000
1918

820
548

2$\dot{7}$20
2466

2540
2466

740
548

1920
1918

200 remainder.

(39)

$\frac{7}{8}$ yds. : $6\frac{3}{11}$ yds. :: $\$\frac{13}{17}$: $\frac{8}{7} \times \frac{69}{11} \times \frac{13}{17} = \frac{7176}{1309} = \$5.482.$

(40)

$I = Prt. = \$4237{\cdot}71 \times {\cdot}065 \times 1{\cdot}67 = \$460{\cdot}0034205.$

(41)

$$t = \frac{A - P}{Pr} = \frac{\$1000 - \$674{\cdot}30}{\$674{\cdot}30 \times {\cdot}085} = \frac{325{\cdot}70}{57{\cdot}3155} = 5{\cdot}68258 \text{ years} =$$

5 years 8 months 5·7288 days.

(42)

By Table, page 260, the amount of $1 for 14 payments at 4 per cent. is $1·73168.

Then $1·73168 × 813·71 = $1409·0853328 = Amount.

Subtract 813·71

Difference = 595·3753328 = Interest.

(43)

$$\begin{array}{rcrcr}
\$300 & \times & 0 & = & 0 \\
700 & \times & 4 & = & 2800 \\
750 & \times & 7 & = & 5250 \\
850 & \times & 9 & = & 7650 \\
400 & \times & 13 & = & 5200 \\
1300 & \times & 19 & = & 24700 \\
\hline
4300 &) & & & 45600
\end{array}$$

(10 months $18\frac{6}{43}$ days.

4300

2600

30

78000 = days.

4300

35000

34400

600

(44)

23 per cent. of \$4200 = $\frac{23}{100}$ of 4200 = \$966·00, and \$4200 — \$966·00 = \$3234·00. E has half as much as A, B, C, and D together; therefore E has *one-third* of \$3234·00, which is \$1078·00.

Deducting E's share, \$1078, from \$3234, the whole sum to be divided, there remains \$2156 to be divided among A, B, C and D. Now D gets a certain amount; C gets \$42·11 more than D; B gets \$61·34 (42·11 + 19·23) more than D; and A gets \$78·44 (61·34 + 17·10) more than D. Together they get, then, *four times* D's share, together with \$42·11 + \$61·34 + \$78·44, or, in other words, four times D's share, together with \$181·89.

That is, four times D's share, together with \$181·89 is equal to \$2156.

Hence \$2156·00 — \$181·89 = \$1974·11 = four times D's share. Then \$1974·11 ÷ 4 = \$493·5275 = D's share.

Add 42·11

Sum \$535·6375 = C's share.
Add 19·23

Sum \$554·8675 = B's share.
Add 17·10

Sum \$571·9675 = A's share.

(45)

$$P=\frac{A}{1+rt}=\frac{\$3786{\cdot}80}{1+1{\cdot}76^{*}}=\frac{3786{\cdot}80}{2{\cdot}76}=\frac{378680}{276}=\$1372{\cdot}02898+$$

(46)

$$\frac{\left\{\left(3\tfrac{3}{7}-2\tfrac{7}{10}\right)\times\cdot\dot{4}\dot{6}\div\tfrac{2}{5}\text{ of }\cdot\dot{1}4285\dot{7}\right\}\div 8\tfrac{1}{2}\text{ times }\left(\tfrac{1}{2}+\tfrac{1}{7}+\tfrac{1}{5}-\tfrac{337}{2310}\right)}{\left\{\left(\cdot 7\dot{3}\times\cdot 123\dot{4}\dot{5}\div\tfrac{679}{750}\right)+\tfrac{3}{7}+9\tfrac{3}{5}+17\tfrac{4}{11}\right\}\div 27{\cdot}4\dot{9}22077\dot{7}}$$

$$=\frac{\left\{\left(3\tfrac{30}{70}-2\tfrac{49}{70}\right)\times\tfrac{46}{99}\div\tfrac{2}{5}\text{ of }\tfrac{1}{7}\right\}\div\tfrac{17}{2}\times\left(\tfrac{35}{70}+\tfrac{10}{70}+\tfrac{14}{70}-\tfrac{387}{2310}\right)}{\left\{\left(\tfrac{66}{90}\times\tfrac{12222}{99000}\div\tfrac{679}{7}\right)+\tfrac{3}{7}+9\tfrac{3}{5}+17\tfrac{4}{11}\right\}\div 27{\cdot}4\dot{9}22077\dot{7}}$$

(Continued on next page.)

$^{*}rt$ = ·16 × 11 = 1·76.

(46 continued.)

$$= \frac{\left(\frac{51}{70} \times \frac{46}{99} \times \frac{5}{2} \times \frac{7}{1}\right) \times \frac{2}{17} \times \frac{2310}{1610}}{\left\{\left(\frac{11}{15} \times \frac{679}{5500}\right) \times \frac{750}{679} + 27\frac{151}{385}\right\} \div 27\cdot4\dot{9}22077\dot{7}}$$

$$= \frac{\frac{391}{66} \times \frac{2}{17} \times \frac{2310}{1610}}{\left(\frac{1}{10} + 27\frac{151}{385}\right) \div 27\cdot4\dot{9}22077\dot{7}} = \frac{1}{27\frac{379}{770} \div 27\cdot4\dot{9}22077\dot{7}}$$

$$= \frac{1}{27.4\dot{9}22077\dot{7} \div 27\cdot4\dot{9}22077\dot{7}} = \frac{1}{1} = 1$$

(47)

312312302 quaternary = 224690 decimal scale.
2312132 quaternary = 11678 decimal scale.

Sum = 236368

4234 quinary = 569 decimal, and 569 × 23011 = 13093259.
236368 × 13093259 = 3094827443312.
555 + 444 + 333 + 222 + 111 senary = 2553 senary = 645 decimal.
3094827443312 — 645 = 3094827442667.
6542 septenary = 2333 decimal.
3094827442667 ÷ 2333 = $1326544124\frac{1375}{2333}$ den.

X.		VIII.
1326544124	=	11704272374
X.		VIII.
1375	=	2537
X.		VIII.
2333	=	4435
X.		VIII.

$\therefore 1326544124\frac{1375}{2333} = 11704272374\frac{2537}{4435}$.

(48)

$\cdot1 = \frac{1}{10}$ and $\left(\frac{1}{10}\right)^2 = \frac{1}{100} = 01$

$\cdot\dot{1} = \frac{1}{9}$ and $\left(\frac{1}{9}\right)^2 = \frac{1}{81} = \cdot\dot{0}1234567\dot{9}$.

FIFTH SERIES.

(50)

Assume 27 | $\cancel{2} \ldots \cancel{9} \ldots \cancel{16} \ldots \cancel{27} \ldots \overset{}{\underset{16}{\cancel{48}}}$ and $\underset{3}{\cancel{81}}$, strike out 2, 9 and 16 since they are contained as factors in the others.

The l. c. m. $= 27 \times 16 \times 3 = 1296$.

(51)

$$t = \frac{\log. n}{\log. (1+r)} = \frac{\log. 7}{\log. (1{\cdot}06)} = \frac{0{\cdot}845098}{0{\cdot}025306} = 33{\cdot}395 \text{ years.}$$

(52)

20 miles $=$ 1267200 inches; and 14 ft. 10 in. $=$ 178 inches.
$1267200 \div 178 = 7119\frac{9}{89}$ times.

(53)

$1749600 = 2^5 \times 3^7 \times 5^2$; increasing each index by unity and multiplying, we have $6 \times 8 \times 3 = 144$.

(54)

$$\tfrac{2}{3} \text{ of } \frac{96}{\frac{5}{6}} \div \frac{\frac{1}{2} \text{ of } 7}{\frac{3\frac{1}{4}}{2}} = \tfrac{2}{3} \times \frac{\frac{96}{1}}{\frac{5}{6}} \div \frac{\frac{7}{2}}{\frac{3\frac{1}{4}}{2}} = \tfrac{2}{3} \times \tfrac{576}{5} \div \frac{\frac{7}{2}}{\frac{13}{8}}$$

$$= \tfrac{2}{3} \times \tfrac{576}{5} \div \tfrac{28}{13} = \tfrac{2}{3} \times \tfrac{576}{5} \times \tfrac{13}{28} = 35\tfrac{23}{35}.$$

(55)

A can do the whole work in 12 days, therefore he can do $\frac{1}{12}$ in 1 day. A and B together can do the work in 5 days, therefore they can do $\frac{1}{5}$ in 1 day. Therefore B can do $\frac{1}{5} - \frac{1}{12} = \frac{7}{60}$ in 1 day, and he will require as many times 1 day to do the whole work as $\frac{7}{60}$ is contained times in 1, i. e. $1 \div \frac{7}{60} = \frac{60}{7}$ $= 8\frac{4}{7}$ days.

(56)

$P = \frac{A}{(1+r)^t}$; log. $P =$ log. A — log. $(1+r) \times t =$ log. 8899·77 — log. (1·06) × 22 = 3·949378 — 0·025306 × 22 = 3·949378 — 0·556732 = 3·392646, and log. 3·392646 = \$2469·71.

By Table, page 260, amount of \$1 at 6 per cent. for 22 payments = 3·60354.

Then \$8899·77 ÷ 3·60354 = \$2469·73 nearly.

(57)

Let the 1st number be 2. Then $2 \times 2 = 4$
$1\frac{1}{3} \times 3 = 4$
$10 - (2 + 1\frac{1}{3}) = 10 - 3\frac{1}{3} = 6\frac{2}{3} \times 4 = 26\frac{2}{3}$, but it should equal 4.

Therefore $26\frac{2}{3} - 4 = +\ 22\frac{2}{3}$ = error.

Let $1\frac{1}{2}$ be the 1st number; then $1\frac{1}{2} \times 2 = 3$
$1 \times 3 = 3$
$10 - (1\frac{1}{2} + 1) = 10 - 2\frac{1}{2} = 7\frac{1}{2} \times 4 = 30$, but it should = 3.

Therefore $30 - 3 = +27$ = error.

Errors.
$+\ 27 \times 2 = 54$
$+\ 22\frac{2}{3} \times 1\frac{1}{2} = 34$

Diff. $= 4\frac{1}{3}$ diff. $= 20$, and $20 \div 4\frac{1}{3} = 4\frac{8}{13}$ = 1st number.

$4\frac{8}{13} \times 2 = 9\frac{3}{13}$ = 1st product.

Second number $= 9\frac{3}{13} \div 3 = 3\frac{1}{13} \times 3 = 9\frac{3}{13}$ = 2nd product.

$10 - 7\frac{9}{13} = 2\frac{4}{13} \times 4 = 9\frac{3}{13}$ = 3rd product.

(58)

Suppose A has 40; then B has 110 — 40 = 70, and C has 130 — 70 = 60.

A and C together have 40 + 60 = 100, but it should be 120.

Therefore 100 — 120 = — 20 = error.

Suppose A has 80; then B has 110 — 80 = 30, and C has 130 — 30 = 100.

A and C together have 80 + 100 = 180, but they should have 120.

Therefore 180 — 120 = + 60 = error.

(Continued on next page.)

(58 continued.)

$$\begin{array}{rcccr} \text{Errors.} & & & & \\ +60 & \times & 40 & = & 2400 \\ -20 & \times & 80 & = & 1600 \\ \hline \text{Sum} = 80 & & & \text{Sum} = & 4000 \end{array}$$

$4000 \div 80 = 50 =$ number A has.

Then B has $110 - 50 = 60$, and C has $130 - 60 = 70$.

$$\frac{50 + 60 + 70}{3} = 60 = \text{each man's share when equally divided.}$$

(59)

Formula I, p. 333. $l = a + (n - 1)\,d = 7 + (47 - 1) \times 4$
$= 7 + (46 \times 4) = 7 + 184 = 191.$

Formula VI, p. 333. $s = \left\{2a + (n - 1)\,d\right\}\frac{n}{2}$

$$= \left\{2 \times 7 + (93 - 1) \times 4\right\} \tfrac{93}{2} = \left\{14 + (92 \times 4)\right\} \tfrac{93}{2}$$

$$= (14 + 368) \times \tfrac{93}{2} = \frac{382 \times 93}{2} = 17763.$$

(60)

$$t = \frac{\log. n}{\log. (1 + r)} = \frac{\log. 21}{\log. (1{\cdot}07)} = \frac{1{\cdot}322219}{0{\cdot}029384} = 44{\cdot}997 \text{ years.}$$

SIXTH SERIES.

(61)

B gets \$196·87 more than C, and A gets \$387 + \$196·87 = \$583·87 more than C, therefore together they get *three* times C's share, together with \$196·87 + \$583·87, i. e. three times C's share, together with \$780·74; but together they get \$3700.

Therefore \$3700 = three times C's share, together with \$780·74, or \$3700 — \$780·74 = \$2919·26 = three times C's share.

Hence \$2919·26 ÷ 3 = \$973·08$\frac{2}{3}$ = C's share.

$$\begin{array}{lr} \text{Add} & 196{\cdot}87 \\ \hline \text{Sum} = & \$1169{\cdot}95\tfrac{1}{3} = \text{B's share.} \\ \text{Add} & 387{\cdot}00 \\ \hline \text{Sum} = & \$1556{\cdot}95\tfrac{2}{3} = \text{A's share.} \end{array}$$

(62)

$5716 = 2^2 \times 1429$

$$\begin{array}{l} 1 \;..\; 2 \;..\; 4 \\ 1 \;..\; 1429 \\ \hline 1 \;..\; 2 \;..\; 4 \;..\; 1429 \;..\; 2858 \;..\; 5716 \end{array}$$

(63)

$$\frac{\{(17\tfrac{7}{12} - 10\tfrac{59}{60}) - (\dot{\cdot 4} + \tfrac{1}{6} + \dot{\cdot 9} - \tfrac{1}{2})\} \div (\cdot 8\dot{3}7\dot{8} \div \tfrac{1}{2} \text{ of } 31)}{\cdot 632\dot{2}63\dot{2} \times \tfrac{1}{2} \text{ of } 9\tfrac{1}{4} \div (\tfrac{1}{5} \text{ of } 4\tfrac{1}{9} \text{ of } \tfrac{1}{11} \text{ of } 85\tfrac{16}{37} \div 101)}$$

$$= \frac{6\tfrac{3}{5} - 1 \div (\tfrac{8370}{9990} \times \tfrac{2}{31})}{\tfrac{6322}{9999} \times \tfrac{1}{2} \times \tfrac{37}{4} \div (\tfrac{1}{5} \times \tfrac{37}{9} \times \tfrac{1}{11} \times \tfrac{3161}{37} \times \tfrac{1}{101})}$$

$$= \frac{5\tfrac{3}{5} \times \tfrac{9990}{8370} \times \tfrac{31}{2}}{\tfrac{6322}{9999} \times \tfrac{1}{2} \times \tfrac{37}{4} \times \tfrac{5}{1} \times \tfrac{9}{37} \times \tfrac{11}{1} \times \tfrac{37}{3161} \times \tfrac{101}{1}}$$

$$= \frac{\tfrac{28}{5} \times \tfrac{37}{31} \times \tfrac{31}{2}}{\tfrac{3161}{101} \times \tfrac{37}{4} \times \tfrac{5}{1} \times \tfrac{1}{37} \times \tfrac{37}{3161} \times \tfrac{101}{1}}$$

$$= \frac{\;\dfrac{14 \times 37}{5}\;}{\tfrac{1}{4} \times \tfrac{5}{1} \times \tfrac{37}{1}} = \frac{\;\dfrac{14 \times 37}{5}\;}{\dfrac{5 \times 37}{4}} = \tfrac{56}{25} = 2\tfrac{6}{25}.$$

(64)

Each child gets 1 child's share, ∴ 17 children get 17 shares.

Each woman gets *three* times a child's share, ∴ 4 women get 12 shares.

Each man gets *six* times a child's share, ∴ 3 men get 18 shares.

And together they get 47 times a child's share.

Therefore \$7200 ÷ 47 = \$153·19$\tfrac{7}{47}$ = a child's share.

\$153·19$\tfrac{7}{47}$ × 3 = \$459·57$\tfrac{21}{47}$,

\$153·19$\tfrac{7}{47}$ × 6 = \$919·14$\tfrac{42}{47}$.

(65)

$25400 = 2^3 \times 5^2 \times 127$. Adding unity to each index and multiplying the results, we get 4 × 3 × 2 = 24.

(66)

$$\tfrac{2}{3}\text{ of }4\tfrac{1}{2}\text{ of }\frac{9\frac{2}{7}}{1\frac{1}{4}}\text{ of }\tfrac{1}{6}\text{ of }£3\ 16s.\ 11\tfrac{1}{2}d. = \frac{\cancel{2}}{\cancel{3}}\times\frac{\overset{3}{\cancel{9}}}{\cancel{2}}\times\frac{\overset{5}{\cancel{65}}\times\overset{2}{\cancel{14}}}{\cancel{7}\times 11}\times\frac{\cancel{1}}{\cancel{6}}$$

$$\times\ \$15\cdot39\tfrac{1}{6} = 6\text{ times }\$15\cdot39\tfrac{1}{6} = \$92\cdot35.$$

$$\tfrac{3}{11}\text{ of }4\tfrac{3}{5}\text{ of }\frac{19\frac{1}{2}}{\frac{3\frac{1}{4}}{1\frac{3}{8}}}\text{ of }\tfrac{95}{117}\text{ of }\tfrac{11}{23}\text{ of }\cdot85\text{ of }\frac{1}{42\frac{1}{2}}\text{ of }\$1783$$

$$= \tfrac{3}{11}\times\tfrac{23}{5}\times\frac{\frac{39}{2}}{\frac{13}{4}}\times\tfrac{95}{117}\times\tfrac{11}{23}\times\tfrac{85}{100}\times\tfrac{2}{85}\text{ of }\$1783.$$

$$= \frac{\cancel{3}}{11}\times\frac{\cancel{23}}{\cancel{5}}\times\frac{\overset{2}{\cancel{78}}}{\cancel{13}}\times\frac{\overset{19}{\cancel{95}}}{\underset{39}{\cancel{117}}}\times\frac{11}{\cancel{23}}\times\frac{\cancel{85}}{\cancel{100}}\times\frac{2}{\cancel{85}}\times\frac{\overset{17\cdot83}{\cancel{1783}}}{1}$$

$$= \$17\cdot83\times4 = \$71\cdot32.\quad \$92\cdot35 - \$71\cdot32 = \$21\cdot03.$$

(67)

$7 : 13 = 7\div13 = \cdot538$
$9 : 16 = 9\div16 = \cdot562$
$8 : 15 = 8\div15 = \cdot533$
$10 : 19 = 10\div19 = \cdot526$

Therefore 9 : 16 is the greatest, and 10 : 19 is the least.

$$\text{Compound ratio} = \frac{7}{13}\times\frac{\overset{3}{\cancel{9}}}{\underset{2}{\cancel{16}}}\times\frac{\cancel{8}}{\underset{3}{\cancel{15}}}\times\frac{\overset{5}{\cancel{10}}}{19} = \frac{21}{247} = 21:247.$$

(68)

$$67\cdot4\dot{3}\dot{2} = 67\tfrac{428}{990} = \tfrac{66758}{990}\text{ and }7\cdot9\dot{0}3\dot{6} = 7\tfrac{9027}{9990} = \tfrac{78957}{9990}$$

$$\frac{66758}{990}\div\frac{78957}{9990} = \frac{66758}{\underset{11}{\cancel{990}}}\times\frac{\overset{111}{\cancel{9990}}}{78957} = \frac{7410138}{868527} = 8.5318452.$$

(69)

9 per. 9 yds. 7 ft. 120 in. = 365628 inches

[illegible] of [illegible] of [illegible] of 35 acres 2 roods = $\frac{3}{35}$ of 35 acres 2 roods = $\frac{3}{35}$ of 222678720 inches

$$\frac{365628}{\frac{3}{35}\text{ of }222678720} = \frac{2559396.}{133607232} = 0\cdot019156118.$$

(70)

Dissimilar.	Similar.
$17\cdot03\dot{4}\dot{2}$	$17\cdot034242\dot{4}\dot{2}$
$27\cdot063\dot{5}\dot{7}$	$27\cdot063575\dot{7}\dot{5}$
$98\cdot123\dot{4}5\dot{6}$	$98\cdot123456\dot{4}5\dot{6}$
$829\cdot6\dot{4}2\dot{3}$	$829\cdot642342\dot{3}4\dot{2}$
$986\cdot123\dot{4}29\dot{8}$	$986.123429\dot{8}42\dot{9}$
$9\cdot\dot{8}7634\dot{2}$	$9\cdot876342\dot{8}7634\dot{2}$
$813\cdot986423\dot{4}56\dot{7}$	$813\cdot986423\dot{4}56\dot{7}$

Similar and Coterminous.

$17\cdot034242\dot{4}24242424242\dot{2}$

$27\cdot063575\dot{7}57575757575\dot{5}$

$98\cdot123456\dot{4}56456456456\dot{6}$

$829\cdot642342\dot{3}42342342342\dot{2}$

$986\cdot123429\dot{8}42984298429\dot{9}$

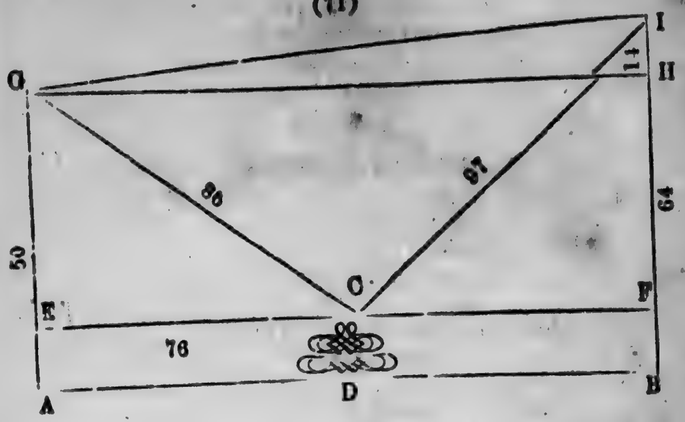
$9\cdot876342\dot{8}76342876342\dot{2}$

$813\cdot986423\dot{4}56745674567\dot{7}$

4 carried

$2781\cdot849813\dot{1}5668982995\dot{7}$

(71)



$$EG = \sqrt{86^2 - 76^2} = \sqrt{1620} = 40 \cdot 249 \text{ feet}$$

Height of Statue $CD = AG - EG = 50 - 40 \cdot 249 = 9 \cdot 751$ ft. $= BF$

$$FI = BI - BF = 64 - 9 \cdot 751 = 54 \cdot 249 \text{ feet}$$

$$CF = \sqrt{CI^2 - FI^2} = \sqrt{97^2 - 54 \cdot 249^2} = \sqrt{6466 \cdot 645999} = 80 \cdot 411 \text{ feet}$$

$GH = EF = EC + CF = 76 + 80 \cdot 411 = 156 \cdot 411$ feet and

$$HI = 64 - 50 = 14 \text{ feet}$$

$$GI = \sqrt{GH^2 + HI^2} = \sqrt{156 \cdot 411^2 + 14^2} = \sqrt{24660 \cdot 400921}$$

$$= 157 \cdot 036 \text{ feet.}$$

(72)

The mixture = spirits + water = $\frac{1}{2}$ of mixture + 25 gal. + $\frac{1}{3}$ of mixture − 5 gal. = $\frac{1}{2} + \frac{1}{3}$ + 20 gal. = $\frac{5}{6}$ + 20 gal. Then 20 gal. = $\frac{1}{6}$ of the mixture, and therefore the mixture contained 6 × 20 = 120 gal.

$$\left.\begin{array}{l} \text{Then } \tfrac{1}{2} \text{ of } 120 = 60 + 25 = 85 \text{ gal.} = \text{spirits} \\ \phantom{\text{Then }} \tfrac{1}{3} \text{ of } 120 = 40 - 5 = 35 \text{ gal.} = \text{water} \end{array}\right\}$$

SEVENTH SERIES.

(73)

401241·3424 (422·33
31
———
132) 412
314
———
1342) 4341
3234
———
13443) 110234
101434
———
140012) 330024
330024

(74)

Suppose father's age = 60, the son's age now = 60 ÷ 5 = 12, and son's age four years ago = 12 — 4 = 8. But the son's age four years ago should, by the question, have been 60 ÷ 7 = $8\frac{4}{7}$.

Therefore $8 - 8\frac{4}{7} = -\frac{4}{7}$ = error.

Suppose father's age = 35; then son's age now = 35 ÷ 5 = 7, and age four years ago = 7 — 4 = 3.

But son's age four years ago should, by question, have been 35 ÷ 7 = 5.

Therefore 3 — 5 = — 2 = error.

Errors.

$-2 \times 60 = 120$
$-\frac{4}{7} \times 35 = \ \ 20$
diff. $1\frac{3}{7}$ diff. = 100

$100 \div \frac{10}{7} = 70$ = father's and son's age = 70 ÷ 5 = 14.

(75)

$$\cdot 72\dot{3}4\dot{7} \div \cdot 00\dot{3}\dot{2} = \frac{72275}{99900} \div \frac{32}{9900} =$$

$$\frac{72275}{\underset{111}{\cancel{99900}}} \times \frac{\overset{11}{\cancel{9900}}}{32} = \frac{795025}{3552} = 223\cdot 82460\dot{5}8\dot{5}$$

(76)

Logarithm of 97294764·372 is 7·988089
7·988089 ÷ 11 = 0 726189
Log. 0·726189 = 5 32341 = 11th root of 97294764·372.

(77)

Assume 43½ for the greater number

$$7\tfrac{1}{4} : 3\tfrac{1}{2} :: 43\tfrac{1}{2} : \frac{43\frac{1}{2} \times 3\frac{1}{2}}{7\frac{1}{4}} = 21 \text{ the less}$$

43½ — 21 = 22½ but it should = 30
Therefore error = 22½ — 30 = — 7½.

Assume 72½ for the greater number

$$7\tfrac{1}{4} : 3\tfrac{1}{2} :: 72\tfrac{1}{2} : \frac{72\frac{1}{2} \times 3\frac{1}{2}}{7\frac{1}{4}} = 35 = \text{the less}$$

72½ — 35 = 37½, but it should = 30
Therefore error = 37½ — 30 = + 7½.

$$\begin{array}{ll} \text{Errors.} & \\ +\,7\tfrac{1}{2} \times 43\tfrac{1}{2} & = 326\tfrac{1}{4} \\ -\,7\tfrac{1}{2} \times 72\tfrac{1}{2} & = 543\tfrac{3}{4} \\ \hline \text{Sum} = 15 & \text{Sum} = 870 \end{array}$$

870 ÷ 15 = 58 greater

$$7\tfrac{1}{4} : 3\tfrac{1}{2} :: 58 : \frac{58 \times 3\frac{1}{2}}{7\frac{1}{4}} = 28 \text{ less.}$$

(78)

$$\begin{array}{r|ccccccc} \text{Assume } 35 & \cancel{35}, & 16, & 18, & \cancel{28}, & 62, & \cancel{63}, & \cancel{40} \\ \hline \text{Assume } 16 & & \cancel{16}, & \cancel{18} & \cancel{4} & \cancel{62} & 9 & \cancel{8} \\ \hline \text{Assume } 9 & & & \cancel{9} & & 31 & \cancel{9} & \\ \hline & & & & & 31 & & \end{array}$$

$l.\,c.\,m. = 35 \times 16 \times 9 \times 31 = 156240.$

(79)

Here $a = 1$, $d = 6$, $n = 101$,

$$s = \left\{2a+(n-1)d\right\}\frac{n}{2} = \left\{2\times 1+(101-1)\times 6\right\}\frac{101}{2}$$

$$= (2+600)\frac{101}{2} = \frac{602\times 101}{2} = 30401.$$

(80)

$$\frac{\overset{4}{\cancel{12}}}{7}\times\frac{\cancel{11}}{\underset{7}{\cancel{56}}}\times\frac{\overset{5}{\cancel{35}}}{\underset{11}{\cancel{121}}}\times\frac{11\frac{3}{7}}{\underset{\cancel{2}}{\cancel{22}}}\times\frac{\cancel{8}}{\cancel{42}}\times\frac{\cancel{4\frac{1}{2}}}{3}=\frac{11\frac{3}{7}\times 4\times 5}{7\times 7\times 11\times 3}=\frac{228\frac{4}{7}}{1617}=228\tfrac{4}{7} : 1617.$$

(82)

$$\frac{\left(\left\{(9\frac{1}{5}+4\frac{1}{12}+3\frac{1}{7}-16\frac{9}{35})\times\cdot\dot{5}\dot{4}\right\}\div 1\frac{1}{7}\right)\times 35 \text{ times } \cdot\dot{1}4285\dot{7}}{\left\{\cdot 0\dot{7}\times\cdot 24\dot{3}7\dot{8}\times(1\frac{1}{44}\times 4\frac{46}{451})\right\}\times(4\frac{3}{11}-2\frac{4}{17})}$$

$$=\frac{\left(\left\{(16\frac{229}{420}-16\frac{108}{420})\times\frac{54}{99}\right\}\div\frac{11}{7}\right)\times 35\times\frac{1}{7}}{\frac{8}{90}\times\frac{24354}{99900}\times\frac{45}{44}\times\frac{1850}{451}\times(4\frac{51}{187}-2\frac{44}{187})}$$

$$=\frac{\frac{121}{420}\times\frac{6}{11}\times\frac{7}{11}\times\frac{35}{1}\times\frac{1}{7}}{\frac{4}{45}\times\frac{451}{1850}\times\frac{45}{44}\times\frac{1850}{451}\times\frac{381}{187}}=\frac{\frac{1}{2}}{\frac{381}{187}}=\frac{187}{762}$$

(83)

Suppose the *hour* hand moves over 4 minutes, then since the minute hand moves 12 times as fast, it will have travelled over 48 minutes. But in order to overtake the hour hand, the minute hand must traverse the entire circle, 60 minutes, plus the 4 minutes we have supposed the hour hand to have moved forward, *i. e.* 64 minutes. Then 48 should equal 64, for we should find the same number by each process; 48 — 64 = — 16 error.

Suppose hour hand moves over 6 minutes, the minute hand moves over 6 × 12 = 72 minutes. But minute hand moves over 60 + 6 = 66 minutes.

Then 72 — 66 = + 6 error.

(Continued on next page.)

(83 continued.)

Errors.

$$\begin{array}{rl} -\ 16 \times 6 = & 96 \\ +\ \ 6 \times 4 = & 24 \\ \hline \text{Sum } 22 & \text{Sum } 120 \end{array}$$

$120 \div 22 = 5\frac{5}{11}$ min. = minutes passed over by the hour hand, hence space passed over by the minute hand $= 5\frac{5}{11} \times 12 = 65\frac{5}{11}$ min. = 1 hour $5\frac{5}{11}$ min. = time.

(84)

$\text{Log. } 5 = \text{log. } 10 - \text{log. } 2 = 1 - 0{\cdot}301030 = 0{\cdot}698970$

$3850000 = 5 \times 7 \times 11 \times 10000.$

$\therefore \text{Log. } 3850000 = \text{log. } 5 + \text{log. } 7 + \text{log. } 11 + \text{log. } 10000$
$= 0{\cdot}698970 + 0{\cdot}845098 + 1{\cdot}041393 + 4 = 6{\cdot}585461.$

$3181{\cdot}\dot{8}\dot{1} = 31{\cdot}\dot{8}\dot{1} \times 100 = 31\frac{9}{11} \times 100 = \frac{350}{11} \times 100.$

$\therefore \text{Log. } 3181{\cdot}\dot{8}\dot{1} = \text{log. } 5 + \text{log. } 7 + \text{log. } 1000 - \text{log. } 11$
$= 0{\cdot}698970 + 0{\cdot}845098 + 3 - 1{\cdot}041393 = 3{\cdot}502675$

$\cdot 0000154 = 2 \times 7 \times 11 \div 10000000$

$\therefore \text{Log. } \cdot 0000154 = \text{log. } 2 + \text{log. } 7 + \text{log. } 11 - \text{log. } 10000000$
$= 0{\cdot}301030 + 0{\cdot}845098 + 1{\cdot}041393 - 7 = \bar{5}.187521.$

$\text{Log. } \frac{1}{77} = \text{log. } 1 - (\text{log. } 7 + \text{log. } 11) = 0 - (0{\cdot}845098 + 1{\cdot}041393) = 0 - 1{\cdot}886491 = 2{\cdot}113509.$

$1{\cdot}\dot{5}7142\dot{8} = 1\frac{4}{7} = \frac{11}{7}.$

$\text{Log. } 1{\cdot}\dot{5}7142\dot{8} = \text{log. } 11 - \text{log. } 7 = 1{\cdot}041393 - 0{\cdot}845098 = 0{\cdot}196295$

$93{\cdot}17 = 9317 \div 100 = 11^3 \times 7 \div 100.$

$\therefore \text{Log. } 9317 = 3 \text{ times log. } 11 + \text{log. } 7 - \text{log. } 100 = 1.041393 \times 3 + 0{\cdot}845098 - 2 = 1{\cdot}969277.$

EIGHTH SERIES.

(85)

Simple Interest $= Prt = \$700 \times \cdot 045 \times 3 = \$94{\cdot}50.$

Amount Compound Interest $= P\,(1 + r)^t = \$700 \times (1.045)^3 = \$700 \times 1{\cdot}14116 = \$798{\cdot}814 - \$700 = \$98{\cdot}814 =$ Comp Int.

$\$98.814 - \$94{\cdot}50 = \$4{\cdot}314.$

(86)

X's gain $= \frac{1}{12}$, and Z's $= \frac{1}{2}$; ∴ Y's gain $= 1 - (\frac{1}{12} + \frac{1}{2})$
$= 1 - \frac{7}{12} = \frac{5}{12}$.

X's gain is $\frac{1}{12}$ for 3 months, therefore for 1 month it is $\frac{1}{36}$.
Y's gain is $\frac{5}{12}$ for 9 months, " " " $\frac{5}{108}$.
Z's gain is $\frac{1}{2}$ for 4 months, " " " $\frac{1}{8}$.
$\frac{1}{8} : \frac{1}{36} :: \$3024 : \$3024 \times \frac{1}{36} \times \frac{8}{1} = \$672$ = X's stock.
$\frac{1}{8} : \frac{5}{108} :: \$3024 : \$3024 \times \frac{5}{108} \times \frac{8}{1} = \$1120$ = Y's stock.

(87)

$$\frac{3}{8} \times \sqrt{1\tfrac{7}{9}} \div (1\tfrac{1}{2})^3 = \frac{3}{8} \times \sqrt{\tfrac{16}{9}} \div (\tfrac{3}{2})^3 = \frac{3}{8} \times \frac{4}{3} \times \frac{8}{27} = \frac{4}{27}.$$

(88)

			80677568161 (4321 cube rt
$4^2 = 16 \times 300$	=	4800	64
$4 \times 3 = 12 \times 30$	=	360	
3^2	=	9	16677
		5169	15507
$43^2 = 1849 \times 300$	=	554700	1170568
$43 \times 2 = 86 \times 30$	=	2580	
2^2	=	4	
		557284	1114568
$432^2 = 186624 \times 300$	=	55987200	56000161
$432 \times 1 = 432 \times 30$	=	12960	
1^2	=	1	
		56000161	56000161

(89)

$$7 = \left\{ 8 - 1 \begin{matrix} 3+4 \\ 1+6 \end{matrix} \right\} = 7$$

4 lbs. at 8d.
1 lb. at 4d. } Make a mixture of 6 lbs. at 7d.
1 lb. at 6d.

$$6 : 112 :: 4 : \frac{112 \times 4}{6} = 74\tfrac{2}{3} \text{ at 8d.}$$

(Continued on next page.)

(89 continued.)

$$6 : 112 :: 1 : \frac{112 \times 1}{6} = 18\tfrac{2}{3} \text{ at 4d.}$$

$$6 : 112 :: 1 : \frac{112 \times 1}{6} = 18\tfrac{2}{3} \text{ at 6d.}$$

(90)

Assume 40 as the sum of the three numbers.
Since 1st + 2nd + 3rd = 40,
And 1st + $\frac{1}{2}$ (2nd + 3rd) = 34 ∴ $\frac{1}{2}$ (2nd
÷ 3rd) = 6 .... ∴ 2nd + 3rd = 12
And 2nd + $\frac{1}{3}$ (1st + 3rd) = 34 ∴ $\frac{2}{3}$ (1st
+ 3rd) = 6 .... ∴ 1st + 3rd = 9
And 3rd + $\frac{1}{4}$ (1st + 2nd) = 34 ∴ $\frac{3}{4}$ (1st
+ 2nd) = 6 .... ∴ 1st + 2nd = 8

Adding, 2 × (1st + 2nd + 3rd) = 29
∴ 1st + 2nd + 3rd = $14\frac{1}{2}$.
But the sum should equal 40.
Hence $14\frac{1}{2} - 40 = -25\frac{1}{2}$.

Assume 48 as the sum of the three numbers.
Since 1st + 2nd + 3rd = 48.
And 1st + $\frac{1}{2}$ (2nd + 3rd) = 34 ∴ $\frac{1}{2}$ (2nd
+ 3rd) = 14 .... ∴ 2nd + 3rd = 28
And 2nd + $\frac{1}{3}$ (1st + 3rd) = 34 ∴ $\frac{2}{3}$ (1st
+ 3rd) = 14 .... ∴ 1st + 3rd = 21
And 3rd + $\frac{1}{4}$ (1st + 2nd) = 34 ∴ $\frac{3}{4}$ (1st
+ 2nd) = 14 .... ∴ 1st + 2nd = $18\frac{2}{3}$

Adding, 2 × (1st + 2nd + 3rd) = $67\frac{2}{3}$
∴ 1st + 2nd + 3rd = $33\frac{5}{6}$

But the sum should equal 48.
Hence $33\frac{5}{6} - 48 = -14\frac{1}{6}$ = error.

Errors.

$$-25\tfrac{1}{2} \times 48 = 1224$$
$$-14\tfrac{1}{6} \times 40 = 566\tfrac{2}{3}$$

Diff. = $11\frac{1}{3}$ Diff. = $657\frac{1}{3}$
$657\frac{1}{3} \div 11\frac{1}{3} = 58$ = the sum of the three numbers.

(Continued on next page.)

(90 continued.)

1st + $\frac{1}{2}$ (2nd + 3rd) = 34 ∴ $\frac{1}{2}$ (2nd + 3rd) = 58 — 34 = 24
∴ 2nd + 3rd = 48.
2nd + $\frac{1}{3}$ (1st + 3rd) = 34 ∴ $\frac{2}{3}$ (1st + 3rd) = 58 — 34 = 24
∴ 1st + 3rd = 36.
1st + 2nd + 3rd = 58, and 2nd + 3rd = 48 ∴ 1st = 10.
1st + 2nd + 3rd = 58, and 1st + 3rd = 36 ∴ 2nd = 22.
2nd + 3rd = 48, and 2nd = 22 ∴ 3rd = 26.

(91)

4 means + 2 extremes = 6 terms.

Formula IX, p. 333. $d = \frac{l - a}{n - 1} = \frac{40 - 1}{6 - 1} = \frac{39}{5} = 7\frac{4}{5}$.

1, $8\frac{4}{5}$, $16\frac{3}{5}$, $24\frac{2}{5}$. $32\frac{1}{5}$, 40.

(92)

$s = 1860040$, $l = 1240029$, and $r = 3$.
Formula XI, p. 340. $a = rl - (r - 1)\,s = 1240029 \times 3 - (2 \times 1860040) = 3720087 - 3720080 = 7$.

(93)

6 apples + 7 pears cost 33 pence ∴ 2 apples + $2\frac{1}{3}$ pears cost 11 pence.
10 apples + 8 pears cost 44 pence ∴ 2 apples + $1\frac{3}{5}$ pears cost $8\frac{4}{5}$ pence.
Subtract, and $2\frac{1}{3} - 1\frac{3}{5}$ pears cost 11d. — $8\frac{4}{5}$d.
That is, $\frac{11}{15}$ of a pear costs $2\frac{1}{5}$d.
If $\frac{11}{15}$ cost $\frac{11}{5}$d., $\frac{1}{15}$ will cost $\frac{1}{11}$ of $\frac{11}{5}$d., which is $\frac{1}{5}$d.
If $\frac{1}{15}$ cost $\frac{1}{5}$d, $\frac{15}{15}$ will cost $\frac{15}{5}$d. = 3d.
6 apples + 7 pears cost 33 pence, and 7 pears cost 21d. ∴ 6 apples cost 12d. and 1 apple costs 2d.

(94)

$$\frac{1}{2} \times \frac{3}{4} \times \frac{5}{9} \times \frac{\frac{57}{8}}{\frac{3}{2}} \times \frac{2}{3} \times \frac{4}{5} \times \frac{3}{4}$$

$$= \frac{1}{2} \times \frac{\cancel{3}}{4} \times \frac{\cancel{5}}{\underset{3}{\cancel{9}}} \times \frac{\overset{19}{\cancel{57}}}{\underset{\underset{2}{\cancel{6}}}{\cancel{12}}} \times \frac{\cancel{2}}{\cancel{3}} \times \frac{\cancel{4}}{\cancel{5}} \times \frac{\cancel{3}}{\cancel{4}} = \frac{19}{2 \times 4 \times 3 \times 2} = \frac{19}{48}$$

(95)

$10 = $\frac{1}{4}$ of 2nd rem. — $20 ∴ $\frac{3}{4}$ of 2nd rem. = $30 ∴ 2nd rem. = $40.

$40 = $\frac{1}{5}$ of 1st rem. — $30 ∴ $\frac{4}{5}$ of 1st rem. = $70 ∴ 1st rem. = $87·50.

$87·50 = $\frac{1}{2}$ of original sum—$50 ∴ $\frac{1}{2}$ of original sum = $137·50 ∴ original sum = $137·50 × 2 = $275.

(96)

$a = 60, n = 17$, and $d = 4$.

Formula VI, p. 333. $s = \left\{ 2a + (n-1)d \right\} \frac{n}{2}$

$$= \left\{ 2 \times 60 + (17-1) \times 4 \right\} \frac{17}{2} = (120 + 64) \times \frac{17}{2}$$

$$= \frac{184 \times 17}{2} = \$1564 = \text{sum received for 17 years.}$$

Formula I, p. 333. $l = a + (n-1)d = 60 + (17-1) \times 4 = 60 + 64 = \$124$ = wages for 17th year.

NINTH SERIES.

(98

£749 16s. 5¾d. = £749.823958 ; £1 sterling = $4 867
£749·823958 × 4·867 = $3649·3932.

(99)

2)177408

2)88704

2)44352

2)22176

2)11088

2)5544

2)2772

2)1386

3)693

3)231

7)77

11

$2^8 \times 3^2 \times 7 \times 11$.

(100)

Formula III, page 354, $r = \sqrt[t]{\frac{A}{P}} - 1 \therefore r + 1 = \sqrt[t]{\frac{A}{P}}$

$$\text{Log.}\ (r+1) = (\text{log.}\ A - \text{log.}\ P) \div t$$

That is, $\text{log.}\ (r+1) = (\text{log}\ 11111{\cdot}11 - \text{log.}\ 704) \div 11$
$= (4.045757 - 2{\cdot}847573) \div 11$
$= 1{\cdot}198184 \div 11 = 0{\cdot}108925$

Therefore $r + 1 =$ natural number corresponding to the logarithm 0·108925 which is 1·285.

Since $r + 1 = 1{\cdot}285$, $r = {\cdot}285 =$ rate per unit and rate per cent. $= {\cdot}285 \times 100 = 28\frac{1}{2}$.

(101)

If 9 be $\frac{1}{13}$, $\frac{13}{13}$ or the whole will equal $9 \times 13 = 117$.

(102)

3 gal. + 4 gal. + 7 gal. = 14 gal.

Hence 14 gal. : 292 gal. :: 3 gal. : $\dfrac{292 \times 3}{14} = 62\frac{4}{7}$ of 1st kind.

14 gal. : 292 gal. :: 4 gal. : $\dfrac{292 \times 4}{14} = 83\frac{3}{7}$ gal. of 2d. "

14 gal. : 292 gal. :: 7 gal. : $\dfrac{292 \times 7}{14} = 146$ gal. of 3d "

(103)

$£\frac{1}{2} + £\frac{1}{3} + £\frac{1}{4} + £\frac{1}{5} = £1\frac{17}{60}$

Then $£1\frac{17}{60} : £500 :: £\frac{1}{2} : £500 \times \frac{1}{2} \times \frac{60}{77} = \dfrac{£15000}{77}$

$= £194\ 16\text{s.}\ 1\frac{19}{77}\text{d.}$

$£1\frac{17}{60} : £500 :: £\frac{1}{3} : £500 \times \frac{1}{3} \times \frac{60}{77} = \dfrac{£10000}{77}$

$= £129\ 17\text{s.}\ 4\frac{64}{77}\text{d.}$

$£1\frac{17}{60} : £500 :: £\frac{1}{4} : £500 \times \frac{1}{4} \times \frac{60}{77} = \dfrac{£7500}{77}$

$= £97\ 8\text{s.}\ 0\frac{48}{77}\text{d.}$

$£1\frac{17}{60} : £500 :: £\frac{1}{5} : £500 \times \frac{1}{5} \times \frac{60}{77} = \dfrac{£6000}{77}$

$= £77\ 18\text{s.}\ 5\frac{23}{77}\text{d.}$

(104)

By Table, page 363, present value of annuity of $1 at 6 per cent. for 23 payments = $12·30338.

Hence present value of $100 = $12·30338 × 100 = $1230·338·

By Formula V, page 361, $v = \frac{a}{r}\left\{1 - \frac{1}{(1+r)^t}\right\}$

$$= \frac{100}{\cdot 06} \times \left(1 - \frac{1}{(1\cdot 06)^{23}}\right)^* = \frac{10000}{6} \times (1 - 0\cdot 261795)$$

$$= \frac{10000}{6} \times 0\cdot 738205 = \frac{738205}{6} = \$1230\cdot 34$$

(105)

Since each loses 1 hour per day for 24 days, the whole hours lost = 24 × 25.

Also, 5 men working 1 hour per day for 12 days make up 5 × 12 × 1 = 60 hours.

Hence they will each have to work as many hours per day as 60 hours is contained times in 24 × 25 hours, *i.e.* $\frac{24 \times 25}{60} = 10$ hours.

(106)

$a = 5$, $s = 161$ and $d = 6$

Then Formula II, p. 333, $l = -\tfrac{1}{2}d + \sqrt{2ds + (a - \tfrac{1}{2}d)^2} = -\tfrac{1}{2} \text{ of } 6 + \sqrt{2 \times 6 \times 161 + (5 - \tfrac{1}{2} \text{ of } 6)^2} = -3 + \sqrt{1932 + 4} = -3 + \sqrt{1936} = -3 + 44 = 41$ years.

(107)

$$6^3 : 10^3 :: 1 \text{ day} : \frac{10^3 \times 1}{6^3} = \frac{1000}{216} = 4\cdot 629 \text{ days.}$$

---

* $\text{Log.} \frac{1}{(1\cdot 06)^{23}} = \log. 1 - \log. 1\cdot 06 \times 23 = 0 - 0\cdot 025306 \times 23$

$= 0 - 0\cdot 582038 = \bar{1}\cdot 417962$

$\therefore \frac{1}{(1\cdot 06)^{23}}$ = natural number corresponding to the logarithm $\bar{1}\cdot 417962$, which is $0\cdot 261795$

(108)

For 12 months he was to receive £8 and a suit of clothes; for 7 months he received £3 13s. 4d. and the suit of clothes; ∴ for 5 months he would have received the difference between £8 and £3 13s. 4d., which is £5 6s. 8d.

Hence for 1 month he would have received £5 6s. 8d. ÷ 5, which is £1 1s. 4d., and hence his wages for the year would have been, in money alone, £1 1s. 4d. × 12, *i.e.*, £12 16s.

Therefore the suit of clothes was valued at £12 16s. — £8 = £4 16s.

TENTH SERIES.

(109)

$\frac{1}{2}+\frac{1}{3}+\frac{1}{4}=\frac{13}{12}$; if $\frac{13}{12}$ of a number $=48$, $\frac{1}{12}$ will $=48\div13=3\frac{9}{13}$.

If $3\frac{9}{13}=\frac{1}{12}$, $\frac{12}{12}$, or the whole number $=3\frac{9}{13}\times12=44\frac{4}{13}$.

(110)

$$6^3 : 8^3 :: 600 : \frac{600\times8^3}{6^3}=\frac{600\times512}{216}=1422\cdot2 \text{ lbs.}$$

(See Art. 33, sec. X.)

(111)

Part of ball remaining after 1st has taken off her share $=\frac{3}{4}$

Then whole ball : remainder :: cube of diameter of whole : cube of diameter of remainder

$1:\frac{3}{4}::5^3:x^3$ hence $x=\sqrt[3]{\frac{3}{4}\times125}=\sqrt[3]{\frac{375}{4}}=\sqrt[3]{93\cdot75}=4\cdot542$

∴ Part taken off by 1st $=5$ in. — $4\cdot542$ in. $=0\cdot458$ in.

After 2nd had taken off her portion $\frac{1}{2}$ of the ball remained.

$1:\frac{1}{2}::5^3:x^3$, hence $x=\sqrt[3]{\frac{1}{2}\times125}=\sqrt[3]{\frac{125}{2}}=\sqrt[3]{62\cdot5}=3\cdot968$ in.

∴ Part taken off by 2nd $=4\cdot542-3\cdot968=0\cdot574$ in.

After 3rd had taken off her share there remained $\frac{1}{4}$ of the ball.

$1:\frac{1}{4}::5^3:x^3$, hence $x=\sqrt[3]{\frac{1}{4}\times125}=\sqrt[3]{31\cdot25}=3\cdot149$ in.

∴ Part taken off by 3rd $=3\cdot968-3\cdot149=0\cdot819$ inches

Remainder $=3\cdot149=$ part taken off by 4th.

(112)

71214·43 ÷ 12·342 = 71214430 ÷ 12342

12342)71214430(5570·238552
62831
72734
62831
88033
87625
3070·0
2468·4
500·50
371·36
118·130
111·067
7·0520
6·2831
·65780
·62831
·028480
·024684
·003685

5570·238552(71·118 = sq. rt.
54
151) 170
151
1521) 18·23
15·21
15221) 3·0285
1·5221
152228)1·406452
1·360051
·036411

NOTE.—Unless the quotient is carried out to *six places* of decimals, i.e., twice as many as are required in the root, the last figure in the root will be 6 or 7.

(113)

1st
$60 × 48 = $2880 for 1 month
$800 × 43 = 34400 for 1 month
$1500 × 4 = 6000 for 1 month
Sum = $43280
= $43280 for 1 month.

2nd
$600 × 48 = $28800 for 1 month
$1800 × 42 = 75600 for 1 month
Sum = $104400
= $104400 for 1 month.

(Continued on next page.)

(113 continued).

$$\text{3rd}\left\{\begin{array}{r}
\$400 \times 48 = \$19200\\
\$500 \times 42 = 21000\\
\$500 \times 36 = 18000\\
\$500 \times 30 = 15000\\
\$500 \times 24 = 12000\\
\$500 \times 18 = 9000\\
\$500 \times 12 = 6000\\
\$500 \times 6 = 3000\\
\hline
\text{Sum} = \$103200
\end{array}\right\} = \$103200 \text{ for 1 month.}$$

$$\text{4th}\left\{\begin{array}{r}
\$900 \times 40 = \$36000\\
\$900 \times 34 = 30600\\
\$900 \times 28 = 25200\\
\$900 \times 22 = 19800\\
\$900 \times 16 = 14400\\
\$900 \times 10 = 9000\\
\$900 \times 4 = 3600\\
\hline
\text{Sum} = \$138600
\end{array}\right\} = \$138600 \text{ for 1 month.}$$

$$\begin{array}{r}
\$43280\\
104400\\
103200\\
138600\\
\hline
\end{array}$$

4 years at \$1·25 per day
= \$1·25 × 4 × 365 = \$1825 = share of 5th.

\$389480 for one month.

\$20000 — \$1825 = \$18175 = sum to be divided among the four.

\$389480 : \$18175 :: \$43280 : \$2019·651 = share of 1st.

\$389480 : \$18175 :: \$104400 : \$4871·803 = " 2nd.

\$389480 : \$18175 :: \$103200 : \$4815·805 = " 3rd.

\$389480 : \$18175 :: \$138600 : \$6467·739 = " 4th.

(114)

Simple Interest, formula IX, p. 248. $t = \dfrac{n-1}{r} = \dfrac{16-1}{\cdot 05} = \dfrac{15}{\cdot 05} = \dfrac{1500}{5} = 300$ years.

Compound Interest, formula V, p. 254. $t = \dfrac{\log. n}{\log. (1+r)}$

$$= \frac{\log. 16}{\log. 1\cdot 05} = \frac{1\cdot 204120}{0\cdot 021189} = \frac{1204120}{21189} = 56\cdot 827 \text{ years.}$$

(115)

For every \$1 the first gave, the second gave \$3, and the third \$6. \$1 + \$3 + \$6 = \$10.

Hence the 1st gave \$1, the second \$3, and the third \$6 as often as \$10 is contained times in \$9202; which is $920\frac{1}{5}$ times.

$\$1 \times 920\frac{1}{5} = \$920 \cdot 20$ = payment of 1st person.
$\$3 \times 920\frac{1}{5} = \$2760 \cdot 60 =$ " 2nd "
$\$6 \times 920\frac{1}{5} = \$5521 \cdot 20 =$ " 3rd "

(116)

$25 + 22 = 47 =$ whole number of men.

$165 \div 47 = 3\frac{24}{47} =$ acres cleared by each man.

$3\frac{24}{47} \times 22 = 77\frac{11}{47}$ acres = acres cleared by company of 22 men.

165 acres — $77\frac{11}{47}$ acres = $87\frac{36}{47}$ acres = acres cleared by company of 25 men.

1st company contains 3 more men than 2nd company and receives \$86 more.

Therefore \$86 pays 3 men. Hence each man gets $\$86 \div 3 = \$28 \cdot 66\frac{2}{3}$.

Each man clears $3\frac{24}{47}$ acres, and receives $\$28 \cdot 66\frac{2}{3}$ for it; therefore cost of 1 acre $= \$28 \cdot 66\frac{2}{3} \div 3\frac{24}{47} = \$8\frac{82}{495}$.

(117)

$15^2 = 225$; $346 - 225 = 121 =$ square of the less.
Hence less $= \sqrt{121} = 11$.

(118)

Formula V, page 248, $A = P\ (1 + rt) = \$1200 \times 1 \cdot 95 = \$2340 \cdot 00$.

(119)

$$\left.\begin{array}{rcr} 24 & : & 496 \\ 9 & : & 11 \\ 7 & : & 4 \\ 465 & : & 337\frac{1}{2} \\ 3\frac{1}{3} & : & 5\frac{3}{5} \\ 2\frac{1}{3} & : & 3\frac{1}{3} \end{array}\right| :: 5\frac{1}{2} : x$$

(Continued on next page.

(119 continued.)

$$\frac{\overset{\cancel{81}}{\overset{\cancel{162}}{\cancel{486}}}}{1}\times\frac{11}{1}\times\frac{4}{1}\times\frac{\overset{\cancel{3}}{\overset{\cancel{9}}{\overset{\cancel{27}}{\overset{1\cancel{135}}{\cancel{675}}}}}}{\cancel{2}}\times\frac{\overset{\cancel{2}}{\overset{\cancel{4}}{\cancel{28}}}}{\cancel{5}}\times\frac{\cancel{7}}{\cancel{2}}\times\frac{\cancel{11}}{\cancel{2}}\times\frac{1}{\underset{\cancel{8}}{\cancel{24}}}\times\frac{1}{\underset{\cancel{3}}{\cancel{9}}}\times\frac{1}{\cancel{7}}\times\frac{1}{\underset{\cancel{81}}{\underset{\cancel{99}}{\cancel{465}}}}$$

$$\times\frac{\cancel{9}}{\cancel{11}}\times\frac{\cancel{9}}{\cancel{7}}=11\times4\times3=132 \text{ days.}$$

(120)

$$\begin{aligned}
A+B+C\phantom{{}+D} &= \tfrac{47}{60}\\
B+C+D &= \tfrac{37}{60}\\
A\phantom{{}+B}+C+D &= \tfrac{42}{60}\\
A+B\phantom{{}+C}+D &= \tfrac{45}{60}\\
\hline
3A+3B+3C+3D &= \tfrac{171}{60}\\
\therefore A+B+C+D &= \tfrac{57}{60}
\end{aligned}$$

$$\begin{aligned}
A+B+C+D &= \tfrac{57}{60}\\
A+B+C\phantom{{}+D} &= \tfrac{47}{60}\\
\therefore D &= \tfrac{10}{60}=\tfrac{1}{6}
\end{aligned}$$

$$\begin{aligned}
A+B+C+D &= \tfrac{57}{60}\\
B+C+D &= \tfrac{37}{60}\\
\therefore A &= \tfrac{20}{60}=\tfrac{1}{3}
\end{aligned}$$

$$\begin{aligned}
A+B+C+D &= \tfrac{57}{60}\\
A\phantom{{}+B}+C+D &= \tfrac{42}{60}\\
\therefore B &= \tfrac{15}{60}=\tfrac{1}{4}
\end{aligned}$$

$$\begin{aligned}
A+B+C+D &= \tfrac{57}{60}\\
A+B\phantom{{}+C}+D &= \tfrac{45}{60}\\
\therefore C &= \tfrac{12}{60}=\tfrac{1}{5}
\end{aligned}$$

$\frac{57}{60}:\frac{60}{60}::\frac{1}{6}:\frac{1}{6}\times\frac{60}{60}\times\frac{60}{57}=\frac{10}{57}=$ D's true share which is therefore $=\frac{10}{57}$ of \$6213 = \$1090.

$\frac{57}{60}:\frac{60}{60}::\frac{1}{3}:\frac{1}{3}\times\frac{60}{60}\times\frac{60}{57}=\frac{20}{57}=$ A's true share which is therefore $=\frac{20}{57}$ of \$6213 = \$2180.

$\frac{57}{60}:\frac{60}{60}::\frac{1}{4}:\frac{1}{4}\times\frac{60}{60}\times\frac{60}{57}=\frac{15}{57}=$ B's true share which is therefore $=\frac{15}{57}$ of \$6213 = \$1635.

$\frac{57}{60}:\frac{60}{60}::\frac{1}{5}:\frac{1}{5}\times\frac{60}{60}\times\frac{60}{57}=\frac{12}{57}=$ C's true share which is therefore $=\frac{12}{57}$ of \$6213 = \$1308.

ELEVENTH SERIES.

(121

$\dot{7}=\frac{7}{9}$; $\cdot\dot{8}\dot{3}=\frac{83}{99}$; $\cdot\dot{7}2\dot{7}=\frac{727}{999}$; $\cdot9\dot{1}32\dot{5}=\frac{91325-9}{99990}=\frac{91316}{99990}=\frac{45658}{49995}$.

$8\cdot67\dot{1}34\dot{7}=8\frac{671347-67}{999900}=8\frac{671280}{999900}=8\frac{11188}{16665}$.

(122)

713 *unden.* = 861 *den.*; 291 *unden.* = 342 *den.*; 3*t*1 *unden.* = 474 *den.*

Then $713\dfrac{291}{3t1}$ *unden.* = $861\frac{342}{474}$ *den.* = $861\frac{57}{79}$ *den.*

12123 *quat* = 411 *den*; 11223 *quat.* = 363 *den.*; 100000 *quat.* = 1024 *den.*

Then $12123\frac{11223}{100000}=411\frac{363}{1024}$ *den.*

(123)

$3\frac{3}{8}$ of $2\frac{1}{5}$ of $7\frac{11}{20}$ of £1 = $\frac{27}{8}$ of $\frac{11}{5}$ of $\frac{151}{20}$ of £1

= £$\frac{44847}{800}$........................ = £56 1 $2\frac{1}{10}$

$9\frac{3}{7}$ of $3\frac{8}{9}$ of 1s. = $\frac{66}{7}$ of $\frac{35}{9}$ of 1s. = $\frac{110}{3}$s ... = 1 16 8

$8\frac{1}{4}$ of $4\frac{1}{8}$ of 1d. = $\frac{33}{4}$ of $\frac{33}{8}$ of 1d. = $\frac{1089}{32}$d... = 0 2 $10\frac{1}{32}$

Sum = £58 0 $8\frac{21}{160}$

$\frac{11}{12}$ of $\frac{5}{14}$ of $\frac{3}{8}$ of $3\frac{1}{2}$d. = $\frac{11}{12}\times\frac{5}{14}\times\frac{3}{8}\times\frac{7}{2}=\frac{55}{128}$d.

£58 0s. $8\frac{21}{160}$d. = $\frac{2228501}{160}$d.

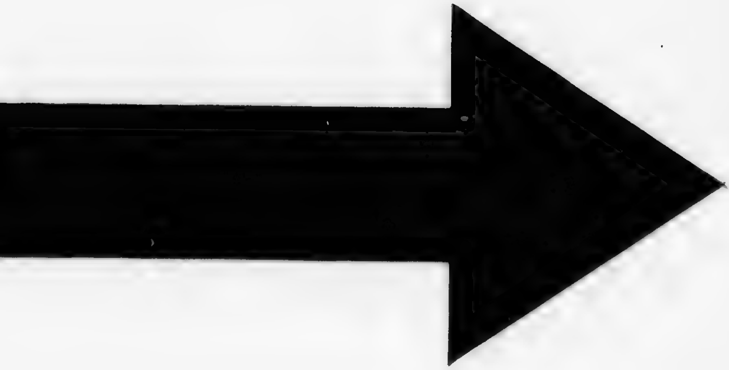
$\frac{2228501}{160}\div\frac{55}{128}=\frac{2228501}{160}\times\frac{128}{55}=\frac{202591}{5}\times\frac{4}{5}=\frac{810364}{25}$

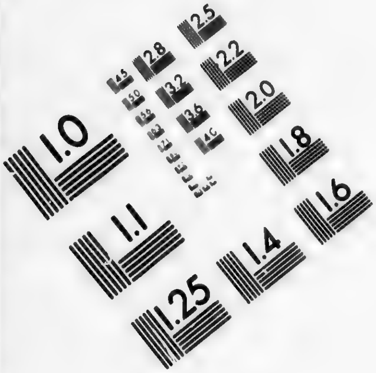
= 32414·56.

(124)

$$\left.\begin{array}{rcl} 24 & : & 90 \\ 2\frac{1}{2} & : & 4\frac{1}{5} \\ 12\frac{1}{2} & : & 9\frac{2}{3} \\ 4\frac{7}{8} & : & 4\frac{1}{7} \\ 8\frac{1}{5} & : & 2\frac{1}{4} \end{array}\right| :: 139\tfrac{1}{4} : x$$

(Continued on next page.)





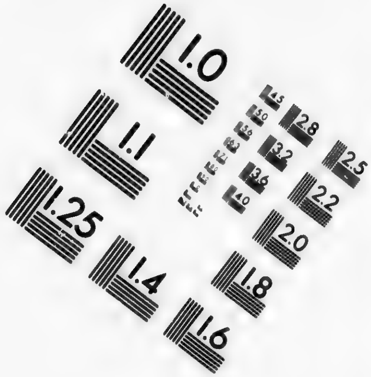
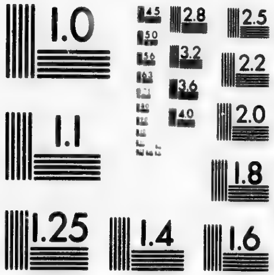
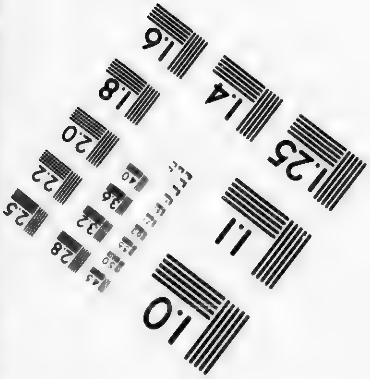
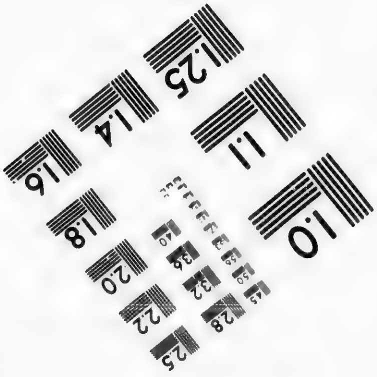


IMAGE EVALUATION TEST TARGET (MT-3)







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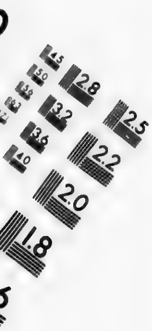

4.5
5.0
2.8
2.5
3.2
3.6
2.2
4.0
2.0
1.8

1.1
1.0

(124 continued.)

$$\frac{\overset{9}{\overset{\cancel{18}}{\cancel{90}}}}{1}\times\frac{\overset{7}{\cancel{21}}}{\cancel{5}}\times\frac{29}{\cancel{8}}\times\frac{\overset{\cancel{9}}{\cancel{9}}}{\cancel{2}}\times\frac{\cancel{5}}{2}\times\frac{\overset{43}{\cancel{559}}}{4}\times\frac{1}{\underset{\cancel{8}}{\cancel{24}}}\times\frac{\cancel{2}}{5}\times\frac{\cancel{2}}{\underset{\cancel{5}}{\cancel{25}}}\times\frac{\cancel{2}}{\underset{\cancel{16}}{\cancel{32}}}$$

$$\times\frac{\cancel{5}}{\underset{4}{\underset{\cancel{8}}{\cancel{16}}}}=\frac{9\times7\times29\times43}{2\times4\times5\times4}=\frac{78561}{160}=491\tfrac{1}{160}.$$

(125)

\$182 is $\frac{91}{100}$ of buying price ∴ \$182 ÷ 91 = \$2 = $\frac{1}{100}$ of buying price ∴ buying price = \$2 × 100 = \$200.

To realize a profit of 7 per cent., he must receive \$1·07 for every \$1 the goods cost; but they cost him \$200, therefore he must sell for \$1·07 × 200 = \$214.

(126)

Simple Interest $t = \dfrac{n-1}{r} = \dfrac{11\frac{1}{2}-1}{.06} = \dfrac{10\cdot5}{.06} = \dfrac{1050}{6}$

$= 175$ years.

Compound Interest $t = \dfrac{\log. n}{\log. (1+r)} = \dfrac{\log. 11\frac{1}{2}}{\log. 1.06} = \dfrac{1.060698}{0.025306}$

$= \dfrac{1060698}{25306} = 41.914$ years.

(127)

An acre contains 4 roods = 160 sq. perches.

∴ 160 ÷ $15\frac{1}{2}$ = $10\frac{10}{31}$ perches = length.

(128)

35 yards = 32 metres ∴ 1 yd. = $\frac{32}{35}$ of a metre.

$69\frac{1}{22}$ miles = $69\frac{1}{22} \times \frac{1760}{1}$ yards = $69\frac{1}{22} \times \frac{1760}{1} \times \frac{32}{35}$ metres

$$=\frac{\overset{217}{\cancel{1519}}}{\cancel{22}}\times\frac{\overset{16}{\overset{\cancel{80}}{\cancel{1760}}}}{1}\times\frac{32}{\underset{1}{\cancel{35}}}=217\times16\times32=111104 \text{ metres.}$$

(129)

7 means + 2 extremes = 9 terms.

Formula XIII, p. 340. $r = \left(\frac{l}{a}\right)^{\frac{1}{n-1}} = \left(\frac{19683}{3}\right)^{\frac{1}{8}} = \left(6561\right)^{\frac{1}{8}} = 3$

Hence means are 9, 27, 81, 243, 729, 2187, and 6561.

(130)

Formula XXI, p. 344. $s = \frac{a}{1-r} = \frac{7}{1-\frac{1}{4}} = \frac{7}{\frac{3}{4}} = \frac{28}{3} = 9\frac{1}{3}$.

(131)

Part remaining after 1st has received his share $= \frac{3}{4}$.

$1 : \frac{3}{4} :: 60^2 : x^2$; whence $x = \sqrt{3600 \times \frac{3}{4}} = \sqrt{900 \times 3}$
$= 30\sqrt{3} = 1{\cdot}732 \times 30 = 51{\cdot}96$ inches.

Hence 1st ground off $60 - 51{\cdot}96 = 8{\cdot}04$ inches.

Part remaining after 2nd had taken off his share $= \frac{1}{2}$.

$1 : \frac{1}{2} :: 60^2 : x^2$; whence $x = \sqrt{3600 \times \frac{1}{2}} = 30\sqrt{2}$
$= 1{\cdot}4142 \times 30 = 42{\cdot}426$.

Hence 2nd ground off $51{\cdot}96 - 42{\cdot}426 = 9{\cdot}534$ inches.

Part remaining after the 3rd had taken off his share $= \frac{1}{4}$.

$1 : \frac{1}{4} :: 60^2 : x^2$; whence $x = \sqrt{3600 \times \frac{1}{4}} = \sqrt{900} = 30$ inches.

Hence 3rd ground off $42{\cdot}426 - 30$ inches $= 12{\cdot}426$ inches, and the 4th ground off remaining 30 inches.

(132)

1 guinea = 21s.
1 half guinea = $10\frac{1}{2}$s.
1 crown = 5s.
1 half crown = $2\frac{1}{2}$s.
1 shilling = 1s.
Sixpence = $\frac{1}{2}$s.
Sum = $40\frac{1}{2}$s.

100 guineas = 2100 shillings.

$2100 \div 40\frac{1}{2} = 51$ times and remainder, 69 half-shillings,

69 half-shil. $= \frac{69}{2}$s. $= £\frac{69}{40} = 1\frac{29}{40}$.

TWELFTH SERIES.

(133)

$$\frac{3}{11}\text{ of }\frac{2}{9}\text{ of }\frac{4}{17}=\frac{8}{561};\ \frac{2\frac{1}{2}}{4\frac{1}{4}}\text{ of }\frac{2}{5}=\frac{10}{17}\text{ of }\frac{2}{5}=\frac{4}{17}.$$

$$\frac{8}{561}:\frac{4}{17}::\$12\tfrac{4}{33}:\$12\tfrac{4}{33}\times\frac{4}{17}\times\frac{561}{8}=\frac{200}{\cancel{33}}\times\frac{\cancel{4}}{\cancel{17}}\times\frac{\overset{\cancel{33}}{\cancel{561}}}{\underset{2}{\cancel{8}}}$$

$$=\$200.$$

(134)

By Formula III, page 354, $r=\sqrt[t]{\frac{A}{P}}-1\ \therefore\ r+1=\sqrt[t]{\frac{A}{P}}$

$$\therefore \text{Log. }(1+r)=(\text{log. }A-\text{log. }P)\div t$$
$$=(\text{log. }1679{\cdot}40-\text{log. }700{\cdot}90)\div 5$$
$$=(3{\cdot}225154-2{\cdot}845656)\div 5.$$
$$=0{\cdot}379498\div 5=0{\cdot}075899.$$

$\therefore 1+r=$ nat. num. corresponding to the logarithm 0·075899 which is 1·19, $\therefore r=\cdot 19=$ rate per unit, and hence rate per cent. = 19.

(135)

Having paid 10 per cent. he had 90 per cent. remaining.

$\frac{90}{100}$ or $\frac{9}{10}$ of his salary = \$1250, $\therefore \frac{1}{10}=\frac{1250}{9}=\$138\frac{8}{9}$.

If $\$138\frac{8}{9}=\frac{1}{10}$, the whole $=\$138\frac{8}{9}\times 10=\$1388{\cdot}888$.

(136)

21 children receive 21 times a child's share
21 women " 42 " "
21 men " 63 " "
Together they receive 126 " "

£3 13s. 6d. ÷ 126 = 7d. = a child's share.
7d. × 2 = 1s. 2d. = a woman's share.
7d. + 1s. 2d. = 1s. 9d. = a man's share.

(137)

A gets 1 time A's share
B " 1 " A's "
C " 2 " A's "
D " 4 " A's "

Together they get 8 times A's share.
$\$200 \div 8 = \$25 =$ A's share ; $\$25 =$ B's share.
$\$25 + \$25 = \$50 =$ C's share ; $\$25 + \$25 + \$50 = \$100$ $=$ D's share.

(138)

$$\sqrt[3]{\tfrac{2}{3}} = \tfrac{1}{3}\sqrt[3]{18} = \tfrac{1}{3} \text{ of } 2{\cdot}62074 = {\cdot}87358$$
$$\sqrt{\tfrac{2}{3}} = \tfrac{1}{3}\sqrt{6} = \tfrac{1}{3} \text{ of } 2{\cdot}44948 = {\cdot}81649$$
$$\text{Difference} = {\cdot}05709$$

(139)

$\frac{3872}{92807}$ when each term is divided by 121, becomes $\frac{32}{767}$.
$17\frac{5}{12} + \frac{4}{15} + 144\frac{11}{21} = 161 + \frac{5}{12} + \frac{4}{15} + \frac{11}{21} = 161 + \frac{175}{420} + \frac{112}{420}$ $+ \frac{220}{420} = 161 + \frac{507}{420} = 161 + 1\frac{87}{420} = 162\frac{87}{420} = 162\frac{29}{140}$.
$2\frac{13}{35} - \frac{7}{25} = 2\frac{65}{175} - \frac{49}{175} = 1\frac{240}{175} - \frac{49}{175} = 1\frac{191}{175}$.

$$\frac{3}{\cancel{4}} \text{ of } \frac{6}{\cancel{7}} \text{ of } \frac{\cancel{4}}{\cancel{15}} \text{ of } \frac{\cancel{15}}{11} \text{ of } \frac{\overset{3}{\cancel{21}}}{23} = \frac{54}{253}.$$

$6347 \div 2\frac{3}{4} = \frac{6347}{1} \div \frac{11}{4} = \frac{6347}{1} \times \frac{4}{11} = 2308.$

(140)

$$\begin{array}{r|l}
 & 884736 \;(96 = \text{cube root.} \\
 & 729 \\
 \cline{2-2}
9^2 = 81 \times 300 \quad = 24300 & 155736 \\
9 \times 6 = 54 \times 30 = 1620 & \\
6^2 = 36 & \\
\cline{1-1}
25956 & 155736 \\
 \cline{2-2}
\end{array}$$

$95951\frac{161}{625} = 95951{\cdot}2576.$

(Continued on next page.)

(140 continued,)

```
       ˙ ˙   ˙ ˙
       95951·2576 (309·76 = square root.
       9
       ────
609)    5951                 ˙  ˙
        5481                 309·76 (17·6 = 17⅗ = fourth root.
        ──────               1
618·7)470·25                 ──
       433·09             27)209
       ──────                189
619·46)37·1676               ──
       37·1676          34·6)2076
       ───────               2076
```

(141)

250
300
400
500

$$1450:250::\$520:\frac{\$520\times250}{1450}=\$89\tfrac{19}{29}=\text{contrib. on 1st village.}$$

$$1450:300::\$520:\frac{\$520\times300}{1450}=\$107\tfrac{17}{29}=\text{“ \quad 2nd \quad ”}$$

$$1450:400::\$520:\frac{\$520\times400}{1450}=\$143\tfrac{13}{29}=\text{“ \quad 3rd \quad ”}$$

$$1450:500::\$520:\frac{\$520\times500}{1450}=\$179\tfrac{9}{29}=\text{“ \quad 4th \quad ”}$$

(142)

By Table on p. 362, the amount of $1 for 34 payments at 3 per cent. = $57·73018.

$$\$57{\cdot}73018\times260=\$15009{\cdot}84.$$

By Formula I, page 361, $A=\dfrac{a\left\{(1+r)^t-1\right\}}{r}$

$$=\frac{a}{r}\left\{(1+r)^t-1\right\}=\frac{260}{\cdot03}\left\{(1{\cdot}03)^{34}-1\right\}$$

$$=\frac{26000}{3}\times(2{\cdot}731855-1)=\frac{26000\times1{\cdot}731855}{3}=\$15009{\cdot}41$$

(143)

By Formula IX, p. 333, $d = \dfrac{l - a}{n - 1} = \dfrac{79 - 2}{6 - 1} = \dfrac{77}{5} = 15\frac{2}{5}$.

Hence the series is 2, $17\frac{2}{5}$, $32\frac{4}{5}$, $48\frac{1}{5}$, $63\frac{3}{5}$, and 79.

Formula I, p. 333. $l = a + (n - 1)\,d = 3 + (9 - 1) \times 4$
$= 3 + (8 \times 4) = 3 + 32 = 35$.

Formula VI, p. 333. $s = \left\{ 2a + (n - 1)d \right\} \dfrac{n}{2}$

$$= \left\{ 2 \times 3 + (207 - 1) \times 4 \right\} \frac{207}{2} = \left\{ 6 + (206 \times 4) \right\} \frac{207}{2}$$

$$= (6 + 824) \times \frac{207}{2} = \frac{830 \times 207}{2} = 85905.$$

(144)

B travels 4 miles per day faster than A, and will therefore gain the circumference of the island in $\frac{73}{4} = 18\frac{1}{4}$ days.

C travels 10 miles per day faster than A, and will therefore gain the whole circumference of the island in $\frac{73}{10} = 7\frac{3}{10}$ days.

Now B cannot be with A except at the end of $18\frac{1}{4}$ days or twice $18\frac{1}{4}$ days, or three times $18\frac{1}{4}$ days, or some other multiple of $18\frac{1}{4}$ days.

Similarly C cannot be with A except at the end of $7\frac{3}{10}$ days, or of some other multiple of $7\frac{3}{10}$ days.

Therefore C and B will both be with A for the first time after the lapse of a number of days expressed by the least common multiple of $18\frac{1}{4}$ and $7\frac{3}{10}$.

The greatest common factor of $18\frac{1}{4}$ and $7\frac{3}{10}$ is $3\frac{13}{20}$.

Hence the l. c. m. of $7\frac{3}{10}$ and $18\frac{1}{4}$ is $\dfrac{7\frac{3}{10} \times 18\frac{1}{4}}{3\frac{13}{20}} = 36\frac{1}{2}$ = number of days when A, B, and C will first be together.

ARITHMETICAL RECREATIONS.

1. The third of 6 = 2, and the fourth of 20 = 5.
Then if 2 becomes 3, what should 5 become? Evidently 7½. *Ans.*

or

$$\left.\begin{matrix}6 : 20\\ \frac{1}{3} : \frac{1}{4}\end{matrix}\right\} :: 3 : x = \frac{3 \times 20 \times \frac{1}{4}}{6 \times \frac{1}{3}} = 7\tfrac{1}{2}.$$

2. The half of 5 = 2½; then if 7 becomes 2½, what will 11 become?

$$\frac{2\frac{1}{2} \times 11}{7} = \tfrac{55}{14}.$$ Lastly, what part of 9 is $\frac{55}{14}$?

$$\frac{\frac{55}{14}}{9} = \tfrac{55}{126}. \quad \textit{Ans.}$$

or

$$\left.\begin{matrix}9 : 5\\ 7 : 11\end{matrix}\right\} :: \tfrac{1}{2} : x = \frac{\frac{1}{2} \times 5 \times 11}{9 \times 7} = \frac{\frac{55}{2}}{\frac{63}{1}} = \tfrac{55}{126}. \quad \textit{Ans.}$$

3. $99\frac{9}{9}$.

4. ⅓ of 2d. = ⅔d. Then ⅔d. is what part of 3d.? *Ans.* $\frac{2}{9}$.

5. 1½d. for a herring and a half is at the rate of 1d. per herring; hence 11 herrings will cost 11d.

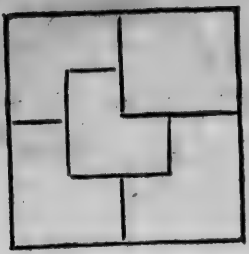
6. 12 apples = 21 pears = 7 cents.
If 12 apples cost 7 cents, what will 100 apples cost?

$$12 : 100 :: 7 : \frac{100 \times 7}{12} = 58\tfrac{1}{3} \text{ cents.}$$

7. If 5 is $\frac{3}{7}$ of a certain number, $\frac{1}{7}$ will be $\frac{1}{3}$ of 5, which is $\frac{5}{3}$. If $\frac{5}{3}$ is $\frac{1}{7}$ of a certain number, the whole number will be $\frac{5}{3} \times 7 = \frac{35}{3} = 11\frac{2}{3}$. *Ans.*

8. The hurdles are arranged so as to form a rectangular enclosure having 49 hurdles on each side and one on each end. Two additional hurdles will give two hurdles to each end, and will thus double the size of the enclosure.

9. The mode of dividing the plot may be learned from the following figure:—



10. 33⅓

11. XIII; rub out the lower half, and there remains the expression VIII = 8.

12. 1st Step: Fill the 3-gallon cask and empty it into the 5-gallon cask.

2nd Step: Again fill the 3-gallon cask out of the 8-gallon cask.

3rd Step: Fill up the 5-gallon cask out of the 3-gallon cask. This will leave one gallon in the latter.

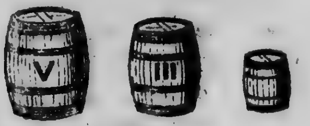
4th Step: Empty the 5-gallon cask into the 8-gallon cask.

5th Step: Pour the one gallon out of the 3-gallon cask into the 5-gallon cask.

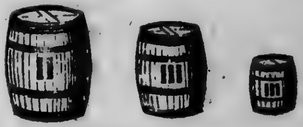
6th Step: Fill the 3-gallon cask out of the 8-gallon cask, and empty it into the 5-gallon cask.

The following diagrams show this more clearly:

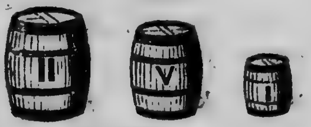
1st Step.



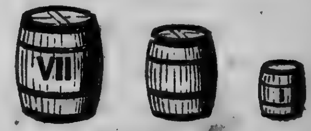
2nd Step.



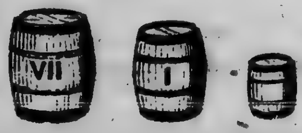
3rd Step.



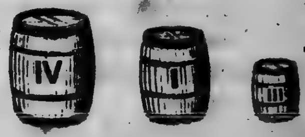
4th Step.



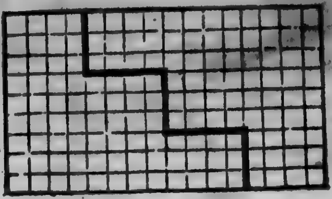
5th Step.



6th Step.



13. The heavy line in the accompanying figure shows how the board is to be cut.



8	1	6
3	5	7
4	9	2

15. Weigh out 7 lbs. as often as possible and there will remain 2 lbs.; add two four pounds and one seven pounds to this, and the sum will be 17 lbs., the share of one.
Weigh 7 lbs. as often as possible out of the remaining 34 lbs. and there will remain 6 lbs., to which add 7 lbs. and 4 lbs., and the sum will be 17 lbs., the share of the second. The remaining 17 lbs. will be the share of the third.

16. The hurdles are, in the first case, placed 12 on a side and one on each end, and then they inclose a space represented by 12 squares whose area is, by the question, 40 square yards. If two hurdles be taken away there will remain 24, and if these be placed in the form of a square, each side containing 6 hurdles, they will enclose a space represented by 36 squares of the same size as the former. Hence they now inclose three times as much space as before, *i. e.* three times 40 square yards, or 120 square yards.

17. He takes the goose to the remote bank and leaves it there, returning, he next carries over the fox, which he leaves, but takes the goose back with him. He now leaves the goose on the first bank, and carries over the oats which he allows to remain on the remote bank with the fox, and returns for the goose.

18. The following diagrams exhibit the solution of this problem:

I.

3	3	3
3	P	3
3	3	3

24

II.

4	1	4
1	P	1
4	1	4

20

III.

2	5	2
5	P	5
2	5	2

28

IV

1	7	1
7	P	7
1	7	1

[illegible]

V.

0	9	0
9	P	9
0	9	0

36

VI.

5	0	4
0	P	0
4	0	5

18

19. XII; rub out the lower half, and VII remains.

20.

17	24	1	8	15
23	5	7	14	16
4	6	13	20	22
10	12	19	21	3
11	18	25	2	9

RULE FOR FILLING MAGIC SQUARES OF ODD NUMBER OF CELLS.

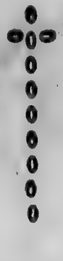
Begin in centre cell of top horizontal row by placing 1 in it; ascend diagonally to the right, and where this carries us beyond the square, transport the next number to the cell at the remote end of the vertical or horizontal band to which it belongs. When in ascending we come to a cell already filled, we place the number in the cell next below the cell last filled. The following is a square of 7 cells in a side filled after this method:

(Continued on next page.)

30	39	48	1	10	19	28
38	47	7	9	18	27	29
46	6	8	17	26	35	37
5	14	16	25	34	36	45
13	15	24	33	42	44	4
21	23	32	41	43	3	12
22	31	40	49	2	11	20

21. Half-a-dozen dozen = 6 × 12 = 72.
Six dozen dozen = 6 × 12 × 12 = 864.
864 − 72 = 792. *Ans.*

22. The following shows the mode of performing this. It will be observed that the two side counters are merely moved one counter higher when the other two are taken away.



23. This problem admits of the following two solutions:

1st Solution.

Persons.	Full bottles.	Hf.-full bottles.	Empty bottles.
1st	2	3	2
2nd	2	3	2
3rd	3	1	3
	7	7	7

Each person has 3½ bottles of wine and 7 bottles.

2nd Solution.

1st	3	1	3
2nd	3	1	3
3rd	1	5	1
	7	7	7

Each person, as before, has 7 bottles and 3½ bottles of wine.

24. There were in all 8 bottles of wine, of which each drank $\frac{8}{3}$, which is $2\frac{2}{3}$. The third person, therefore, drank $\frac{1}{3}$ of a bottle belonging to him who had but 3 bottles, and $\frac{7}{3}$ of a bottle belonging to him who owned the 5 bottles. Hence the latter should have *seven* times as much of the money as the former, or, in other words, the latter gets 7 shillings, and the former 1 shilling.

25. This problem is merely to find some number between 50 and 100 which is exactly divisible by 2 and by 3, but which divided by 5 leaves a remainder 3.

The only numbers between 50 and 100 that are divisible by both 2 and 3, are 54, 60, 66, 72, 78, 84, 90, and 96, and by inspection the only one of these which gives a remainder 3 when divided by 5 is 78; therefore the basket contained 78 eggs.

26. *Ans.* 1 lb., 3 lbs., 9 lbs., and 27 lbs.

For 1 lb. = 1 lb.; 2 lbs. = 3 lbs. — 1 lb., i. e. 3 lbs. in one scale and 1 lb. in the other; 3 lbs. = 3 lbs.; 4 lbs. = 3 lbs. + 1 lb.; 5 lbs. = 9 lbs. — (3 lbs. + 1 lb.); 6 lbs. = 9 lbs. — 3 lbs.; 7 lbs. = 9 lbs. + 1 lb. — 3 lbs.; 8 lbs. = 9 lbs. — 1 lb.; 9 lbs. = 9 lbs.; 10 lbs. = 9 + 1 lb.; 11 lbs. = 9 lbs. + 3 lbs. — 1 lb.; 12 lbs. = 9 lbs. + 3 lbs.; 13 lbs. = 9 lbs. + 3 lbs. + 1 lb.; 14 lbs. = 27 lbs. — (9 lbs. + 3 lbs. + 1 lb.); 15 lbs. = 27 lbs. — (9 lbs. + 3 lbs.); 16 lbs. = 27 lbs. + 1 lb. — (9 lbs. + 3 lbs.); 17 lbs. = 27 lbs. — (9 lbs. + 1 lb.); 18 lbs. = 27 lbs. — 9 lbs.; &c., &c.

27. In order to fill seven out of the eight points, it is merely requisite to remember that the second counter must be carried to the point from which the first *started*, the third to the point from which the second started, &c.

Thus if the first counter is carried from 1 to 4 and there deposited, the second must be taken from 6 to 1 and there deposited; the third from 3 to 6; the fourth from 8 to 3, the fifth from 5 to 8; the sixth from 2 to 5; and the seventh either from 7 to 2 or from 2 to 7.

28. The mouth fills the reservoir in 6 hours, therefore it fills $\frac{1}{6}$ in 1 hour; the right eye fills it in 38 hours, therefore it fills

$\frac{1}{48}$ in 1 hour; the left eye fills it in 72 hours, therefore it fills $\frac{1}{72}$ in 1 hour; the foot fills it in 96 hours, therefore it fills $\frac{1}{96}$ in 1 hour. Hence together they fill $\frac{1}{6} + \frac{1}{48} + \frac{1}{72} + \frac{1}{96} = \frac{61}{288}$ in 1 hour, and to fill the reservoir they require $1 \div \frac{61}{288} = \frac{288}{61} =$ 4 hours 43 min. $16\frac{44}{61}$ sec.

29. The person who thinks of the numbers must proceed as follows: He must multiply the 1st by 2 and add 5 to the product; he must next multiply this sum by 5 and add the second number to the product; he must next multiply this result by 10 and add the third number to the product; lastly, he must subtract 250 and name the remainder.

The three digits of the remainder will be the three numbers thought of, and will be in the order in which they were thought of.

The reason is obvious: let $a =$ 1st, $b =$ 2nd, and $c =$ 3rd number thought of.

$a \times 2 + 5 = 2a + 5.$

$(2a + 5) \times 5 + b = 10a + b + 25.$

$(10a + b + 25) \times 10 + c = 100a + 10b + c + 250.$

$(100a + 10b + c + 250) - 250 = 100a + 10b + c =$ a in hundreds' place, b in tens' place, and c in units' place.

30. Since each man possesses 63 square rods of land more than his son, we must form three pairs of numbers, such that the difference of their squares shall be 63.

The difference of the squares of two numbers is equal to their sum multiplied by their difference, and hence 63 must be divided into two factors in three distinct ways, thus:

$63 = 63 \times 1 = 21 \times 3 = 9 \times 7.$

If sum $= 63$ and difference $= 1$, the numbers are 32 and 31.

If sum $= 21$ and difference $= 3$, the numbers are 12 and 9.

If sum $= 9$ and difference $= 7$, the numbers are 8 and 1.

Hence the squares of Jones, Brown, and Smith, are respectively 32 rods, 12 rods, and 8 rods on the side, and the son's squares are respectively 31, 9, and 1 yards on the side.

Jones' piece was 23 rods longer on each side than Tom's, and since the difference between 32 and 9 is 23, we may conclude that Jones' square was 32 rods to the side, and Tom's 9 rods on a side.

Brown's piece was 11 rods longer on a side than Harry's, and since if the above numbers 12 and 1 have 11 for their difference, we may conclude that Brown's piece was 12 rods on a side, and Harry's piece 1 rod.

Hence Tom was Brown's son, Harry was Smith's son, and Ned was Jones' son

31. The mode of arranging the crew may be remembered by attention to the vowels in the following line:

Populeam virgam mater regina ferebat.

The vowels refer to the crew as follows, $a = 1$, $e = 2$, $i = 3$, $o = 4$, and $u = 5$.

We begin with 4 whites because the first vowel is o, next $u = 5$ blacks, next $e = 2$ whites, next $a = 1$ black, next $i = 3$ whites, next $a = 1$ black, next $a = 1$ white, next $e = 2$ blacks, next $c = 2$ whites, next $i = 3$ blacks, &c., as follows, o standing for a white and + for a black.

oooo+++++oo+ooo+o++oo+++o++oo+

32. You select the multiplier or the multiplicand, such that the sum of its digits shall be exactly divisible by nine. Hence upon the principle of the proof by casting out the nines, the product has the sum of its digits exactly divisible by nine. By subtracting the sum of the digits of the remainder from the next higher multiple of 9 you determine the digit crossed out.

Thus suppose you select 117, and he takes for multiplicand 21613. Then 21613 × 117 = 2528721. Now, suppose he crosses out the 7; upon reading you the remaining digits 252821, you find that their sum = 20, which taken from 27 the next higher multiple of 9 leaves 7 the digit he crossed out.

If he crosses out a 0 or a 9, you cannot determine which, but in all other cases you can tell the exact figure.

33. You write the second, fourth, sixth, &c. lines in such a manner as to make the sum of the first pair, the sum of the second pair, &c. an exact number of 9's. Then having settled the number of pairs, you get the answer by multiplying by that number a row of 9's containing as many digits as there are to be figures in the line.

Thus suppose you agree to write 5 lines each, and that each line is to contain 5 digits, or not more than 5 digits. Then 99999 × 5 = 499995 will be the answer. This is shown as follows :

$$\left.\begin{array}{l}
\left.\begin{array}{lr}\text{Suppose he writes} & 41113\\ \text{You write} & 58886\end{array}\right\} = 99999\\
\left.\begin{array}{lr}\text{Suppose he writes} & 61451\\ \text{You write} & 38548\end{array}\right\} = 99999\\
\left.\begin{array}{lr}\text{Suppose he writes} & 6500\\ \text{You write} & 93499\end{array}\right\} = 99999\\
\left.\begin{array}{lr}\text{Suppose he writes} & 1\\ \text{You write} & 99998\end{array}\right\} = 99999\\
\left.\begin{array}{lr}\text{Suppose he writes} & 99999\\ \text{You write} & 00000\end{array}\right\} = 99999
\end{array}\right\} = 99999 \times 5.$$

$$\text{Sum} = 499995$$

THE END.

ARITH

t each
Then
wn as

× 5.